Dynamical Properties of Two Semiconductor Lasers with Bidirectional Optoelectronic Coupling

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Outline

I Introduction

II Model

III Stability and Route to Chaos

IV "Death by Delay"

V Conclusions
and future trends

Introduction

- Unidirectional delayed coupled lasers → *Anticipating synchronization, chaotic communications.*

- Mutual delayed coupled lasers → *spontaneous symmetry breaking, leader-laggard behaviour.*

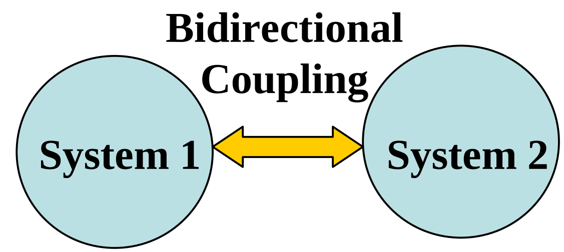
- The study of two bidirectionally coupled lasers is the first step to understand more complicated structures (ring configurations).

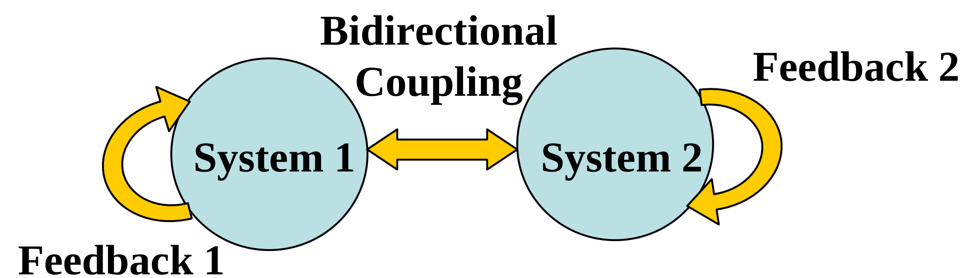
- To know the role of different delay times in bidirectionally coupled lasers. Route to chaos.

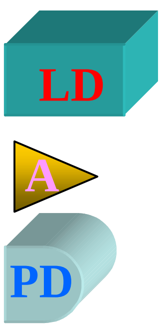
- Observation of some general phenomena in delayed coupled oscillators ("Death by delay").

Model

❑ Topology of the connections of the system.



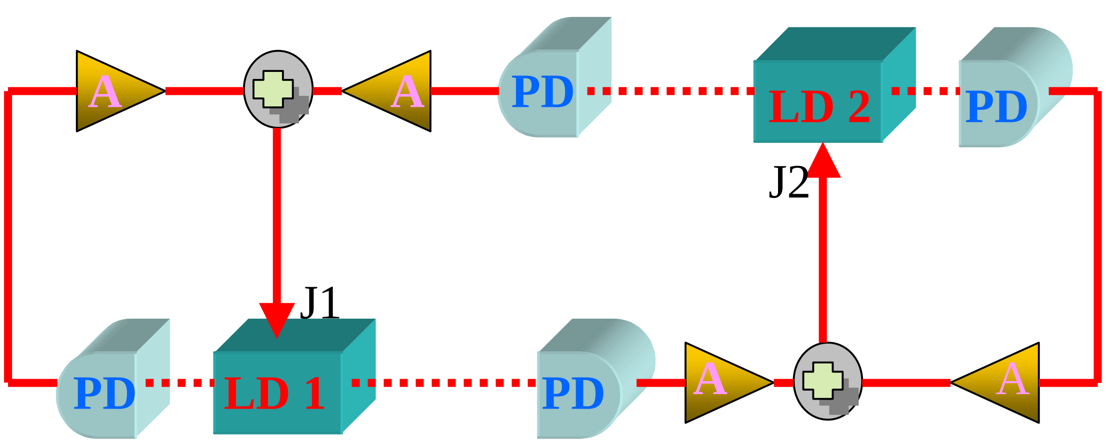




Laser Diode

Amplifier

Photodiode



❑ Experimental setup for the optoelectronic case.

Model

Optoelectronic

$$\frac{dS_1}{dt} = -\gamma_c S_1 + \Gamma g_1 S_1$$

$$\frac{dN_1}{dt} = \underbrace{\frac{J_1}{ed} + \xi_{f1} S_1(t-\tau_{f1}) + \xi_{c1} S_2(t-\tau_{c1})}_{J_{eff}} - \gamma_s N_1 - g_1 S_1$$

$$\frac{dS_2}{dt} = -\gamma_c S_2 + \Gamma g_2 S_2$$

$$\frac{dN_2}{dt} = \frac{J_2}{ed} + \xi_{f2} S_2(t-\tau_{f2}) + \xi_{c2} S_1(t-\tau_{c2}) - \gamma_s N_2 - g_2 S_2$$

Incoherent optical

$$\frac{dS_1}{dt} = -\gamma_c S_1 + \Gamma g_1 S_1$$

$$\frac{dN_1}{dt} = \frac{J_1}{ed} - \gamma_s N_1 - g_1 \left(S_1 + \xi_{f1} S_1(t-\tau_{f1}) + \xi_{c1} S_2(t-\tau_{c1})\right)$$

$$\frac{dS_2}{dt} = -\gamma_c S_2 + \Gamma g_2 S_2$$

$$\frac{dN_2}{dt} = \frac{J_2}{ed} - \gamma_s N_2 - g_2 \left(S_2 + \xi_{f2} S_2(t-\tau_{f2}) + \xi_{c2} S_1(t-\tau_{c2})\right)$$

- These are the coupled delayed rate equations for the photon and carrier densities.

- Expressions for the optical gain and the different strengths.

$$g \approx g_0 + g_n(N-N_0) + g_p(S-S_0)$$

$$\xi = a\eta_{ext} \frac{\eta c \alpha_m}{2n_g}$$

- Both positive and negative values of ξ are studied.

Stability and Route to Chaos

- The system has four fixed points:

  - F.P. I ⇨ Both lasers are lasing.
  - F.P. II ⇨ Off-state.
  - F.P. III and IV ⇨ One laser is lasing while the other is turned-off. (Stability → $J_{eff} < J_{th}$)

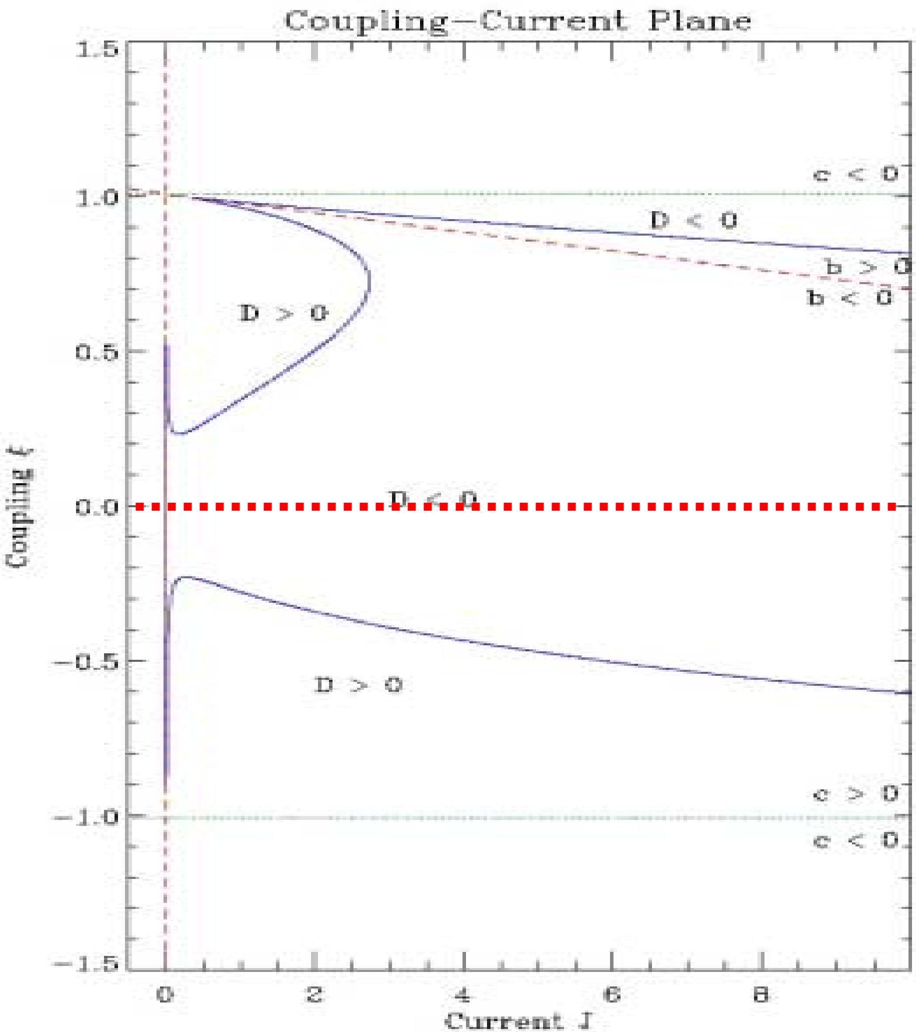
- The stability of these fixed points is studied by evaluating the eigenvalues of the characteristic equation $\det \Delta(\lambda) = 0$ with

$$\Delta(\lambda) \equiv -\lambda I + \left.\frac{\partial f}{\partial x}\right|_{x_{st}} + \left.\frac{\partial f}{\partial x_{\tau_f}}\right|_{x_{st}} \exp(-\lambda \tau_f) + \left.\frac{\partial f}{\partial x_{\tau_c}}\right|_{x_{st}} \exp(-\lambda \tau_c)$$

as function of the <u>bias current</u> <u>**J**</u>, <u>coupling strength</u> <u>ξ</u> and the <u>delay time</u> <u>**τ**</u>.

Stability and Route to Chaos (No Feedback)

- After impossing $\lambda = i\omega$, it is analytically found the stability diagram in the Coupling-Current plane



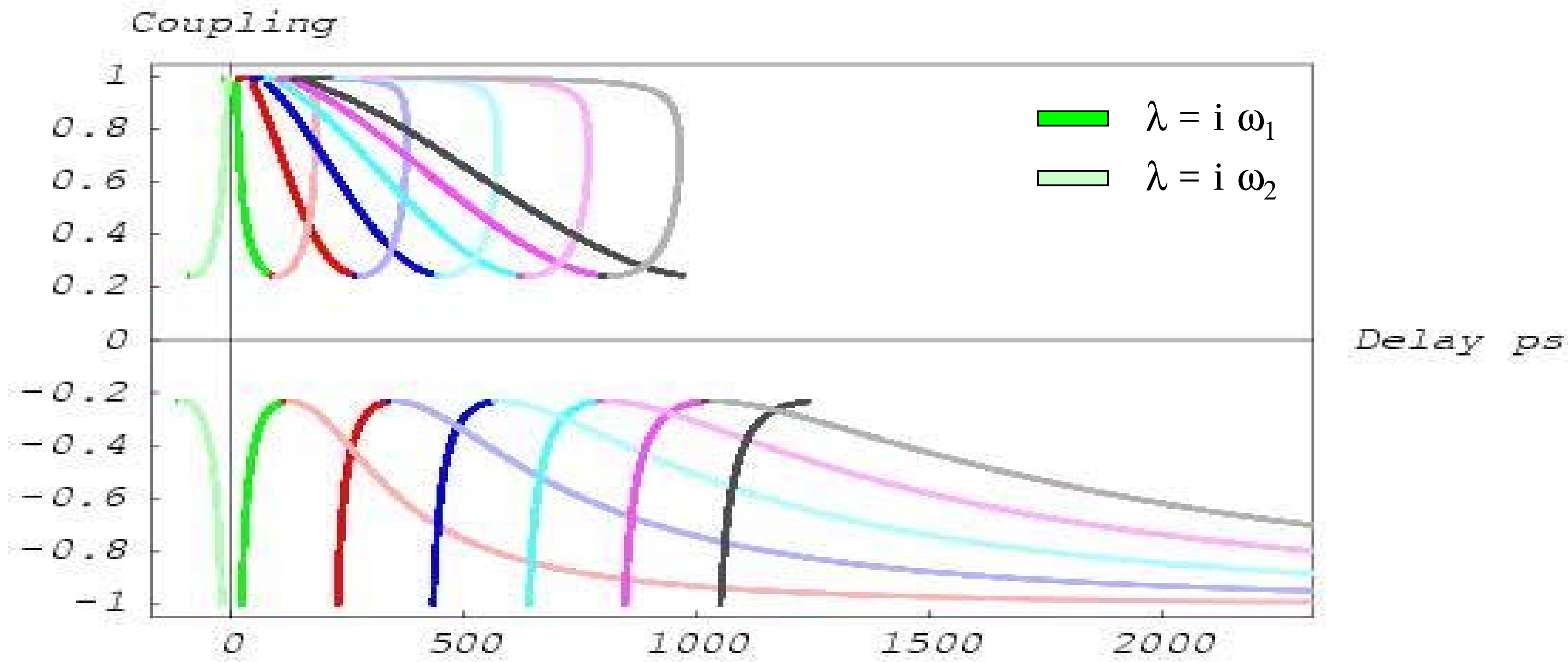
- There is a maximum current beyond which instabilities are forbidden.
- It is easier to destabilize the system with inhibitory couplings than with excitatory ones.
- Saturation largely reduces the size of the unstable regions for $\xi > 0$.

2 solutions $\omega_{1,2}$ whose value depend on $b(J,\xi)$, $c(J,\xi)$ and $D \equiv b^2-4c$

D > 0	c > 0	c = 0	c < 0
b > 0	$\omega_1, \omega_2 \notin \mathbb{R}$	$\omega_1=0, \omega_2 \notin \mathbb{R}$	$\omega_1 \in \mathbb{R}, \omega_2 \notin \mathbb{R}$
b < 0	$\omega_1, \omega_2 \in \mathbb{R}$	$\omega_1=0, \omega_2 \in \mathbb{R}$	$\omega_1 \in \mathbb{R}, \omega_2 \notin \mathbb{R}$

Stability and Route to Chaos (No Feedback)

❑ Fixing the current, we are also able to find the Hopf curves in the Coupling-Delay plane



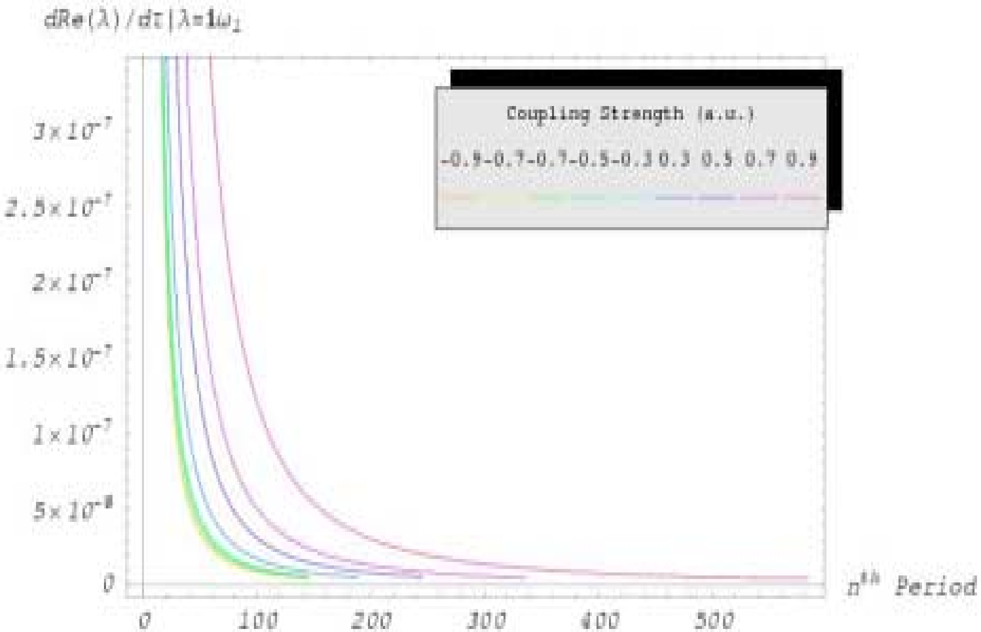
• The Hopf curves are periodic in the delay time:

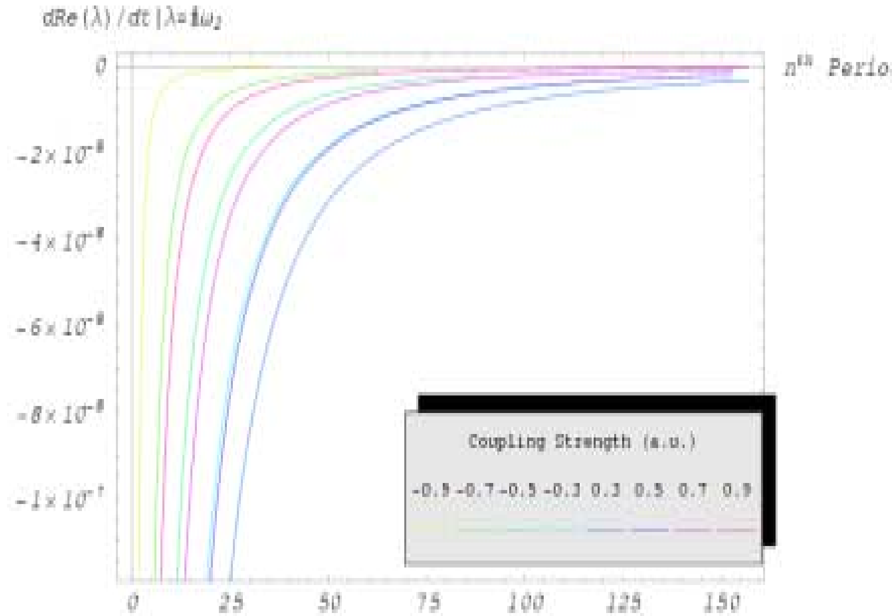
$$\tau \to \tau + \frac{2\pi}{\omega_{1,2}}$$

• It is also found that for $\xi > 0$, $\omega_1 \approx$ ROF while for $\xi < 0$, $\omega_2 \approx$ ROF.

Stability and Route to Chaos (No Feedback)

- Direction and "velocity" of the transitions





- From implicit derivation of the characteristic equation it is concluded that:

$$\left.\frac{d\,\mathrm{Re}(\lambda)}{d\tau}\right|_{\lambda=i\omega_1} > 0, \qquad \left.\frac{d\,\mathrm{Re}(\lambda)}{d\tau}\right|_{\lambda=i\omega_2} < 0$$

- What implies that ω_1 is a destabilizing eigenfrequency while ω_2 is a stabilizing one.

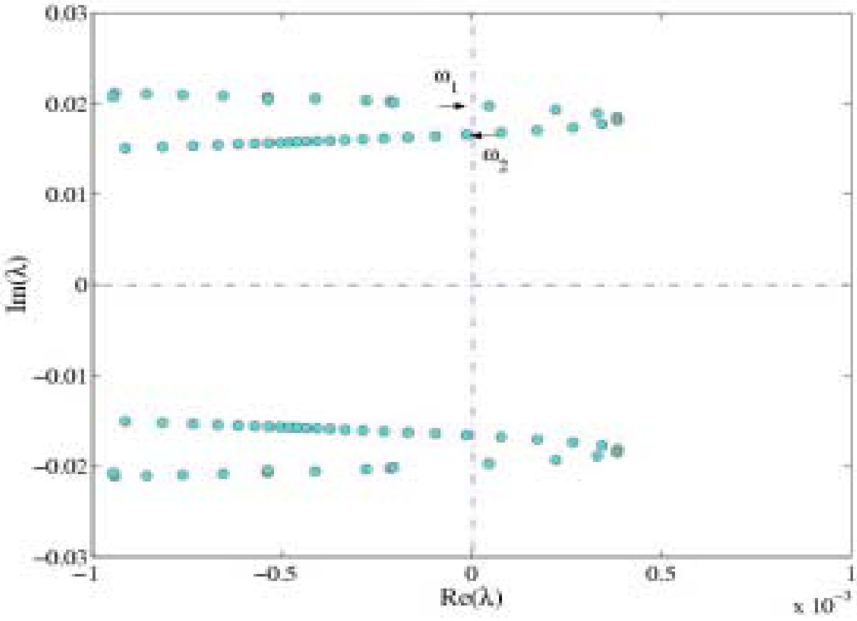
- Since $2\pi/\omega_1 < 2\pi/\omega_2 \rightarrow$ # of unstable eigenvalues grows with τ. Although their magnitude decreases with τ.

- More unstable directions in the phase space but the stretching is smaller in those directions.

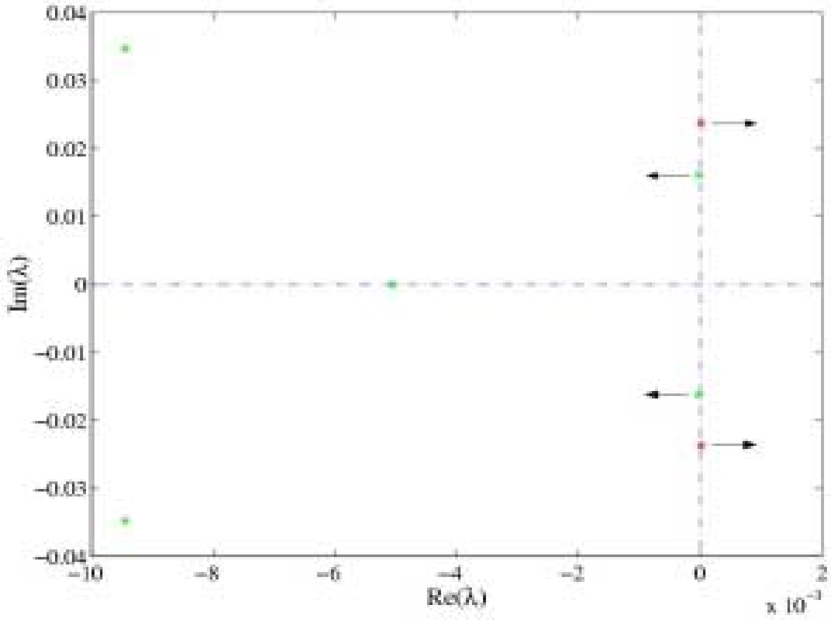
- $D_F \propto \tau$, $H_{KS} \neq H_{KS}(\tau)$

Stability and Route to Chaos (No Feedback)

❑ At the crossing points of two Hopf curves, a codimension two bifurcation appears:



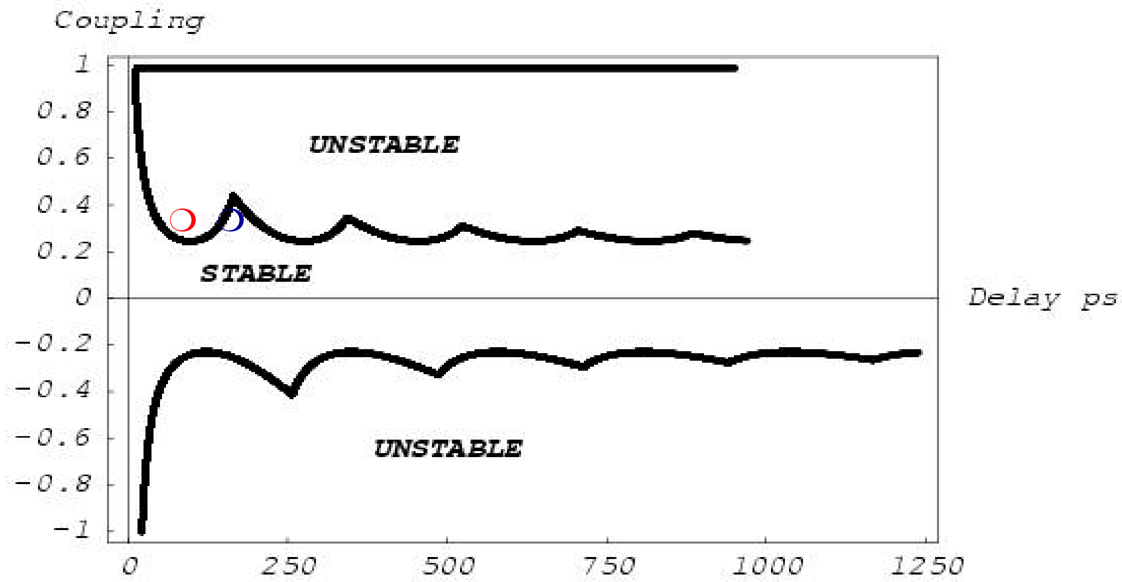
• The eigenvalues follow a closed path in the complex plane.

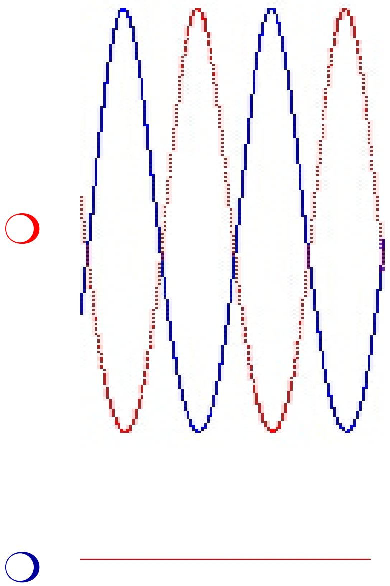


• A pair of complex conjugate eigenvalues is crossing the imaginary axis towards the right while at the same time another is crossing towards the left.

Stability and Route to Chaos (No Feedback)

❑ Finally, the stability diagram in the Coupling-Delay plane is constructed from the exterior borders of the Hopf curves:



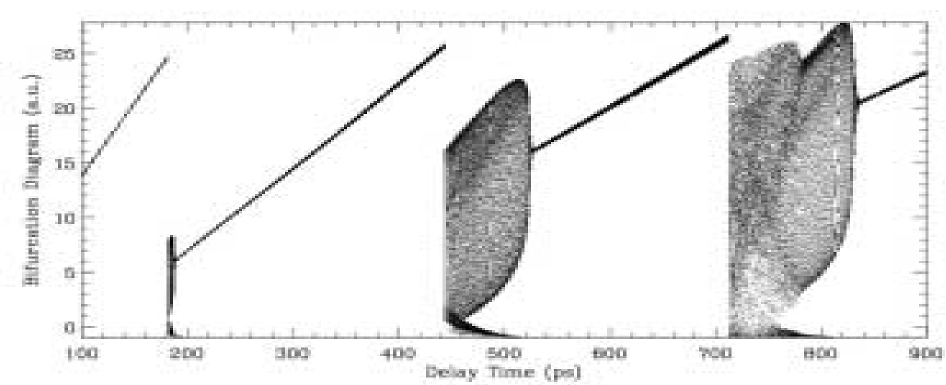


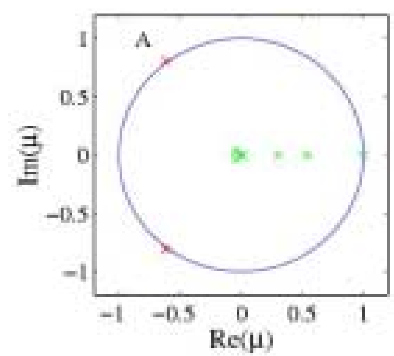
- For short delays, the system is very sensitive to small changes in the distance between lasers.

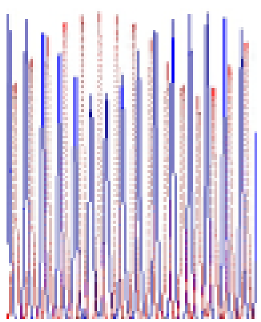
- Assymptoticaly, the delay seems to play a minor role in the details of the stability diagram.

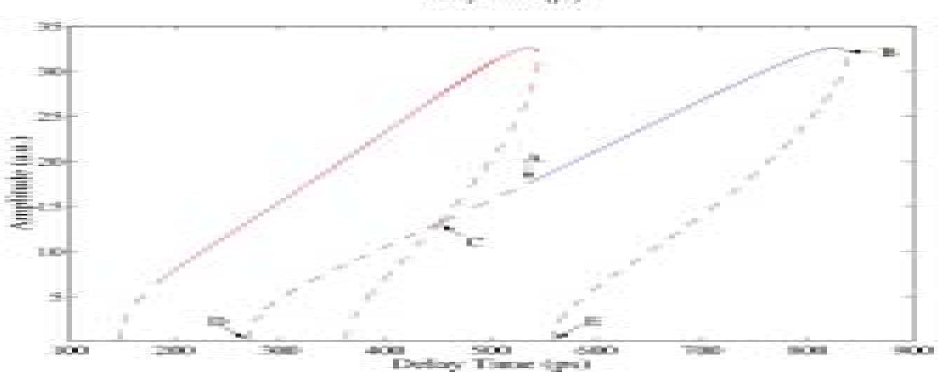
Stability and Route to Chaos (No Feedback)

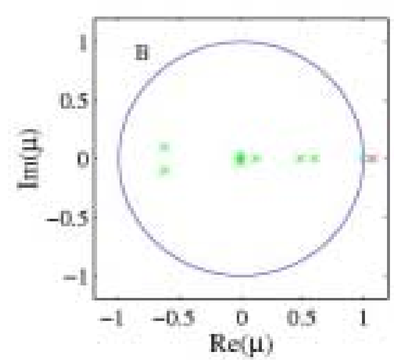
- To completely understand the transition to chaos in this system it is essential the study of the stability of the limit cycles and their interaction. Curry-Yorke quasiperiodic scenario and boundary crisis seems to be present in the road to chaos followed by the system.

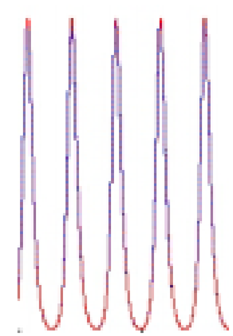






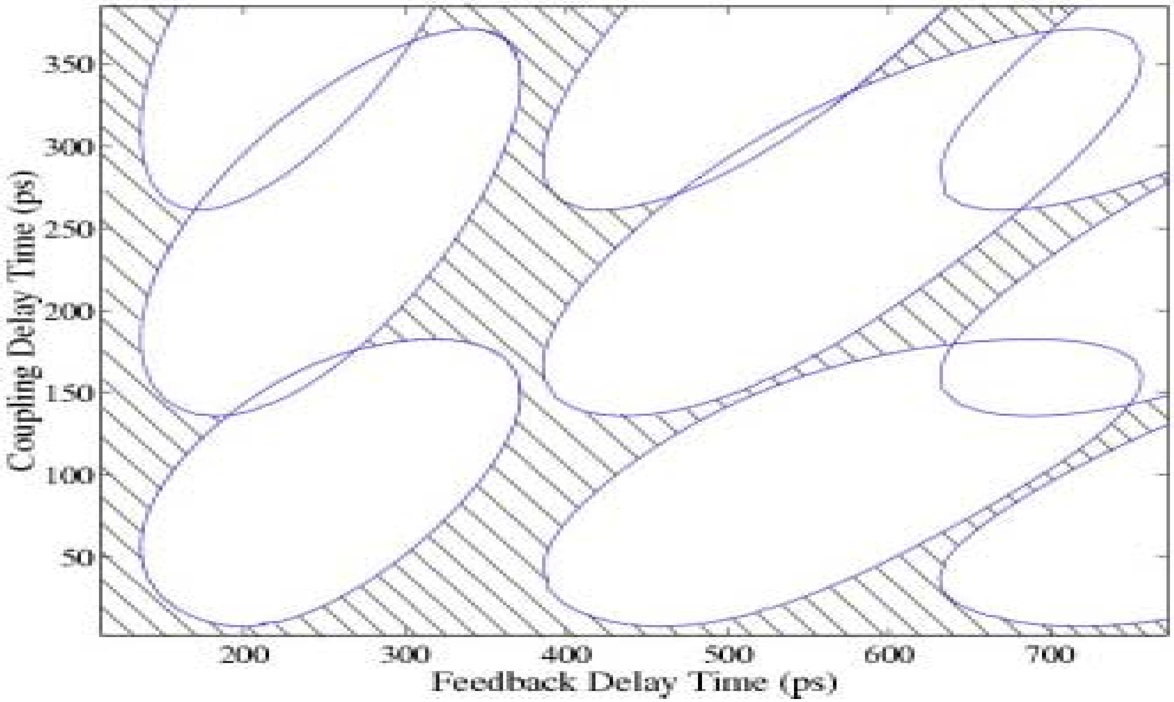


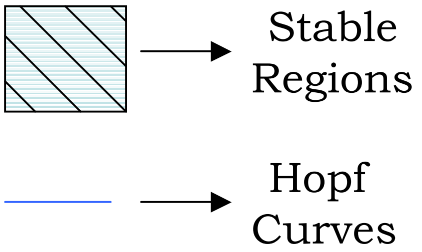




Stability and Route to Chaos (Feedback)

❑ Now, we consider the case in which a feedback loop is present in each laser. Fully analytical results are hopeless in this case. Numerical results suggest that the same route to chaos is followed when increasing either feedback or the coupling time. The Hopf curves in the Coupling-Feedback delay time are formed by closed curves.





Feedback and coupled delay times seem to play similar roles in the dynamics.

“Death by delay”

- With its own feedback both lasers can be operated as self-sustained oscillators when they are decoupled and we can try to observe the recently discovered phenomena named “Death by delay”. (Ramana et al., “Phys. Rev. Lett.” 80, 5109, 1998)

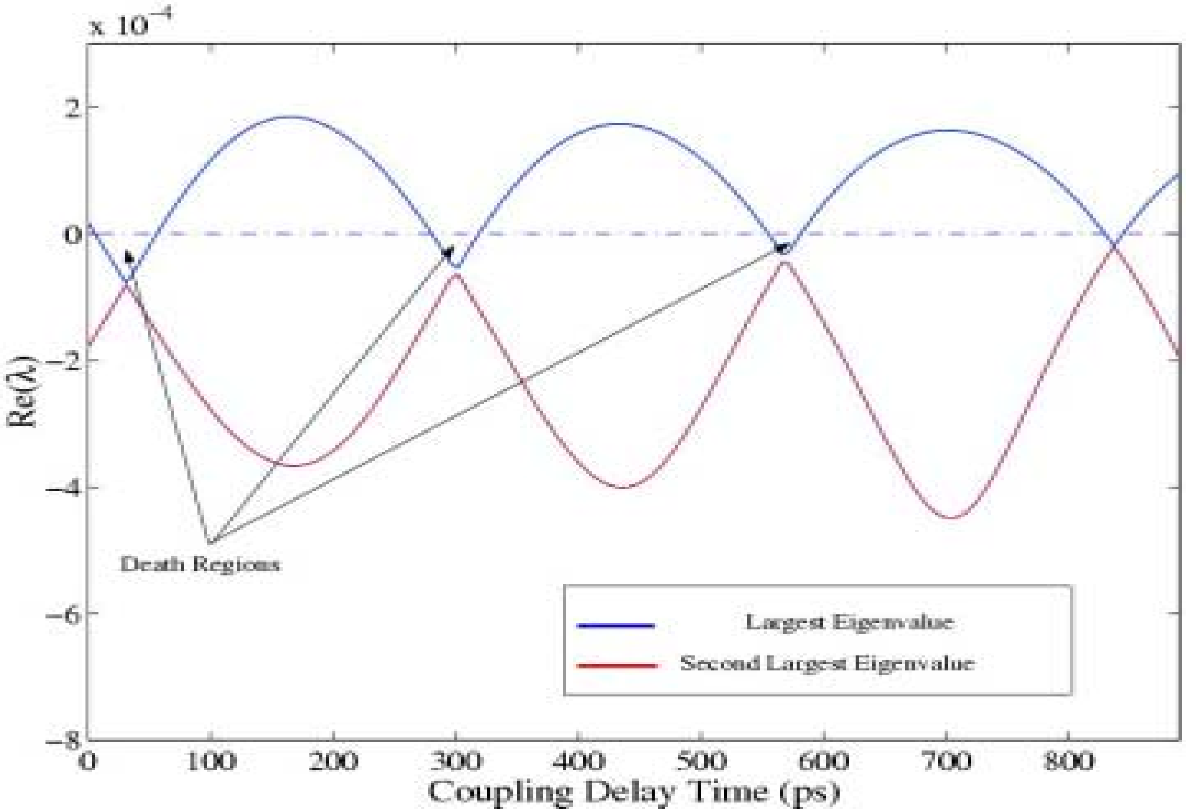
- It consists of the quenching of the oscillations of two coupled oscillators when the delay in the coupling is varied.

- The strong restrictions needed to observe the phenomenon when the delay is not taken into account disappear:

Condition	No delay	Delay
Large detuning	✔	✘
Large coupling	✔	✘
Diffusive coupling	✔	✘

"Death by delay"

- This effect can be predicted in our system by looking at the stabilization of the largest eigenvalue as function of the delay time.



Islands of amplitude death appear while increasing the delay time.

Their size is smaller as the coupling time becomes larger.

Conclusions and Future work

- Curry-Yorke quasiperiodic route to chaos has been identified for this system.

- Crisis events are also presents in the road to chaos.

- "Death by delay" has been predicted for this system when both lasers are operating as self-sustained oscillators.

- Similarity of the roles of feedback and coupling delays.

- Extension of these results to other systems like the incoherent optical coupling.