Global coherence induced by noise or diversity in excitable systems




C.J. Tessone, A. Scirè, R. Toral, P. Colet

FisEs'05 **Madrid, June 27th 2005**

http://www.imedea.uib.es/PhysDept

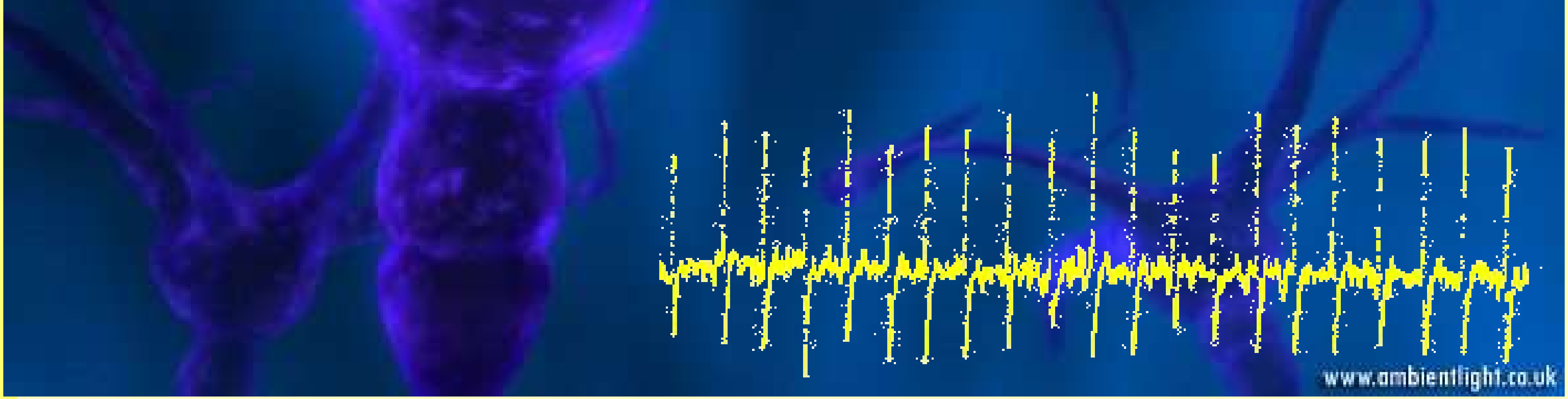
tessonec@imedea.uib.es

Introduction

- <u>Definition of excitable system</u>

An excitable system remains in a stable configuration in absence (or in presence of small) perturbations.

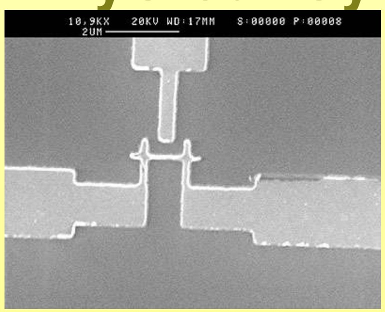
If the perturbations surpass a threshold, the system performs an excursion in phase space (in most cases independent on the strength of the perturbation), returning back to the original state.

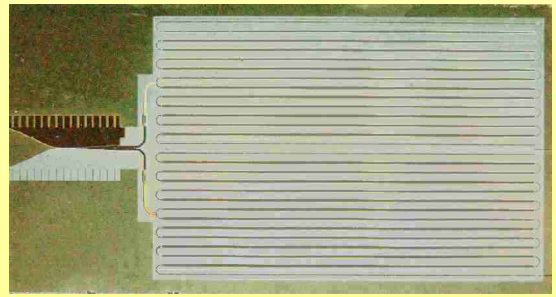


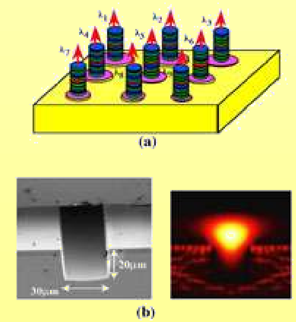
Introduction

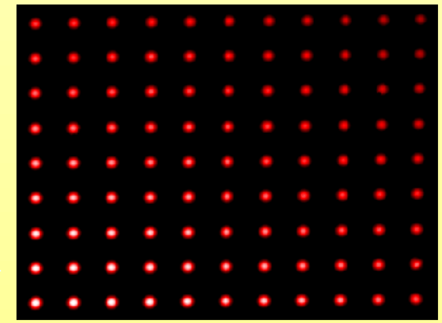
- <u>Examples of excitable systems</u>

Physical systems: Josephson junctions, lasers

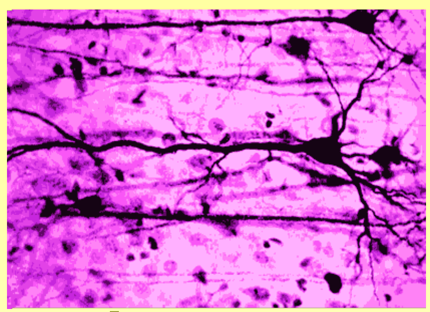


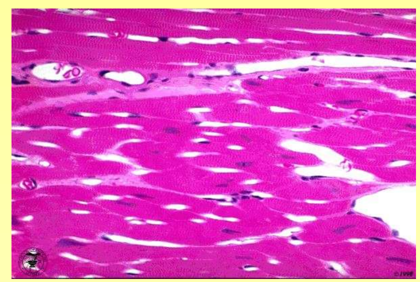


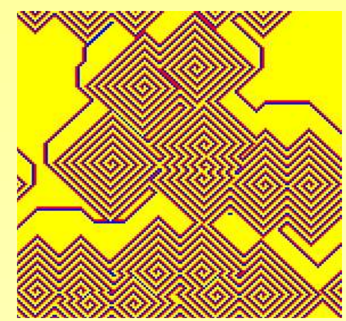




Biological systems: infection propagation, neurons, pancreatic and cardiac cells…





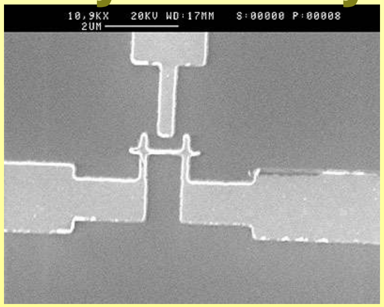


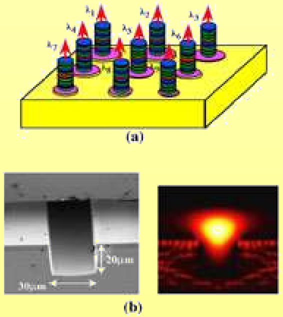
Chemical systems: Belousov-Zhabotinsky reaction

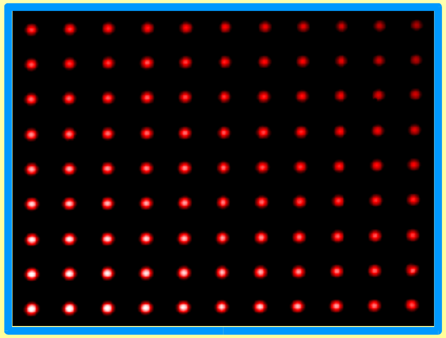
Introduction

• Examples of excitable systems

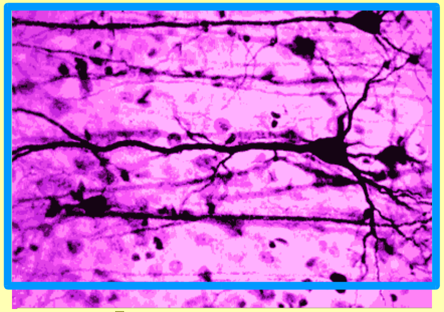
Physical systems: Josephson junctions, lasers

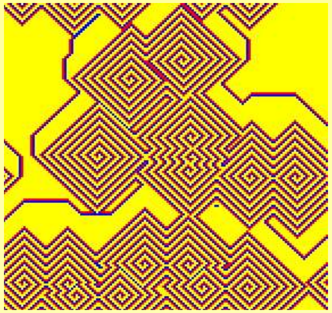






Biological systems: infection propagation, neurons, pancreatic and cardiac cells…





Chemical systems: Belousov-Zhabotinsky reaction

Studied system

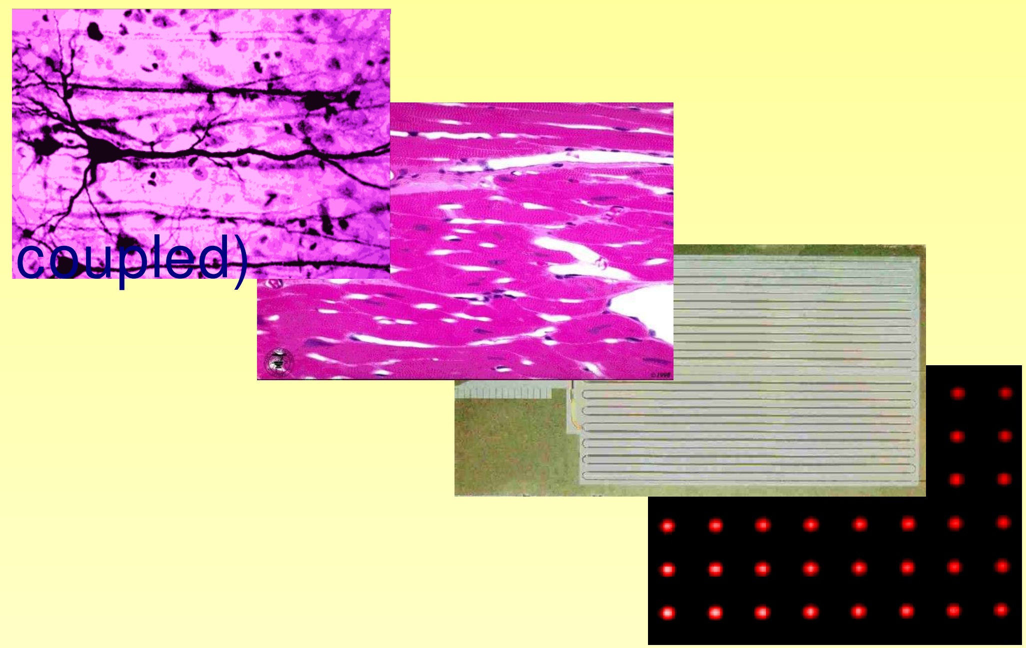
- Globally coupled active rotators

The dynamic of a single unit is given by:

$$\dot{\phi} = \omega - \sin(\phi) + \sqrt{D}\xi$$

This system is useful to model systems like:

- Neuron dynamic
- Cardiac tissues (when coupled)
- Josephson junctions
- Laser dynamics



Studied system

- Globally coupled active rotators

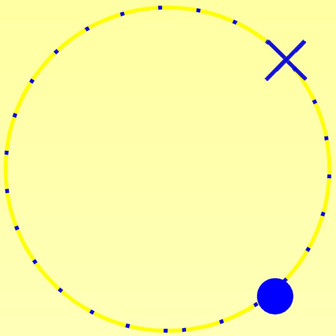
The dynamic of a single unit is given by:

$$\dot{\phi} = \omega - \sin(\phi) + \sqrt{D}\xi$$

D is the noise intensity

ω natural frequency

$\omega < 1$ the system is excitable



$\omega \geq 1$ is oscillatory

I·M·E·D·E·A
Institut Mediterrani d'Estudis Avançats
CSIC

Studied system

- Globally coupled active rotators

$$\dot{\phi}_j = \omega_j - \sin\phi_j + \frac{C}{N}\sum_{k=1}^{N}\sin\left(\phi_k - \phi_j\right) + \sqrt{D}\xi_j$$

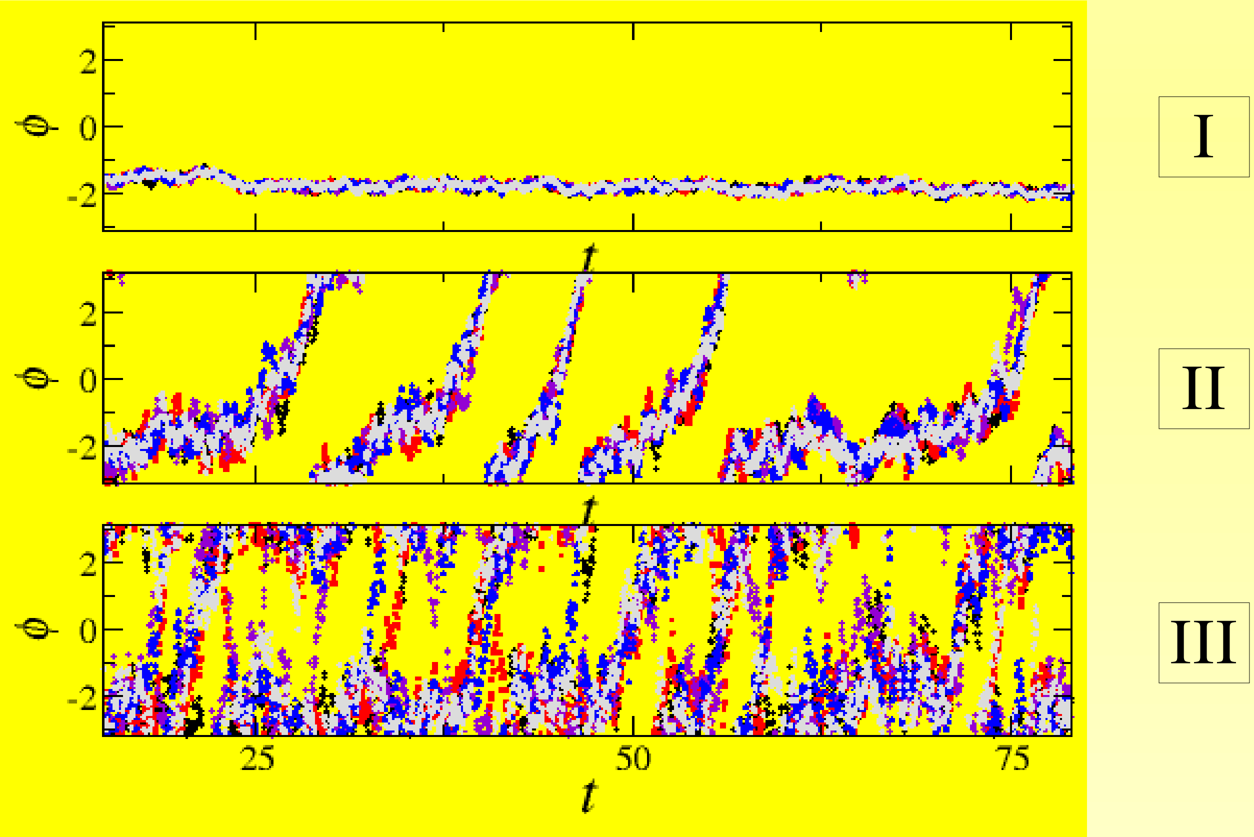
- N: size of the system
- D: noise intensity $\langle \xi_j(t)\xi_k(t)\rangle = \delta(t-t')\delta_{jk}$
- C: coupling strength
- ω_j: natural frequency of *j*-th oscillator distributed according to a function $g(\omega_j)$, with mean value ω and standard deviation σ^2

Studied system

- Globally coupled active rotators

$$\dot{\phi}_j = \omega_j - \sin\phi_j + \frac{C}{N}\sum_{k=1}^{N}\sin(\phi_k - \phi_j) + \sqrt{D}\xi_j$$

increasing D



Studied system

- Order Parameters

i) Kuramoto

Given the expression $\rho(t)e^{i\Psi(t)} = \frac{1}{N}\sum_{k=1}^{N} e^{i\phi_k(t)}$

is computed the time-average $\rho \equiv \langle \rho(t) \rangle$

- $\rho = 1$ when, during the time evolution of the system *all* the units have the same phase: $\phi_j(t) = \phi_k(t), \forall j, k$
- When all the oscillators are uniformily distributed in the circle,

$$\rho \to 0$$

Studied system

- ## Order Parameters

ii) Shinomoto-Kuramoto

$$\zeta = \left\langle \left| \rho(t) e^{i\Psi(t)} - \left\langle \rho(t)\, e^{i\Psi(t)} \right\rangle \right| \right\rangle$$

- In the case in which all the oscillators are at rest, it is equal to 0.
- When the oscillators pulse unsynchronously, it is equal to 0.
- In case of total synchronization with uniform velocity (different from zero), it is equal to 1.
- In the case of coherent pulsations, it takes non-zero values.

Studied system

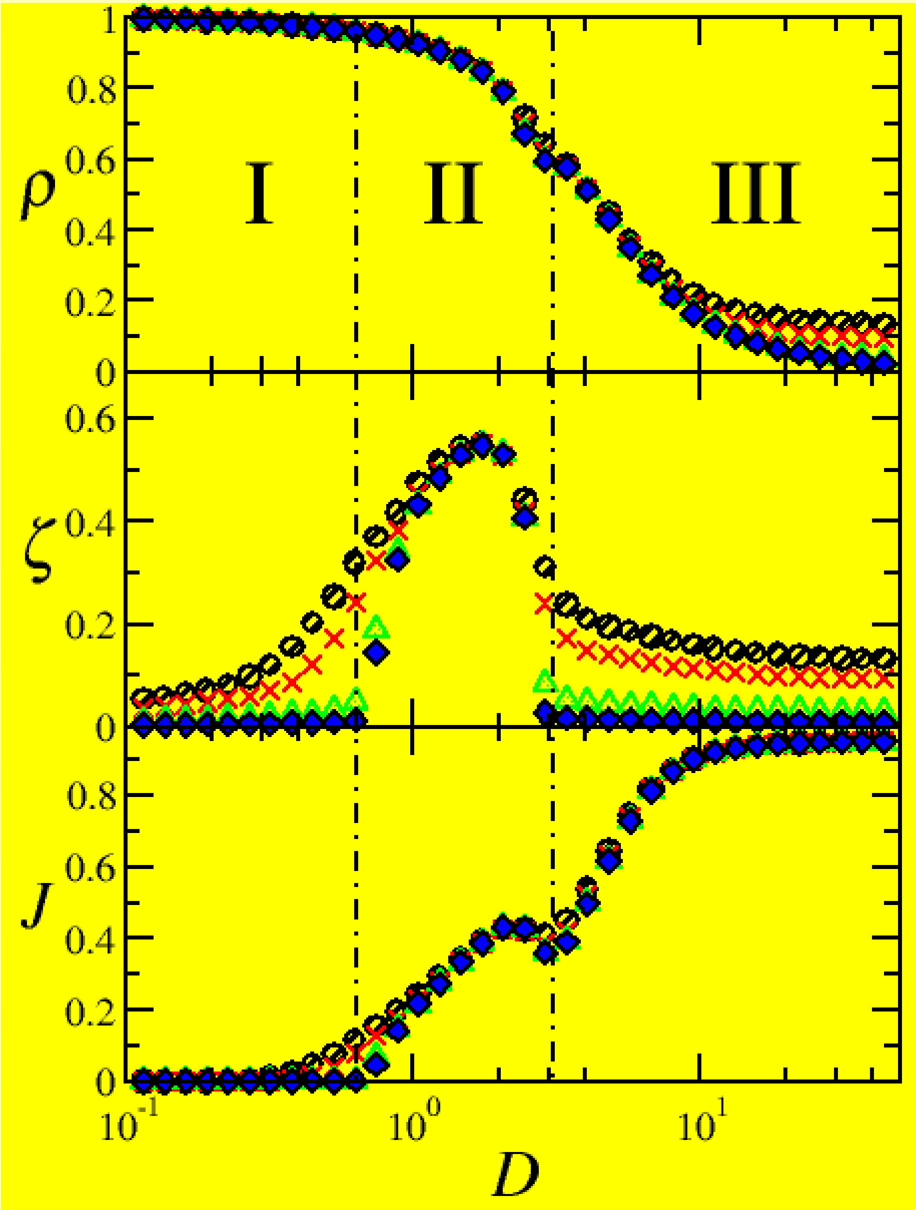
- Order Parameters

iii) Current

$$J = \frac{1}{N}\sum_{k=1}^{N}\left\langle \dot{\phi}_k(t) \right\rangle$$

- Being equal to 0 in this system, means that the units are at rest

Numerical results



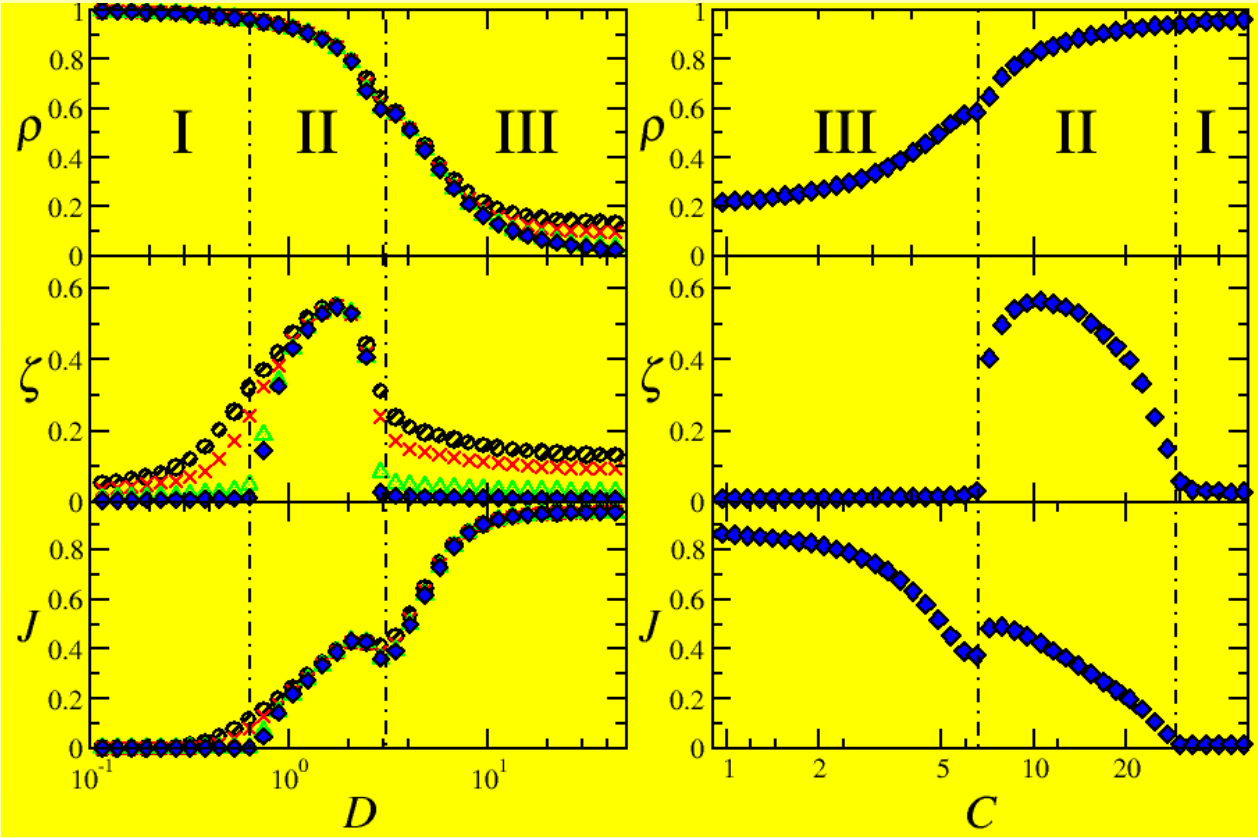
- Regime **I**: the oscillators fluctuate around the fixed point
- Regime **III**: the oscillators pulse in an incoherent fashion.
- Regime **II**: the oscillators pulse coherently.

In egimes **I** and **III** have a steady probability distribution $P(\phi)$.

But the current is different in both cases

Numerical results

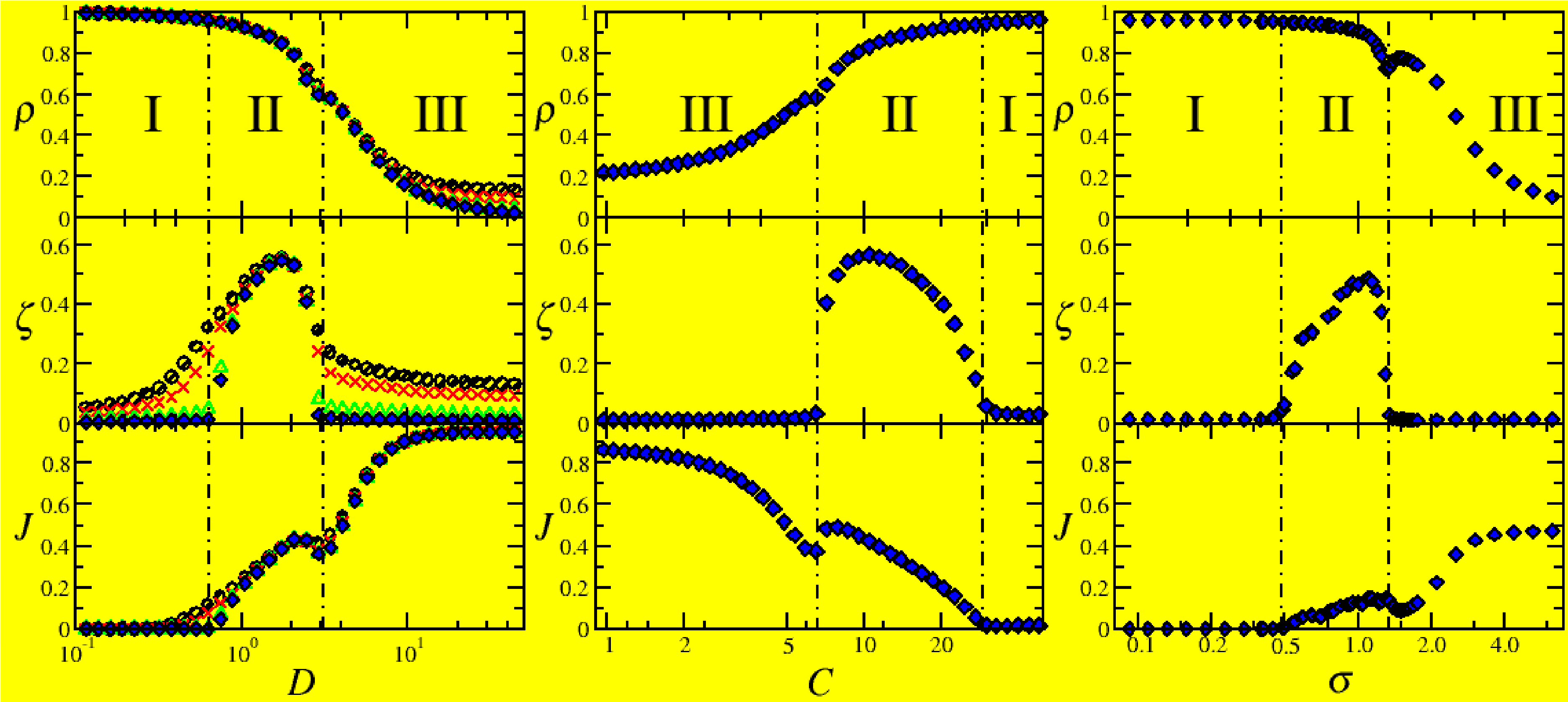
- Dependence with respect to parameters



...the same behaviors are present...

Numerical results

- Dependence with respect to parameters



...the same behaviors are present...

Theoretic results

Macroscopic equation

$$\dot{\phi}_j = \omega_j - \sin\phi_j + \frac{C}{N}\sum_{k=1}^{N}\sin(\phi_k - \phi_j) + \sqrt{D}\xi_j$$

Averaging the dynamic equation over the whole population

$$\frac{1}{N}\sum_{k=1}^{N}\dot{\phi}_j = \omega - \rho(t)\sin\Psi(t) + \sqrt{\frac{D}{N}}\xi(t) \qquad \langle\xi(t)\xi(t')\rangle = \delta(t-t')$$

... and computing the time derivative of the Kuramoto order parameter, we obtain...

$$\dot{\rho}(t) + i\,\rho(t)\dot{\Psi}(t) = \frac{i}{N}\sum_{k=1}^{N}\dot{\phi}_k e^{i\delta_k(t)}$$ where, $\delta_j(t) = \phi_j(t) - \Psi(t)$.

...what we get is an expression for the dynamics of the global quantities...

$$\dot{\Psi}(t) = \frac{\omega}{\rho} - \sin\Psi(t). \qquad \dot{\rho}(t) = \mathcal{O}(\delta_k^2)$$

I·M·E·D·E·A Institut Mediterrani d'Estudis Avançats CSIC

Theoretic results

- Macroscopic equation

$$\dot{\Psi}(t) = \frac{\omega}{\rho} - \sin \Psi(t).$$

- The dynamics of the global phase are also excitable***, but***

The natural frequency is also rescaled by ρ

- The excitability threshold is given by the relation $\rho = \omega$
- ***Disorder in position*** of the oscillators is responsible of the appearance of global pulsations

I·M·E·D·E·A
Institut Mediterrani d'Estudis Avançats
CSIC

Theoretic results

- Computation of ρ

$$\rho = \int d\omega g(\omega) \int_0^{2\pi} d\Psi P(\Psi;\rho) \int_0^{2\pi} d\phi P_{\rm st}(\phi;\Psi,\rho,\omega)\, \cos(\phi-\Psi)$$

- over the probability distribution $g(\omega)$
- over the distribution of global phases

$$P(\Psi;\rho) = \begin{cases} (1/2\pi)\sqrt{\omega^2-\rho^2}/(\omega-\rho\sin\Psi) & \rho<\omega \\ \delta(\Psi-\arcsin(\omega/\rho)) & \rho\geq\omega \end{cases}$$

- over the distribution of the units

$$P_{\rm st}(\phi;\Psi,\rho,\omega) = Z^{-1}e^{-2V(\phi)/D}\int_0^{2\pi} d\phi'\, e^{2V(\phi'+\phi)/D}$$

where... $V(\phi;\Psi,\rho,\omega) = -\omega\phi - \cos(\phi) - C\rho\cos(\Psi-\phi)$

I·M·E·D·E·A Institut Mediterrani d'Estudis Avançats CSIC

Theoretic results

- Computation of order parameters

In this approximation the Shinomoto-Kuramoto order parameter can be directly computed...

$$\zeta = \frac{2}{\pi}\sqrt{2(\omega - \sqrt{\omega^2 - \rho^2})(\omega + \rho)}K\left(\frac{2\rho}{\rho - \omega}\right)$$

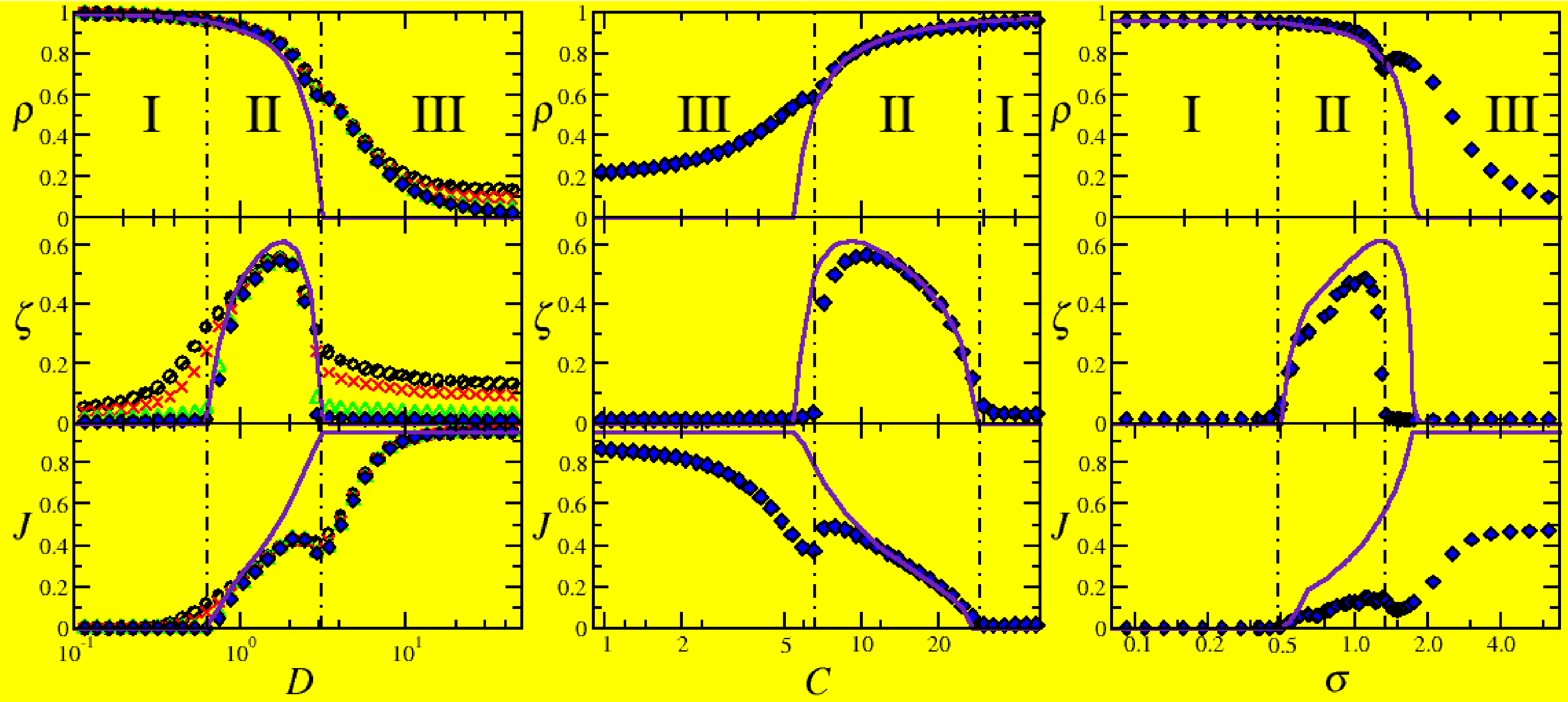
And also the current...

$$J = \frac{\omega^2 - \rho^2}{T}\int_0^T \frac{dt}{\omega - \rho\cos\Omega t} = \sqrt{\omega^2 - \rho^2}.$$

where $\Omega = \sqrt{(\omega/\rho)^2 - 1}$

Theoretic results

- Comparison with numerical results



I·M·E·D·E·A
Institut Mediterrani d'Estudis Avançats
CSIC

Conclusions

- We have studied the effect that **noise**, **coupling** and **diversity** have in the emergence of collective oscillations in a set of globally coupled active rotators
- In this system appear three different dynamical regimes: fluctuations around the fixed point, coherent pulsations and incoherent pulsations
- We developed a simple, yet insightful, theory that shows these regimes and allows to compute theoretically the relevant order parameters
- It has been demonstrated that increasing the disorder in the position of the units has the counterintuitive effect of generating coherent pulsations
- This disorder in positions can be produced by increasing noise, or diversity in natural frequencies, or decreasing the coupling strength

http://www.imedea.uib.es/~tessonec