IMEDEA

Bidirectionally Coupled Semiconductor Lasers

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Coupled oscillators

Fundamental Phenomena

• **Synchronization** of coupled oscillators: neurons, crickets, clapping, clocks, lasers, ...
K. Coffman, et al. PRL 56, 999 (1986) ; S. H. Strogatz et al. Sci. Am. 269, 68 (1993).

• **Kuramoto** model for phase-oscillators

The sound of many hands clapping: "the rhythmic applause."

Surrounded by clocks, Georgia Tech physicist Kurt Wiesenfeld ponders the phenomenon of mutual synchronization. Wiesenfeld's groundbreaking study of synchronized oscillations has sparked interest in its potential in the field of superconductivity.

• Intrinsic **delay** in the coupling between distant subsystems

Role of Delay? Self-sustained oscillations but also inhibitor
"Death by delay", S.H. Strogatz, Nature 394 (1998).

Mutually coupled semiconductor lasers

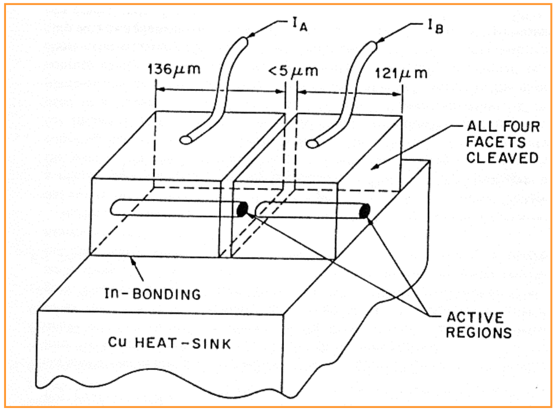
Old concepts (80s):

Cleaved compound-cavity lasers (C^3-lasers) or multi-section lasers

D. Marcuse. IEEE JQE **21**, 154 (1985).

G.P. Agrawal and N.K. Dutta, “Semiconductor lasers”

Two electric contacts device

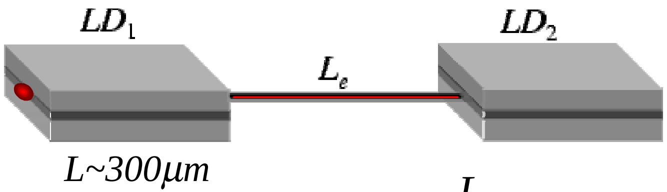


Short length ($L \sim L_e$) and Strong coupling

$\Rightarrow$ single laser with two sections

New features:

Spatially separated lasers *($L_e \gg L$)*



Large separation and Weak coupling

$\Rightarrow$ envisioned as “Transmitter - Receiver System”

Goal:

Modeling - Rate equation model and limits of validity?
Instabilities - Role of the delay, synchronization properties?

Outline

Fourier domain

1. Modeling

- Electromagnetic problem
- Carrier equations
- SVA approximation

2. Monochromatic Steady-State Solutions

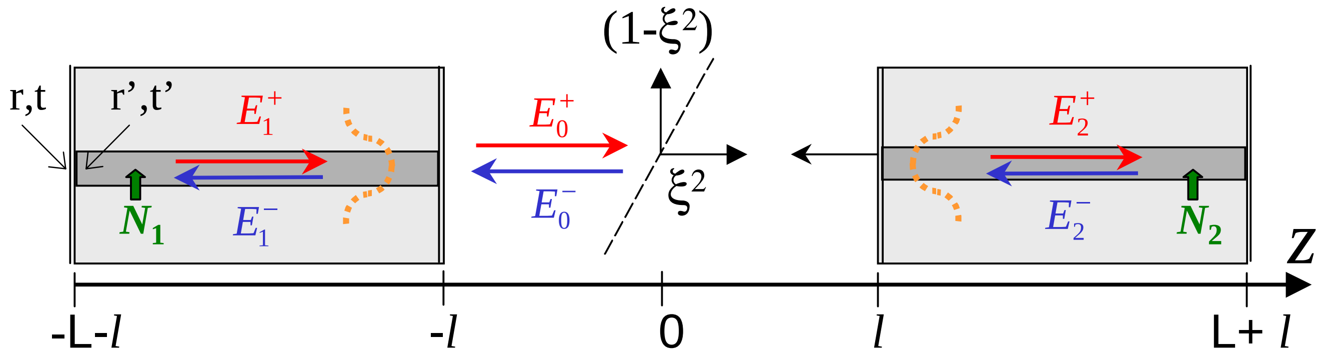
SVA F^{-1}

Time domain

3. Dynamical Properties

- Coupling induced-instabilities
- Synchronization properties
- Symmetric and asymmetric operation

Electromagnetic problem



Coupling time: $\tau = 2l / c$

Cleaved facets
r=-r', tt'=1-r²
external and internal'

Boundary Conditions
$z=-L-l,\ -l,\ l,\ L+l$

Maxwell's Wave Equation

Counter-propagating waves @ $k_{1,2,0}$ and freq. ω

Relationships
Outgoing **SVE** field amplitudes in laser 1,2
E_1^- and E_2^+

Dispersion relations in media *1,2*
$k_{1,2}(\omega, N_{1,2})$

Expansion around solitary emission

J. Mulet, C. Masoller and C.R. Mirasso, Phys. Rev. A **65**, 063815 (2002).

Electromagnetic problem

• **Field equations (in frequency domain):**

Hypothesis

• Solitary lasers single-longitudinal around $\Omega_{1,2}$
• SVA around the central frequency $\Omega=(\Omega_1+\Omega_2) / 2$

$$\left[1 - r^2 \hat{\xi}^2 e^{i2u\tau} - \left(1 - \hat{\xi}^2 e^{i2u\tau}\right) e^{i\Delta\theta_{1,2}}\right] \tilde{A}_{1,2} = \frac{(1-r^2)}{r} \hat{\xi} e^{iu\tau} e^{i\Delta\theta_{2,1}} \tilde{A}_{2,1}$$

• Variation in propagation constants: *(with respect to solitary operation)*

cavity decay rate

$$\Delta\theta_{1,2} \approx i\,\tau_{in} \left[\pm i\Delta - iu - \frac{1}{2}(1 - i\alpha)\gamma\,(\mathcal{G}_{1,2} - 1)\right]$$

Internal round-trip time

$u \equiv \omega - \Omega$

slow frequency

linewidth enhancement factor

$\Delta \equiv (\Omega_2 - \Omega_1)/2$

relative detuning

J. Mulet, C. Masoller and C.R. Mirasso, Phys. Rev. A **65**, 063815 (2002).

Interaction with active media

- $N_j(\vec{r})$ Density of excited electron-hole pairs

current density

carrier diffusion

$$\frac{\partial N_j(\vec{r},t)}{\partial t} = \frac{J_j(\vec{r})}{ed} - \gamma_e N_j + \mathcal{D}\frac{\partial^2 N_j}{\partial z^2} - \frac{i}{\hbar}\overbrace{\left[\mathcal{P}_j^{nl}(z,t)\mathcal{E}_j^*(z,t) - \mathcal{P}_j^{nl*}(z,t)\mathcal{E}_j(z,t)\right]}^{\text{stimulated recombination}}$$

spontaneous recombination

active material polarization

for $-(L+l)<z<L+l$, and appropiate BC

- Mean field approximation
  - *Neglecting diffusion*
  - *Homogeneously distributed gain*
  - *D_j = Total carrier number in laser j / transparency - 1*

Variation in standing wave / solitary

scaled pump

$$\dot{D}_j(t) = \gamma_e\left[\mu_j - D_j - \mathcal{G}_j\frac{\Gamma_j}{\Gamma^{sol}}|A_j(t)|^2\right]$$

gain

$$\mathcal{G}_j \equiv \frac{aD_j}{1+\varepsilon|A_j|^2}$$

gain suppression

$j=1,2$

J. Mulet, C. Masoller and C.R. Mirasso, Phys. Rev. A **65**, 063815 (2002).

Monochromatic solutions

- We impose the conditions:

$$A_1(t) = Q_1 \exp(-i\omega t) \qquad D_1(t) = \overline{D}_1$$

$$A_2(t) = Q_2 \exp(-i\omega t + \phi) \quad \xleftarrow{\text{Relative phase}} \qquad D_2(t) = \overline{D}_2$$

Quite involved problem!!!
(5 real nonlinear equations)

- Simplification:

Identical Lasers, $\Delta=0$ and $\mu_1 = \mu_2$

Symmetric solutions $k_1 = k_2$, i.e.
$D_1=D_2$, $Q_1=Q_2$, but

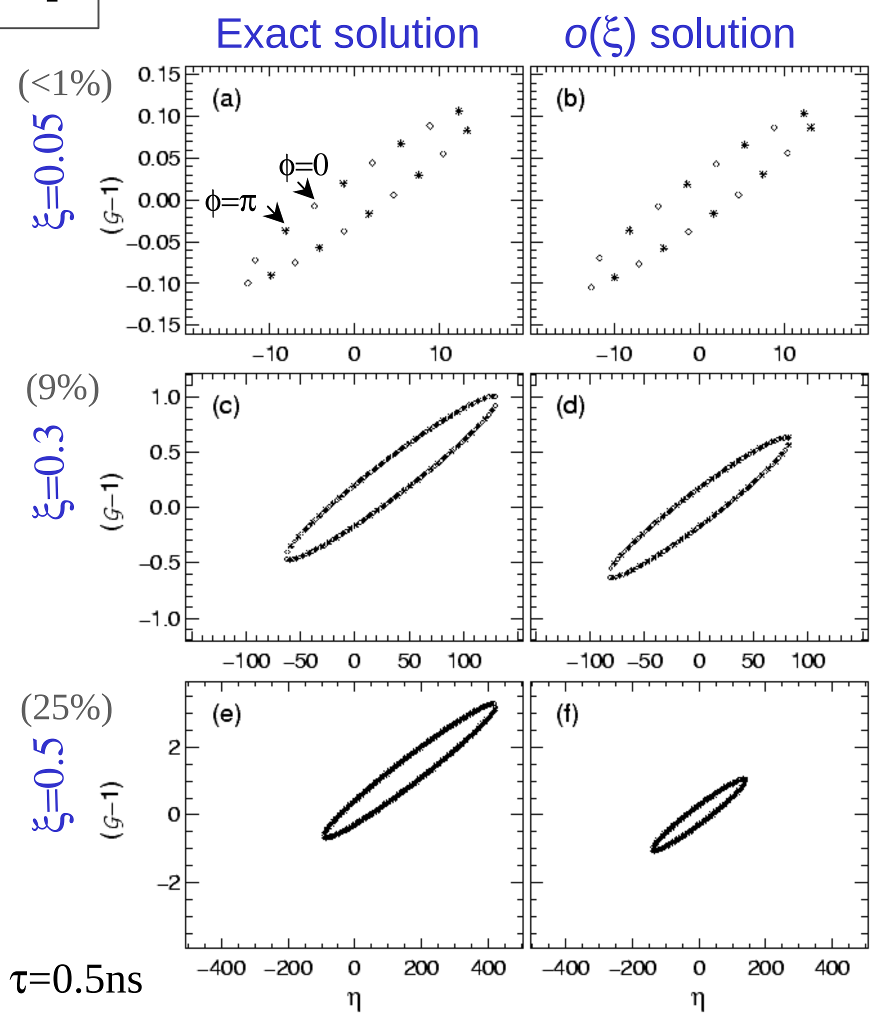
$\phi=0$ **inphase solutions**

$\phi=\pi$ **antiphase solutions**

- Dependence with coupling strength

  o(ξ) approx.

  Phenomenological model



Dynamics: Phenomenological model

- *Assumption*: weak coupling. We expand field equation to **first o(ξ)**

$$
\begin{aligned}
d_t A_{1,2}(t) &= \underbrace{\mp i\Delta A_{1,2} + \frac{1}{2}(1-i\alpha)\gamma\left[\mathcal{G}_{1,2}-1\right]A_{1,2}}_{\text{solitary laser contribution}} + \underbrace{\boxed{\hat{\kappa}_c A_{2,1}(t-\tau)}}_{\text{delayed mutual injection}} \\
d_t D_{1,2}(t) &= \gamma_e\left[\mu_{1,2} - D_{1,2} - \mathcal{G}_{1,2}|A_{1,2}|^2\right] \\
\mathcal{G}_{1,2} &= \frac{a D_{1,2}}{1+\varepsilon|A_{1,2}|^2},
\end{aligned}
$$

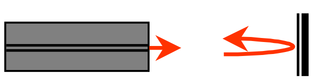
$$\hat{\kappa}_c = \frac{(1-r^2)}{r\tau_{in}}\xi\, e^{i\Omega\tau}$$

J. Mulet et al. Proc SPIE 4283, 293 (2001);
A. Hohl et al. PRA 59, 3941 (1999); T. Heil et al. PRL 86, 795 (2001).

Interpretation :

Conventional feedback,
ω-Filtered feedback,

~ Linear feedback

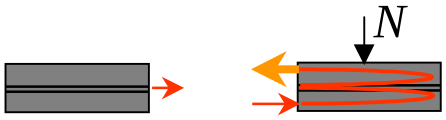


versus

Bidirectional
injection/coupling

~ Nonlinear feedback

$\left\{\begin{array}{l}\text{Nonlinear amplification} \\ \text{interaction with carriers}\end{array}\right.$



Results: Symmetric operation

Conditions: $\Delta=0$ *and* $\mu_1=\mu_2$
long coupling times: $\tau \sim 4$ ns

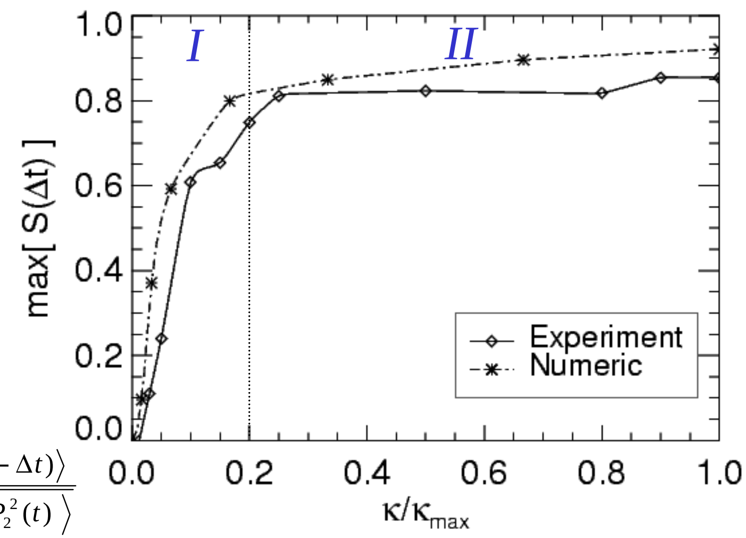
•**<u>Twofold threshold behavior:</u>**

I. Onset of coupling-induced instabilities (irregular pulsations with small correlation)

II. Transition to correlated dynamics

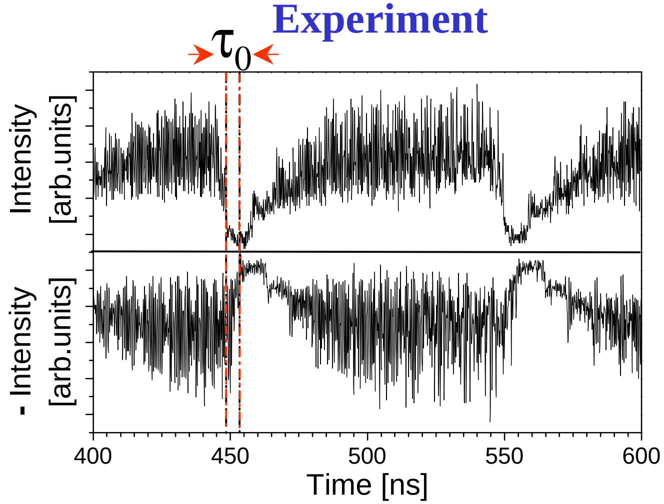
Excellent agreement!

$$S(\Delta t) = \frac{\langle \delta P_1(t)\, \delta P_2(t-\Delta t)\rangle}{\sqrt{\langle \delta P_1^2(t)\rangle \langle \delta P_2^2(t)\rangle}}$$

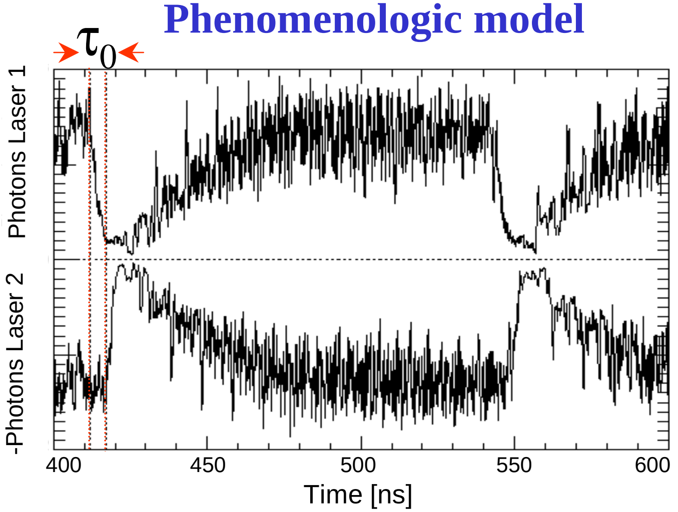


• Behavior in regime *II* *(current close to threshold)*

Experiment



Phenomenologic model



$\mu_1=\mu_2=\mu_{th}^{sol}$

synchronized LFF (power dropouts) but with a time lag

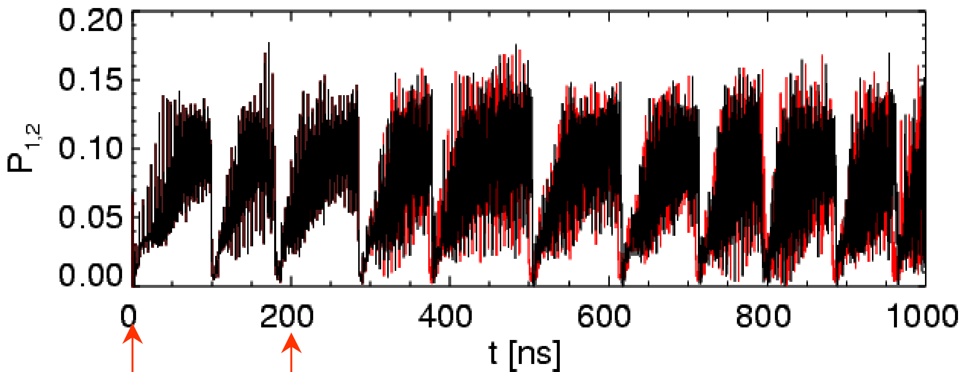
(Achronal synchronization)

T. Heil et al. PRL **86**, 795 (2001).

Symmetric operation

(Unstable) Isochronal state

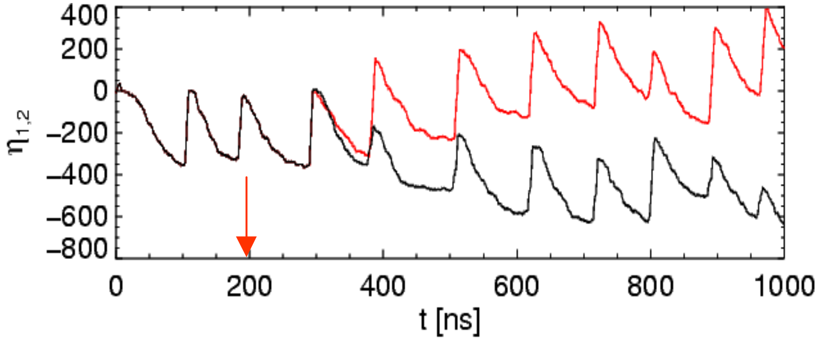
Deterministic numerical simulation



Same Initial conditions both lasers

Small perturbation applied → Achronal solution

Phase instability



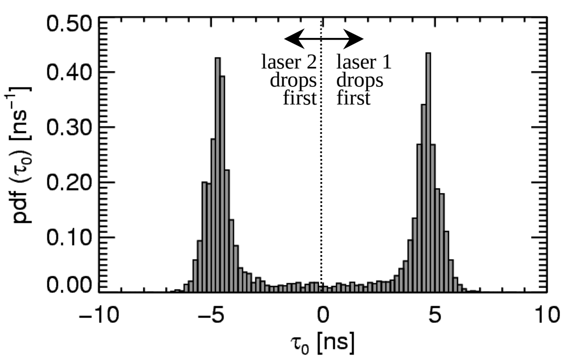
The injection phases Unlock

$$\eta_1(t) = \varphi_2(t-\tau) - \varphi_1(t)$$

$$\eta_2(t) = \varphi_1(t-\tau) - \varphi_2(t)$$

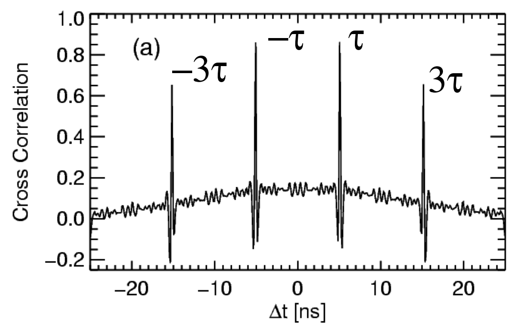
Statistical quantities *(over an achronal state)*

p.d.f. time between drops



Xcorrelation

$$S(\Delta t) = \frac{\langle \delta P_1(t)\, \delta P_2(t-\Delta t)\rangle}{\sqrt{\langle \delta P_1^2(t)\rangle \langle \delta P_2^2(t)\rangle}}$$

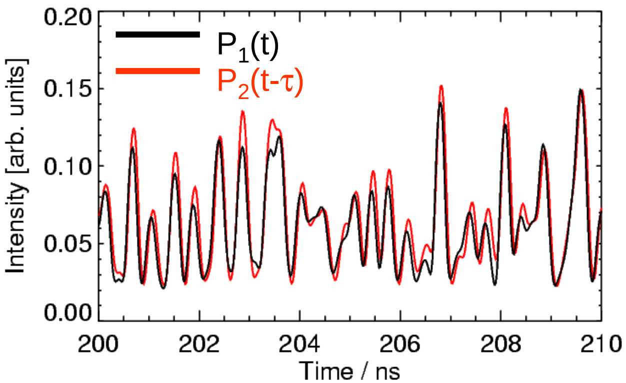


Symmetric operation

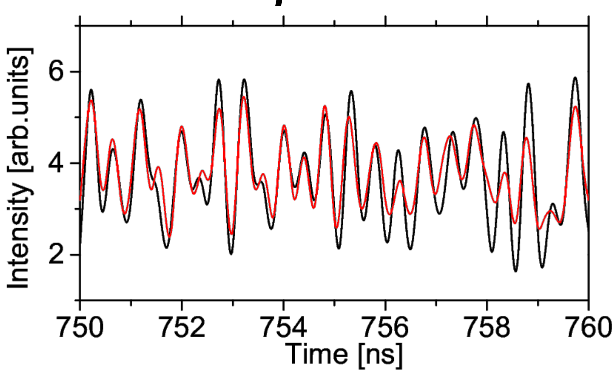
Subnanosecond fluctuations

⇒ generalized synchronization

Numeric

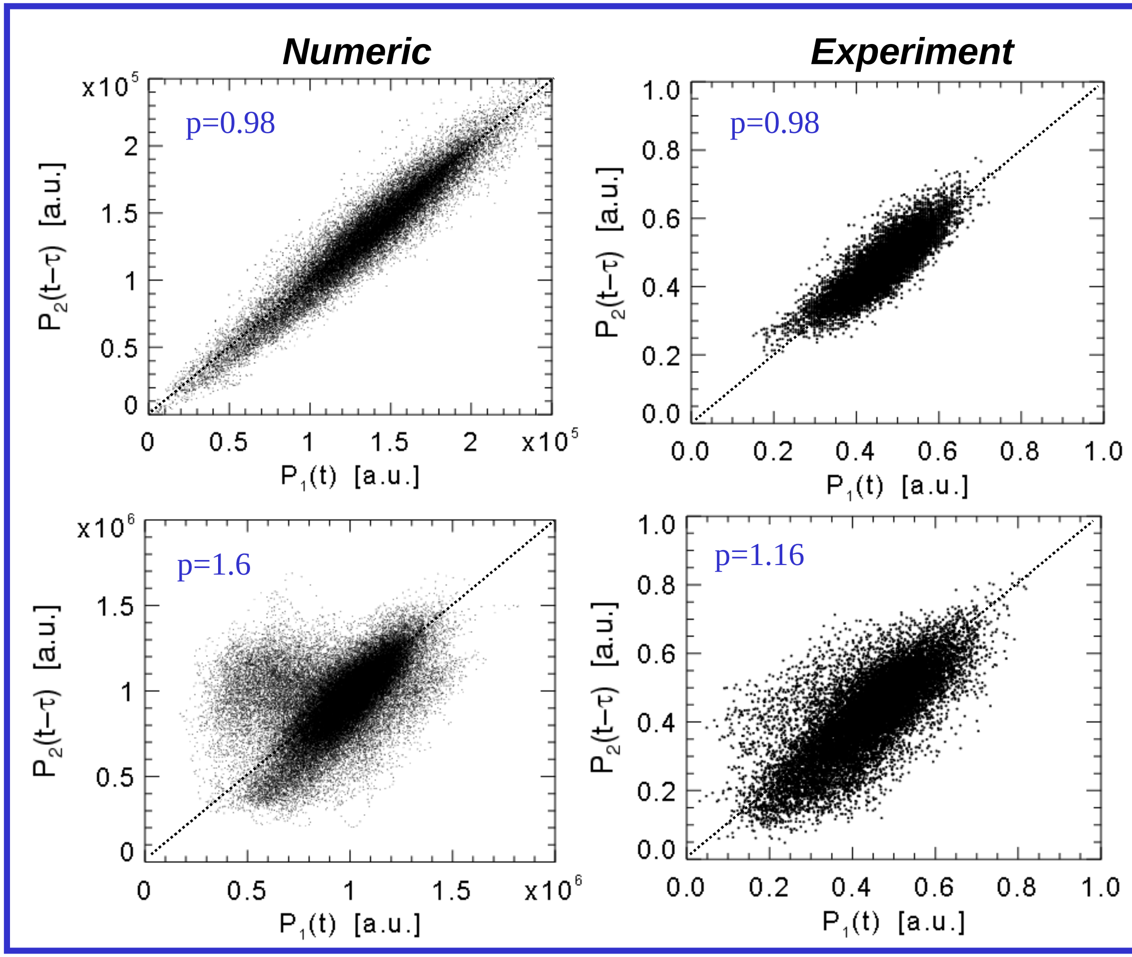


Experiment



Return plots

⇒ relatively high correlation degree Σ ~ 0.8-0.9



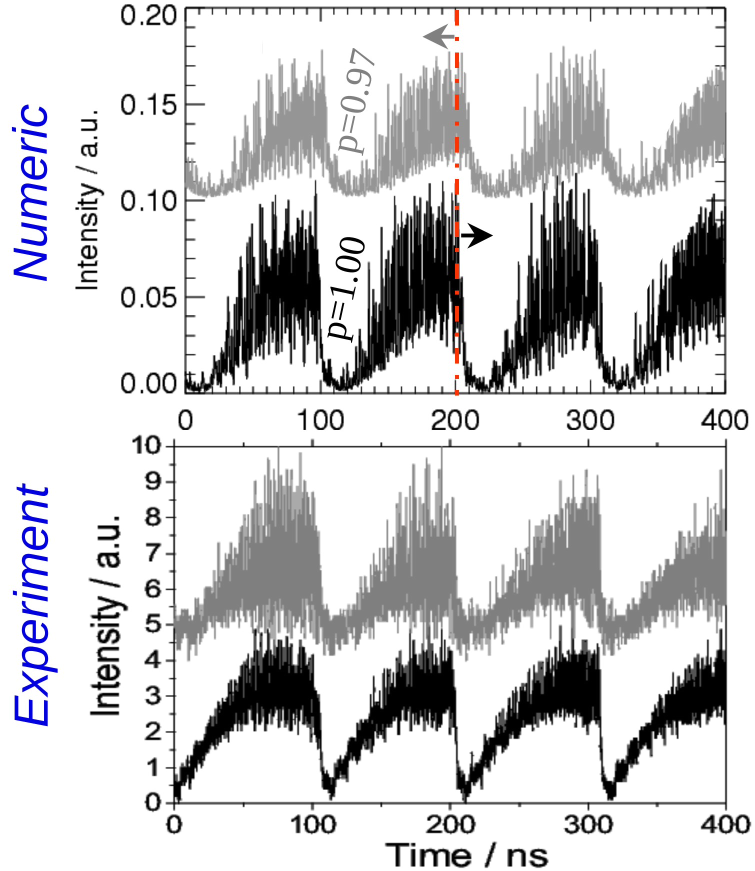
$p = I / I_{th}^{sol}$

J. Mulet, et al. SPIE Proc. **4283**, 293-302 (2001).

Asymmetric Current Injection

Conditions: $\Delta=0$ *but* $\mu_1 \neq \mu_2$ coupling time: $\tau \sim 4$ ns

- weak influence in correlation degree
- strong impact on dynamical properties
  - More periodic power dropouts
  - Laser with higher pump drops first

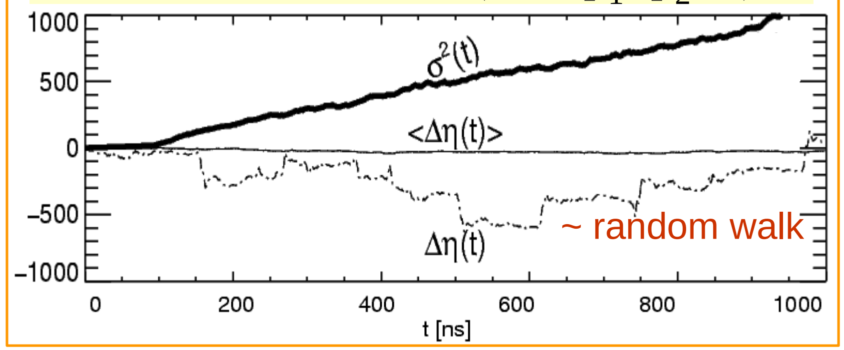


Difference in injection phases:

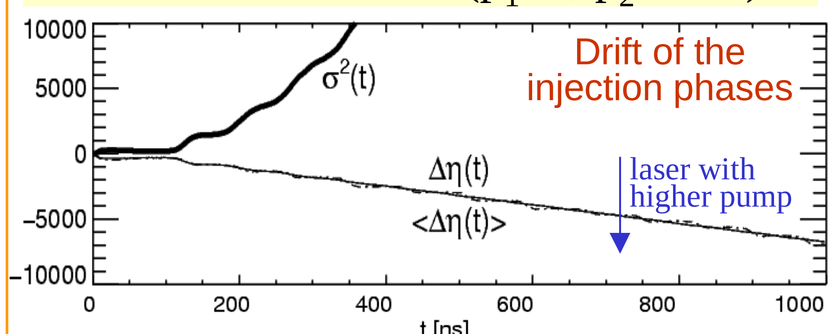
$$\Delta\eta(t) = [\varphi_2(t-\tau) - \varphi_1(t)] - [\varphi_1(t-\tau) - \varphi_2(t)]$$

- Isochronal solution $\Rightarrow \Delta\eta(t)=0$

- Achronal solution (with $p_1=p_2=1$)



- Achronal solution ($p_1=1$, $p_2=0.97$)



Dynamical model with higher-order terms

- Phenomenological o(ξ) model:
- Role of higher-order terms: passive feedback, etc...

$$\tilde{R}_{1,2}(u) \equiv \frac{\left[1 - e^{i\Delta\theta_{1,2}}\right]}{\tau_{in}} \tilde{A}_{1,2}(u)$$

M. Giudici, et al. JOSA B 16, 2114 (1999).

$$\begin{aligned} \hat{\kappa}_c &= (1 - r^2)\hat{\xi}/(r\tau_{in}) \\ \hat{\kappa}_f &= (1 - r^2)\hat{\xi}^2/\tau_{in} \\ \hat{\sigma} &= (1 - r^2)\hat{\xi}/r \end{aligned}$$

$$\begin{aligned} d_t A_{1,2}(t) &= \mp i\Delta A_{1,2}(t) + \frac{1}{2}(1 - i\alpha)\gamma[\mathcal{G}_j(t) - 1]A_{1,2}(t) \\ &\quad \boxed{+R_{1,2}(t)}\,, \\ \boxed{R_{1,2}(t) = \hat{\kappa}_c A_{2,1}(t-\tau)} &- \hat{\kappa}_f A_{1,2}(t - 2\tau) + \\ &\quad \hat{\xi}^2 R_{1,2}(t - 2\tau) - \hat{\sigma} R_{2,1}(t - \tau)\,, \\ \dot{D}_j(t) &= \gamma_e \left[\mu_j - D_j - \mathcal{G}_j \boxed{e^{-\lambda\frac{\tau_{in}}{2}\gamma[\mathcal{G}_j(t)-1]}} |A_j|^2\right]\,, \\ \mathcal{G}_j &= \frac{aD_j}{1 + \varepsilon|A_j|^2}\,. \end{aligned}$$

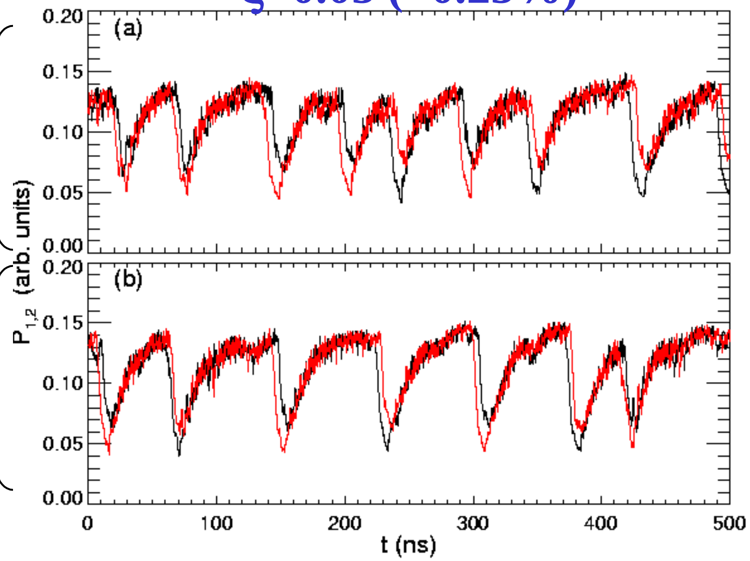
J. Mulet, et al., PRA **65**, 063815 (2002).

- **Comparison:**

ξ=0.05 (=0.25%)

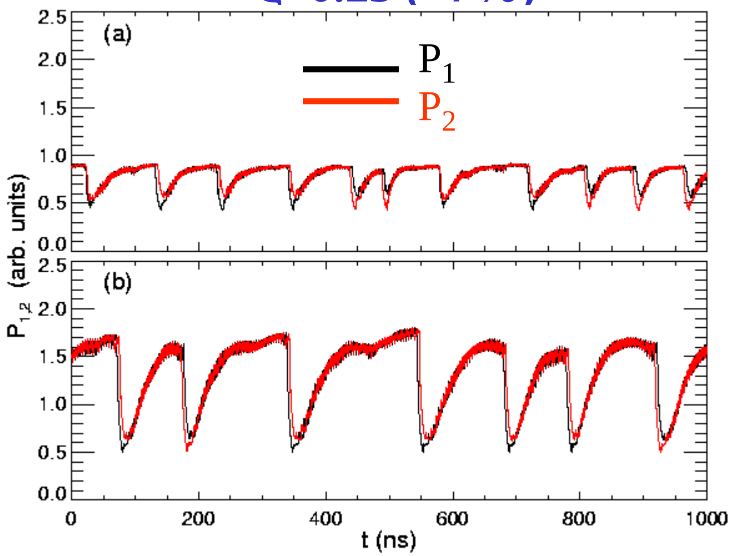
complete

phenomenologic



τ=4ns

ξ=0.25 (=7%)



Conclusions - Summary

- **Modeling**
  - Systematic derivation of the governing equations
  - Monochromatic solutions
  - Phenomenological model obtained for weak coupling
  - Dynamical model including higher-order terms

- **Dynamical Behavior**
  - Onset of (synchronized) coupling-induced instabilities
  - Unstable isochronal solution
  - Phase dynamics under symmetric or asymmetric conditions
  - Agreement with experimental results
  - Phenomenological model (<5% coupler transmitivity)