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Quantum Image processing in Type II-TW-SHG

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Project funded by the Future and Emerging Technologies arm of the IST Programme
FET-Open scheme

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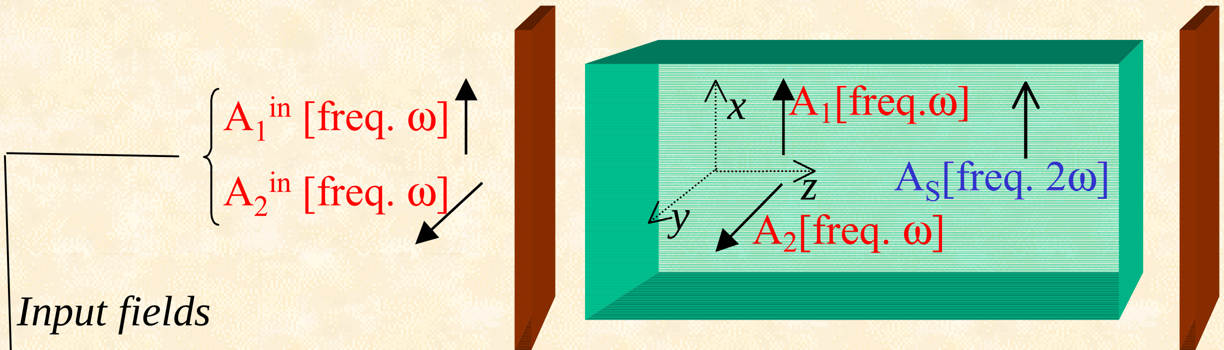
Previous work: Image processing with intracavity SHG

P. Colet, Besancon, Quantim meeting 2001



A_1^{in}: Optical Image
A_2^{in}: Plane wave pump

Characterization of spatial distribution of intracavity fields

2 different image processing regimes

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Image processing with intracavity SHG

P. Colet, Besancon, Quantim meeting 2001

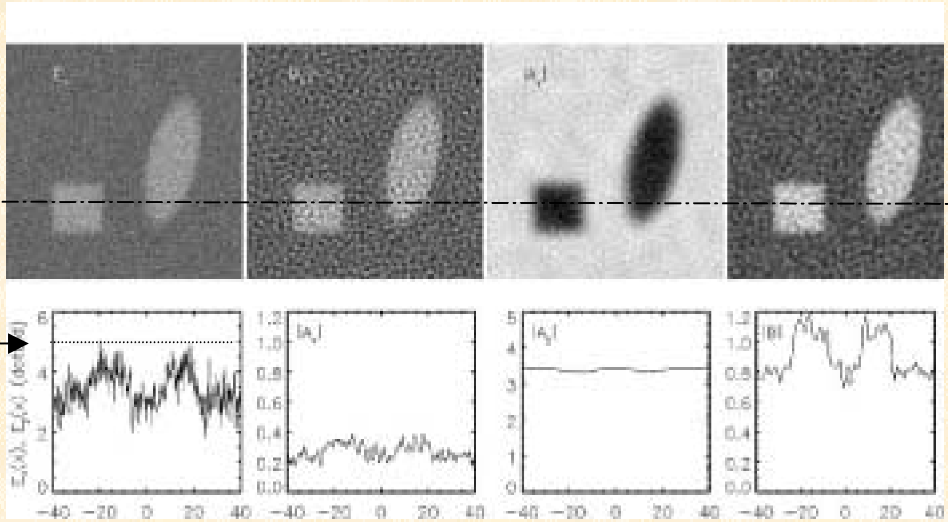
IF everywhere $A_1^{in} < A_2^{in}$:

Image transfer from ω to 2ω

IF image A_1^{in} crosses the reference level A_2^{in}

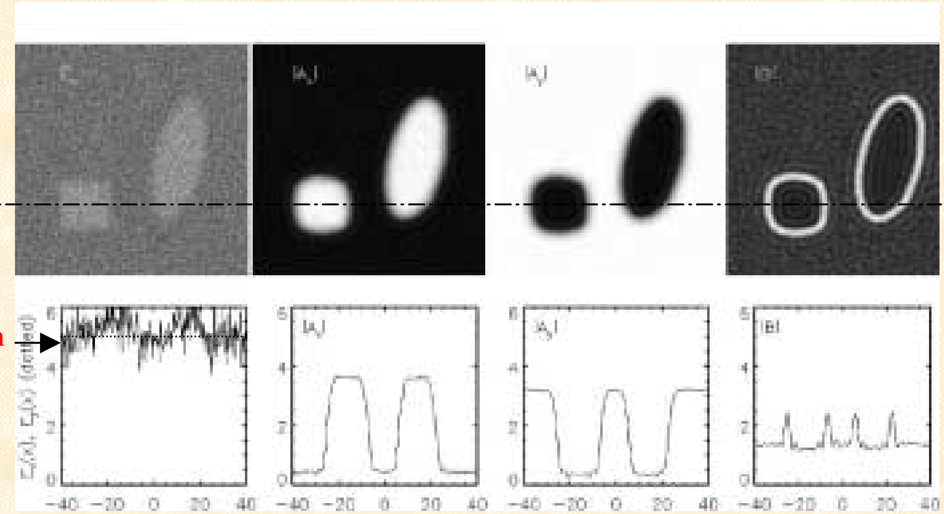
Contrast enhancement and contour recognition

A_1^{in} (input image) $A_1[\omega]$ $A_2[\omega]$ $A_S[2\omega]$



A_2^{in}

A_1^{in} (input image) $A_1[\omega]$ $A_2[\omega]$ $A_S[2\omega]$

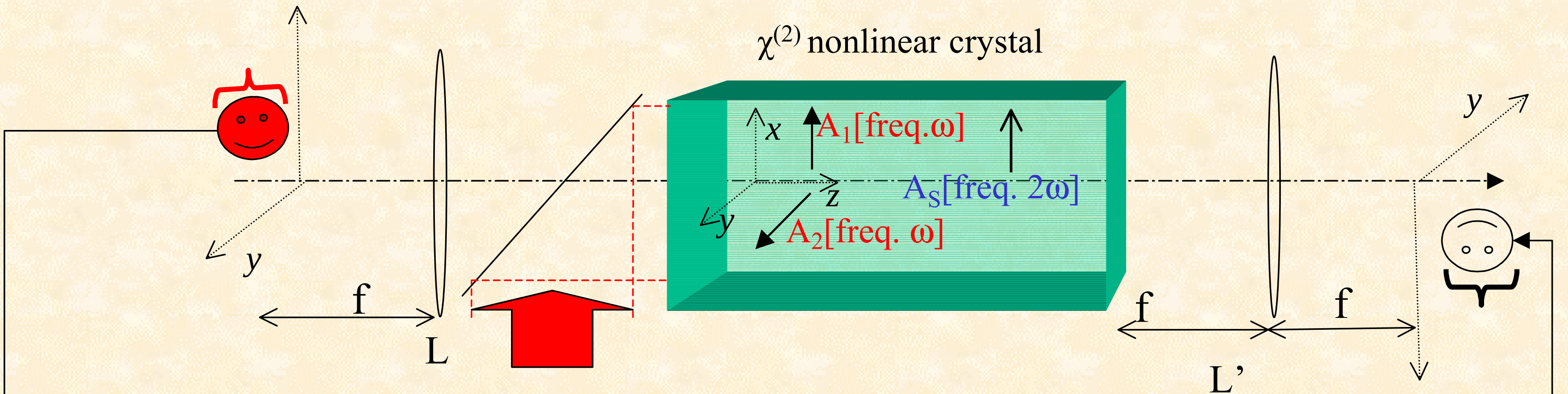


A_2^{in}

+ noise filtering

Type II- Travelling Wave Second Harmonic Generation

Type I-SHG, P. Scotto & M. San Miguel, PRA 65,043811 (2002)



Strong homog. pump field at freq. ω

Input image at freq. ω

Image processing

Output ??

depends -on the polarization of the pump
-on the polarization of the image

Polarization degree of freedom important!

Characterization:

-Spatial intensity distributions

at freq. ω with different polarisations

at freq. 2ω

–Quantum Noise behaviour

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Propagation equations for the Quantum fields

$\hat{A}_\sigma(z,q,\Omega)$: Field amplitude op. associated with a wave with transv. wave vector q, freq. $\omega_\sigma+\Omega$, and longit. wave number $k_\sigma^z(q,\Omega)$

$(\sigma = 1,2,S)$

Notation: $\bar{q} = (q,\Omega)$

Propagation equations

$$\partial_z \hat{A}_S(z,\bar{q}) = K \int d\bar{q}' \hat{A}_1(z,\bar{q}') \hat{A}_2(z,\bar{q}-\bar{q}') e^{i(k_1^z(\bar{q}') + k_2^z(\bar{q}-\bar{q}') - k_S^z(\bar{q}))z}$$

$$\partial_z \hat{A}_1(z,\bar{q}) = -K \int d\bar{q}' \hat{A}_2^+(z,\bar{q}') \hat{A}_S(z,\bar{q}+\bar{q}') e^{i(k_S^z(\bar{q}+\bar{q}') - k_2^z(\bar{q}') - k_1^z(\bar{q}))z}$$

$$\partial_z \hat{A}_2(z,\bar{q}) = -K \int d\bar{q}' \hat{A}_1^+(z,\bar{q}') \hat{A}_S(z,\bar{q}+\bar{q}') e^{i(k_S^z(\bar{q}+\bar{q}') - k_1^z(\bar{q}') - k_2^z(\bar{q}))z}$$

conservation of transverse momentum and energy

Excess of long. momentum

3-wave processes

Type II-SHG

[ω](q,Ω)$_{\text{x-polarized}}$+ [ω](q-q',Ω–Ω)$_{\text{y-polarized}}$ –> [2ω] (q, Ω)

[2ω] (q+q', Ω+Ω') –>[ω](q,Ω)$_{\text{x-polarized}}$+ [ω](q',Ω')$_{\text{y-polarized}}$

In real Space

$$\hat{A}_\sigma(z,q,\Omega) = e^{-i(k_\sigma^z(q,\Omega)-k_\sigma)z} \int dt \int d\rho\, e^{i(\Omega t - q\rho)} A_\sigma(z,\rho,t)$$

$$k_\sigma^z(q,\Omega) \approx k_\sigma + \frac{1}{u_\sigma}\Omega + \frac{1}{2}k_\sigma''\Omega^2 - \frac{1}{2k_\sigma}q^2 + \rho_\sigma q_y$$

(quasimonochr. & paraxial approx.)

$$\partial_z A_1(z,\rho,t) - \frac{1}{u_1}\partial_t A_1(z,\rho,t) + \frac{i}{2}k_1''\partial_t^{\,2} A_1(z,\rho,t) - \frac{i}{2k_1}\nabla^2 A_1(z,\rho,t) + \rho_1 \partial_y A_1(z,\rho,t) = -K A_2^+(z,\rho,t) A_S(z,\rho,t) e^{-i\Delta k.z}$$

Propagation along z

Group velocity dispersion

Diffraction

Walk off

Nonlinear coupling

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Linearization

Ansatz $\hat{A}_\sigma(z,q,\Omega) = \delta^{(2)}(q)\delta(\Omega)c_\sigma(z) + \hat{a}_\sigma(z,q,\Omega)$ **into propagation eq.**

Total fields = Strong classical monochromatic fields generated by the pump + (weak) input signal + quantum fluctuations

Propagation eq. for an input signal/quantum fluctuations

$$\partial_z \hat{a}_S(z,\bar{q}) = +\sqrt{2}c_2(z)e^{+iD_1(\bar{q})z}\hat{a}_1(z,\bar{q}) + \sqrt{2}c_1(z)e^{+iD_2(\bar{q})z}\hat{a}_2(z,\bar{q})$$

$$\partial_z \hat{a}_1(z,\bar{q}) = -c_S(z)e^{-i\Delta_1(\bar{q})z}\hat{a}_2^+(z,-\bar{q}) - \sqrt{2}c_2^*(z)e^{-iD_1(\bar{q})z}\hat{a}_S(z,\bar{q})$$

$$\partial_z \hat{a}_2(z,\bar{q}) = -c_S(z)e^{-i\Delta_2(\bar{q})z}\hat{a}_1^+(z,-\bar{q}) - \sqrt{2}c_1^*(z)e^{-iD_2(\bar{q})z}\hat{a}_S(z,\bar{q})$$

Classical eq. of nonlinear optics for the strong homogeneous fields

$$\partial_z c_S(z) = 2c_1(z)c_2(z)e^{i\Delta k.z}$$

$$\partial_z c_1(z) = -c_2^*(z)c_S(z)e^{-i\Delta k.z}$$

$$\partial_z c_2(z) = -c_1^*(z)c_S(z)e^{-i\Delta k.z}$$

Relevant phase mismatch factors

$\Delta k = k_1 + k_2 - k_S$

$\Delta_1(\bar{q}) = k_1^z(\bar{q}) + k_2^z(-\bar{q}) - k_S$ — twin photon emission

$\Delta_2(\bar{q}) = k_1^z(-\bar{q}) + k_2^z(\bar{q}) - k_S$

$D_1(\bar{q}) = k_1^z(\bar{q}) + k_2 - k_S^z(\bar{q})$ — freq.up/down conversion

$D_2(\bar{q}) = k_1 + k_2^z(\bar{q}) - k_S^z(\bar{q})$

$D_1(\bar{q}=0) = D_1(\bar{q}=0) = \Delta_1(\bar{q}=0) = \Delta_1(\bar{q}=0) = \Delta k$

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Case 1: y- polarized pump

(x-polarized image)

Pumping → $c_1(0)=0$, $c_2(0)=1$, $c_S(0)=0$ => Classical equations of nonlinear optics $\forall z \geq 0$ $c_1(z)=0$, $c_2(z)=1$, $c_S(z)=0$

Type II phase matching: no SH field generated since pumping only in one pol.

Prop. eq. for quantum fields

$$\hat{a}_S(z,\bar{q})=\frac{\sqrt{2}}{\sqrt{2+\left(\frac{D(\bar{q})}{2}\right)^2}}\sin\left(\sqrt{2+\left(\frac{D(\bar{q})}{2}\right)^2}\,z\right)e^{i\frac{D(\bar{q})z}{2}}\hat{a}_1(0,\bar{q})+\left(\cos\left(\sqrt{2+\left(\frac{D(\bar{q})}{2}\right)^2}\,z\right)-i\frac{\frac{D(\bar{q})}{2}}{\sqrt{2+\left(\frac{D(\bar{q})}{2}\right)^2}}\sin\left(\sqrt{2+\left(\frac{D(\bar{q})}{2}\right)^2}\,z\right)\right)e^{i\frac{D(\bar{q})z}{2}}\hat{a}_S(0,\bar{q})$$

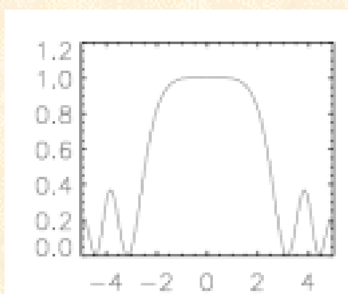
Conversion of an x-polarized input image at freq. ω up to freq. 2ω

Up-Conversion Rate: $$\eta(z,\bar{q})\equiv\frac{\left\langle\hat{a}_S^+(z,\bar{q})\hat{a}_S(z,\bar{q})\right\rangle}{\left\langle\hat{a}_1^+(0,\bar{q})\hat{a}_1(0,\bar{q})\right\rangle}=\frac{2}{2+\left(\frac{D(\bar{q})}{2}\right)^2}\sin^2\left(\sqrt{2+\left(\frac{D(\bar{q})}{2}\right)^2}\,z\right)$$

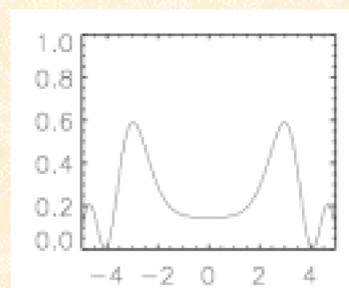
Collinear phase matching $\Delta k = k_1 + k_2 - k_S = 0$

$\eta(z,\bar{q})=1$

if: $\begin{cases} D(\bar{q})=0 \\ z=(2k+1)\dfrac{\pi}{2\sqrt{2}} \end{cases}$



$z \neq (2k+1)\dfrac{\pi}{2\sqrt{2}}$



Classical optics !

Frequency addition in the parametric approximation

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Quantum Novelties

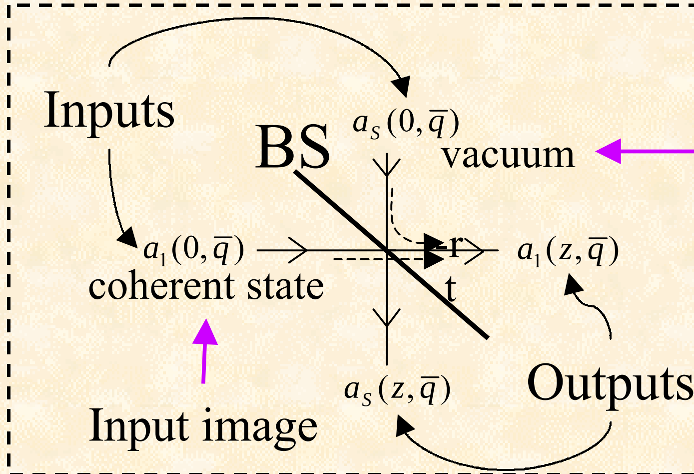
- Characterize the noise behaviour of the optical device
- Improvement of image processing by appropriate tayloring of quantum noise

<u>Input-output transformation</u>

$$\hat{a}_S(z,\bar{q}) = e^{i\frac{\tilde{D}(\bar{q})z}{2}} \left(r(\bar{q})\hat{a}_1(0,\bar{q}) + t(\bar{q})\hat{a}_S(0,\bar{q}) \right)$$

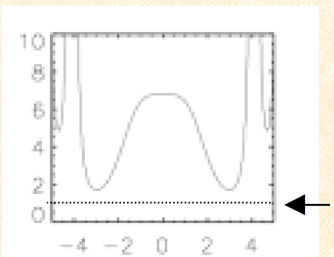
$$\hat{a}_1(z,\bar{q}) = e^{-i\frac{\tilde{D}(\bar{q})z}{2}} \left(t^*(\bar{q})\hat{a}_1(0,\bar{q}) - r(\bar{q})\hat{a}_S(0,\bar{q}) \right)$$

<u>Equivalent to the transformation performed by a Beam Splitter !</u>



1) **Noise figure** F = SNR(input)/SNR(output)

F



←F=1

$$F = \frac{1}{\eta(\bar{q})}$$

Previous case of partial up-convertion

2) **Improvement:** if input state at SH freq = <u>squeezed vacuum with properly chosen squeezed quadrature</u>

Squeezing the pump has no effect on the performance of the device!!!

(Pump field fluctuations decouple) $\partial_z \hat{a}_2(z,\bar{q}) = 0 \Rightarrow \hat{a}_2(z,\bar{q}) = \hat{a}_2(0,\bar{q})$

Case 2: 45º- polarized pump

Collinear phase matching

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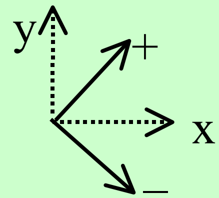
Pumping → $c_1(0)=c_2(0)=1/\sqrt{2}$, $c_S(0)=0$ => Classical equations of nonlinear optics $\forall z \geq 0$ $c_1(z)=\mathrm{sech}(z)/\sqrt{2}$, $c_2(z)=\mathrm{sech}(z)/\sqrt{2}$, $c_S(z)=\tanh(z)$ SHG.

Prop.eq.

New polarization basis



$$\hat{a}_+(z,\bar{q})=\frac{\hat{a}_1(z,\bar{q})+\hat{a}_2(z,\bar{q})}{\sqrt{2}}$$

$$\hat{a}_-(z,\bar{q})=\frac{\hat{a}_1(z,\bar{q})-\hat{a}_2(z,\bar{q})}{\sqrt{2}}$$

$$\begin{cases}
\partial_z\hat{a}_S(z,\bar{q})=-\sqrt{2}c_+(z)\left(\dfrac{e^{-iD_1(\bar{q})z}+e^{-iD_2(\bar{q})z}}{2}\right)\hat{a}_+(z,\bar{q})+\sqrt{2}c_+(z)\left(\dfrac{e^{-iD_1(\bar{q})z}-e^{-iD_2(\bar{q})z}}{2}\right)\hat{a}_-(z,\bar{q}) \\
\partial_z\hat{a}_+(z,\bar{q})=-c_S(z)\left(\dfrac{e^{-i\Delta_1(\bar{q})z}+e^{-i\Delta_2(\bar{q})z}}{2}\right)\hat{a}_+^{+}(z,-\bar{q})+c_S(z)\left(\dfrac{e^{-i\Delta_1(\bar{q})z}-e^{-i\Delta_2(\bar{q})z}}{2}\right)\hat{a}_-^{+}(z,-\bar{q})-\sqrt{2}c_+^*(z)\left(\dfrac{e^{-iD_1(\bar{q})z}+e^{-iD_2(\bar{q})z}}{2}\right)\hat{a}_S(z,\bar{q}) \\
\partial_z\hat{a}_-(z,\bar{q})=-c_S(z)\left(\dfrac{e^{-i\Delta_1(\bar{q})z}-e^{-i\Delta_2(\bar{q})z}}{2}\right)\hat{a}_+^{+}(z,-\bar{q})+c_S(z)\left(\dfrac{e^{-i\Delta_1(\bar{q})z}+e^{-i\Delta_2(\bar{q})z}}{2}\right)\hat{a}_-^{+}(z,-\bar{q})-\sqrt{2}c_+^*(z)\left(\dfrac{e^{-iD_1(\bar{q})z}-e^{-iD_2(\bar{q})z}}{2}\right)\hat{a}_S(z,\bar{q})
\end{cases}$$

Simplification: assume that

$$D_1(\bar{q})=D_2(\bar{q})\equiv D(\bar{q})$$
$$\Delta_1(\bar{q})=\Delta_2(\bar{q})\equiv\Delta(\bar{q})$$

⟺

$$k_1^z(\bar{q})+k_2-k_S^z(\bar{q})=k_1+k_2^z(\bar{q})-k_S^z(\bar{q})$$
$$k_1^z(\bar{q})+k_2^z(-\bar{q})-k_S=k_1^z(-\bar{q})+k_2^z(\bar{q})-k_S$$

(Condition exactly fulfilled for $q=\Omega=0$)

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Case 2: 45º- polarized pump

$\hat{a}_-(z,\bar{q})$ decouples from $\{\hat{a}_+(z,\bar{q}),\hat{a}_S(z,\bar{q})\}$

$$\partial_z\hat{a}_-(z,\bar{q})=-c_S(z)e^{-i\Delta(\bar{q})z}\hat{a}_-^+(z,-\bar{q})$$

Type I OPA with z-dependent pump

$$\left(c_S\left(z\right)=\tanh(z)\right)$$

$$\partial_z\hat{a}_+(z,\bar{q})=-c_S(z)e^{-i\Delta(\bar{q})z}\hat{a}_+^+(z,-\bar{q})-c_+^*(z)e^{-iD(\bar{q})z}\hat{a}_S(z,\bar{q})$$

$$\partial_z\hat{a}_S(z,\bar{q})=-c_+^*(z)e^{-iD(\bar{q})z}\hat{a}_+(z,\bar{q})$$

Type I SHG

Type II SHG [a_1,a_2,a_S] = Type I-SHG [a_+,a_S] * Type I-OPA [a_-]

Similar to TW case without transverse effects: P. Kumar (1994)
Cavity-case (Z.Y. Ou, Phys. Rev. A **49** (1994), 4902)

Implications for Type II-SHG image processing

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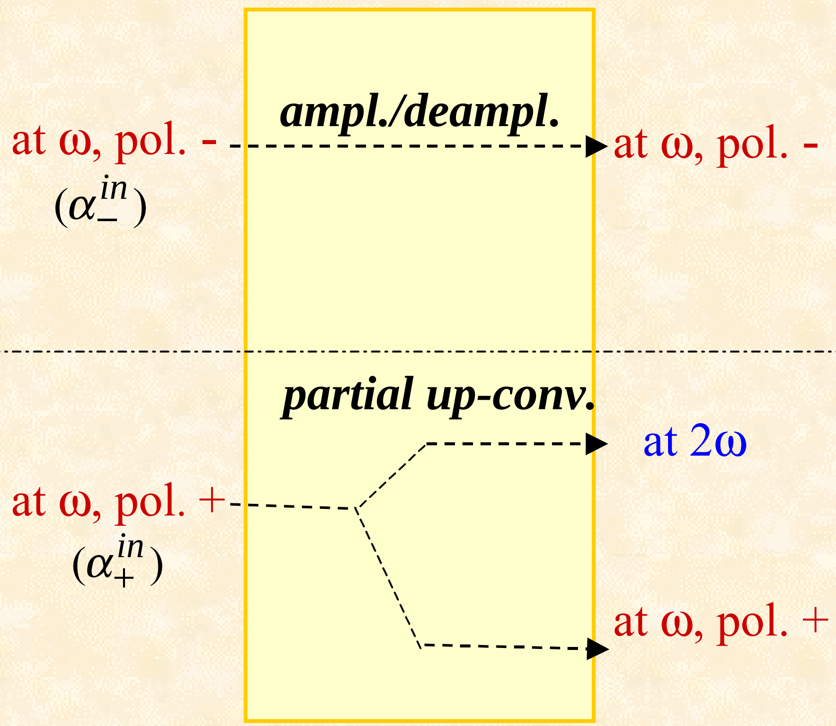
Schematic represention

Input signal

Output



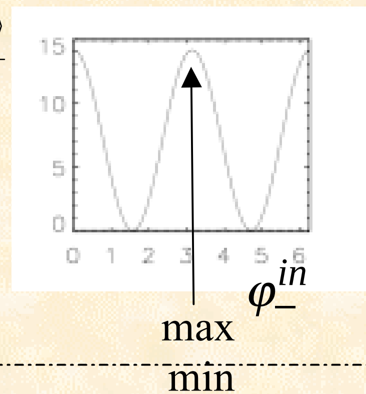
Phase sensitivity

(symmetrical input)

$$\left(\alpha_\pm^{in} = \left|\alpha_\pm^{in}\right| e^{i\varphi_\pm^{in}}\right)$$

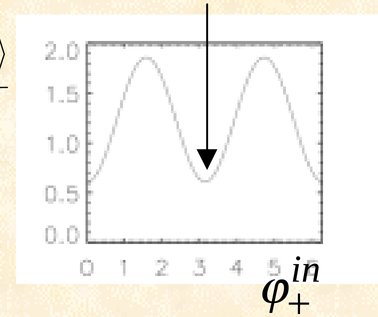
GAIN

$$\frac{\langle \hat{N}_-^{out} \rangle}{\langle \hat{N}_-^{in} \rangle}$$



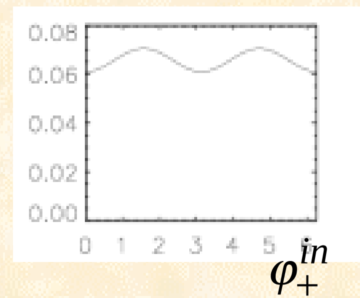
UP-CONV. RATE

$$\frac{\langle \hat{N}_S^{out} \rangle}{\langle \hat{N}_+^{in} \rangle}$$



TRANSM. RATE

$$\frac{\langle \hat{N}_+^{out} \rangle}{\langle \hat{N}_+^{in} \rangle}$$



Implications for Type II-SHG image processing

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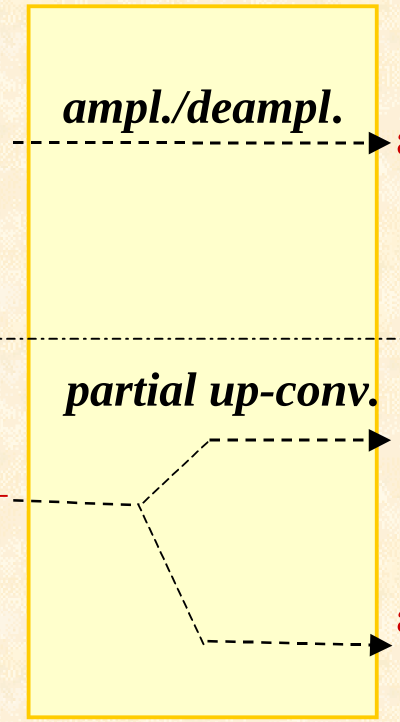
Schematic representation

Input signal

Output



at ω, pol. - (α_-^{in}) → at ω, pol. -

at ω, pol. + (α_+^{in}) → at 2ω ; at ω, pol. +

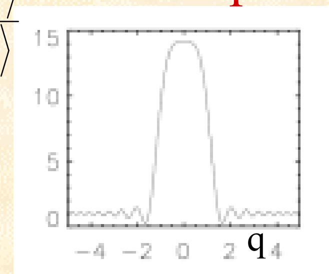
Collinear phase matching

$\Delta k = k_1 + k_2 - k_S = 0$

Spatial behavior

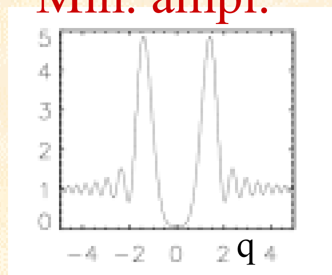
Max. ampl.

$\frac{\langle \hat{N}_-^{out} \rangle}{\langle \hat{N}_-^{in} \rangle}$



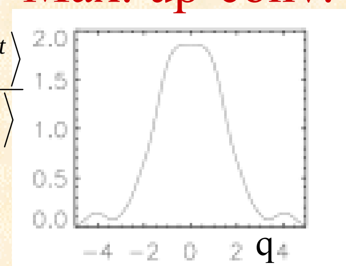
Min. ampl.

$\frac{\langle \hat{N}_-^{out} \rangle}{\langle \hat{N}_-^{in} \rangle}$



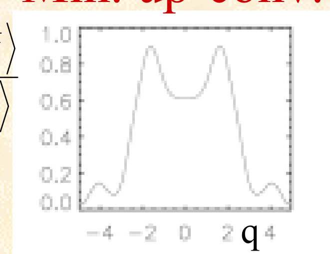
Max. up-conv.

$\frac{\langle \hat{N}_S^{out} \rangle}{\langle \hat{N}_+^{in} \rangle}$

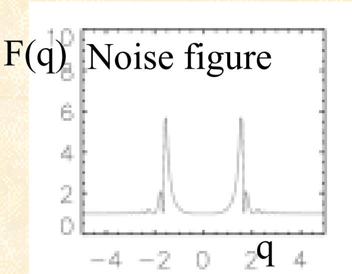


Min. up-conv.

$\frac{\langle \hat{N}_S^{out} \rangle}{\langle \hat{N}_+^{in} \rangle}$

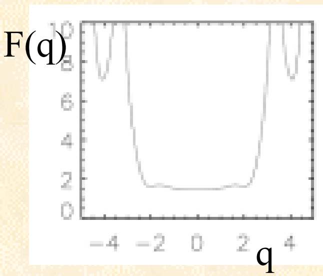


Noise behavior

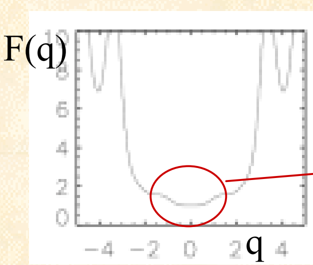


noiseless operation

Image processing efficient on a disk-shaped region centred on the optical axis of the system



excess noise



noiseless operation

Image processing as a function of the polarisation of the input image (Pump 45º)

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Field at freq. ω

Field at freq. 2ω

Arrows: direction: polarization

modulus: amplitude

F=1 ⇔ SNR(Output)=SNR(input)

For sufficiently large crystal:
almost no output with polarization +
Transformation:

$$\{\alpha_-(\bar{q}),\alpha_+(\bar{q})\}\rightarrow\{\alpha_-(\bar{q}),\alpha_S(\bar{q})\}$$

INPUT | OUTPUT (φ ^{in} :OPA max) | OUTPUT (φ ^{in} :OPA min)

1) Input image is x-polarized

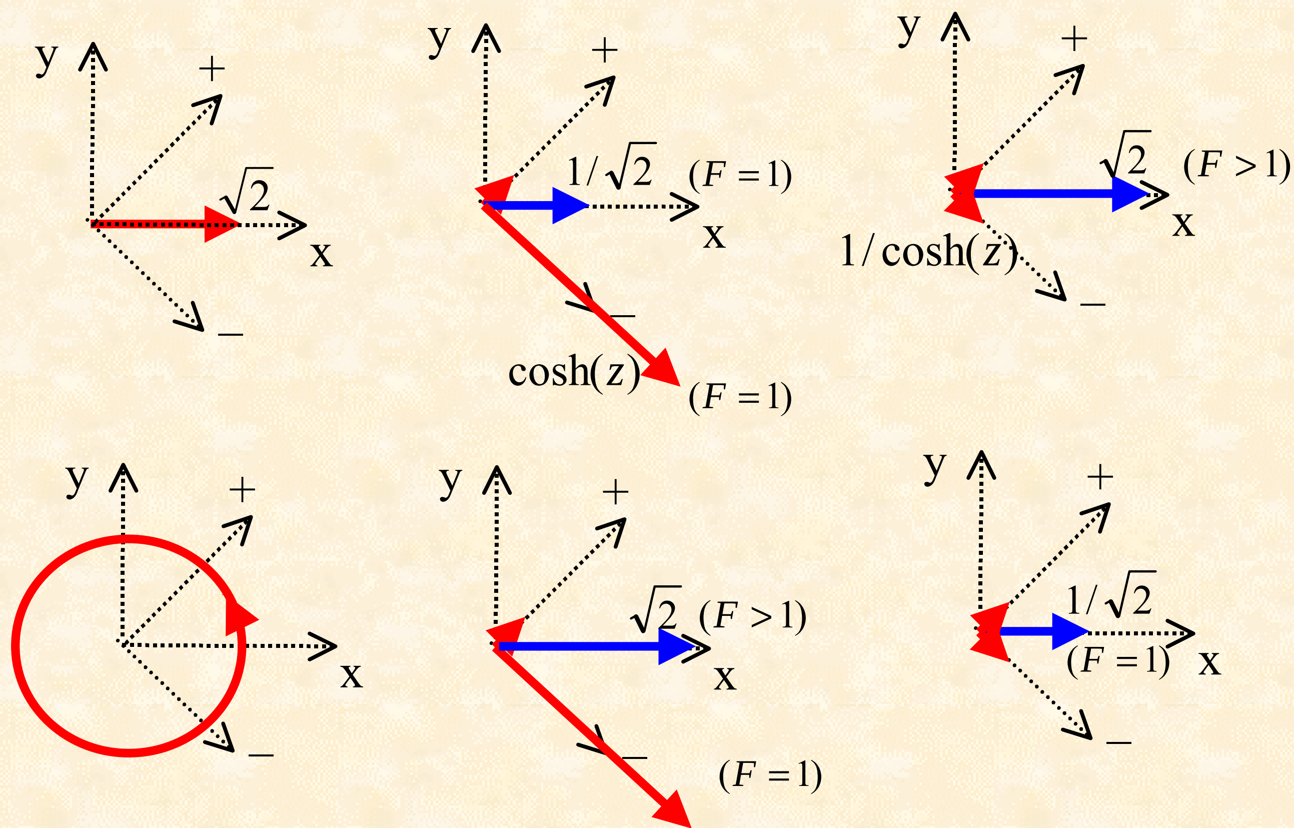
$$\alpha_+^{in}(\bar{q})=\alpha_-^{in}(\bar{q})$$

same phase φ^{in}

2) Input image is circularly-polarized

$$\alpha_+^{in}(\bar{q})=\pm i\alpha_-^{in}(\bar{q})$$

phase shift=π/2



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Conclusions

TW Type II SHG is interesting for quantum image processing.

Pump y-polarized and image x-polarized

- Upconversion of the input image
- Conversion rate in general < 1 => Degradation of signal-to-noise ratio

BUT it is possible to enhance the performance by using nonclassical light.

Pump linearly polarized at 45º

- Image processing properties taylored by tuning the polarisation of the image
- Upconversion of input image
- **AND** noiseless amplification of input image in rotated polarisation

Outlook

Effects of - different parameters for ordinary and extraordinary polarization
- walk off

Other solutions of the nonlinear equations of classical optics

Cavity case: Ongoing investigations of the singly resonant case

Quantum treatment in the nonlinear regime