QUANTIM

Quantum Imaging IST-2000-26019

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Numerical approach to Superresolution

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FET-Open scheme

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Background

Rayleigh, 1879 : *Diffraction limits the resolution of optical devices*

Mathematicians showed that diffraction is not a fundamental limit.

Bertero, Pike, 1982: *Reconstruction is possible, but strongly limited by the signal to noise ratio in the measured image*

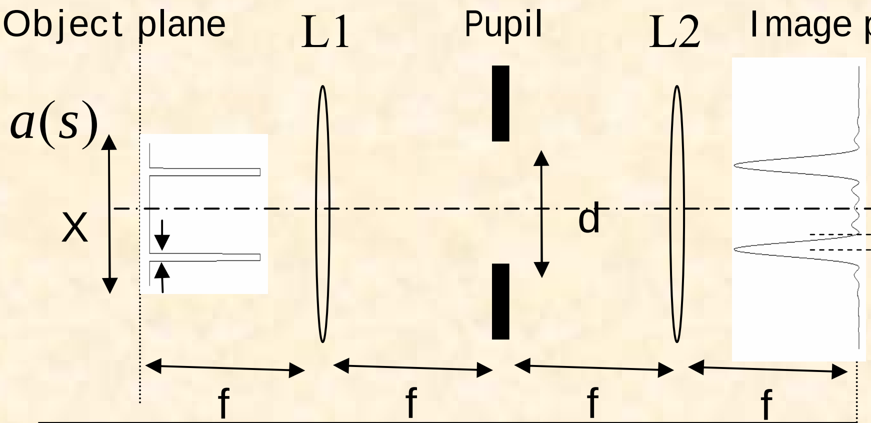
M.Bertero & E.R. Pike, Opt. Acta.**29**, 727 (1982)

Kolobov, Fabre, 2000: *The noise of quantum origin can be reduced by using nonclassical light and hence the reconstruction procedure improved.*

M.Kolobov & C. Fabre, Phys. Rev.Lett.**85**, 3789 (2000)

QUANTIM, 2002: *Practical implementation?*

Effect of diffraction on imaging.



$$\geq l_{Rayl} = \frac{\lambda f}{d}$$

Rayleigh resolution length

to compare with

X Width of the object support

Input-output transf.

$$e(s) = \int_{-1}^{1} \frac{\sin[c(s-s')]}{\pi(s-s')} a(s')ds' \qquad c = \frac{\pi}{2} S$$

(s and s´ expressed in units of X/2)

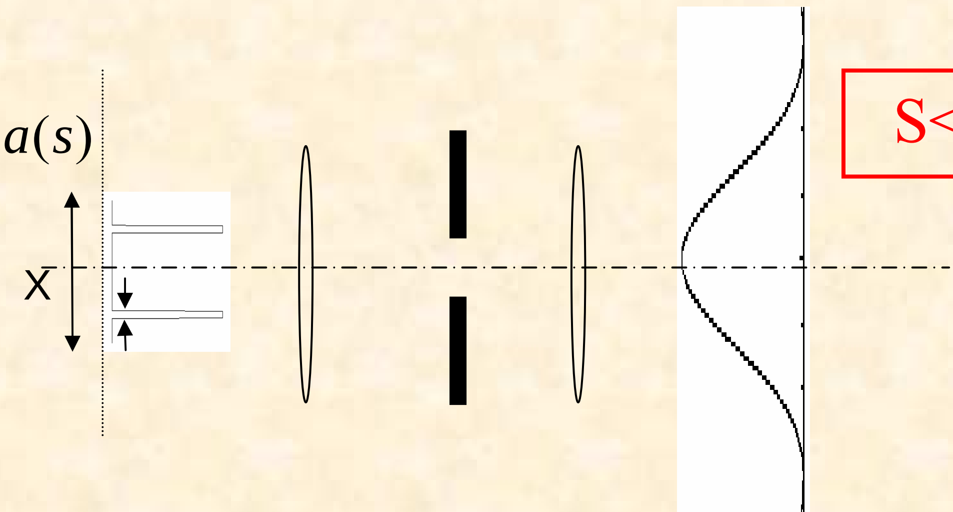
Shannon-number

$$S = \frac{dX}{\lambda f}$$

max.# pixels resolved *classically*

S>>1

S<1



No resolution of any detail in the object in the sense of Rayleigh

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Beyond Rayleigh...

Possibility of object reconstruction.

Knowledge of output field distribution => determination of object

Condition: object confined in a finite region of space

Method: expansions in terms of "eigenfunctions" of the input-output transformation for the optical system

$$\int_{-1}^{1} \frac{\sin[c(x-x')]}{\pi(x-x')} \psi_k(x')dx = \lambda_k \psi_k(x)$$

Solution: the **prolate spheroidal functions**

- Double orthogonality

$$\left(\int_{-\infty}^{+\infty} \psi_k(x)\psi_l(x)dx = \delta_{kl}\right)$$
$$\left(\int_{-1}^{+1} \psi_k(x)\psi_l(x)dx = \lambda_k \delta_{kl}\right)$$

- on $[-\infty,+\infty]$ $\{\psi_k(s)\}_{k=0,1,...}$ **Basis / bandlimited functions with bandwidth 2c**
- on [-1,+1] $\left\{\varphi_k(s) = \sqrt{\lambda_k}^{-1/2} \psi_k(s)\right\}_{k=0,1,...}$ **Basis in L^2(-1,1)**

- are their own "finite" Fourier transform: $\int_{-1}^{+1} ds\, \varphi_k(s) e^{i\omega s} = i^k \left(\frac{2\pi\lambda_k}{c}\right)^{1/2} \varphi_k\left(\frac{\omega}{c}\right)$

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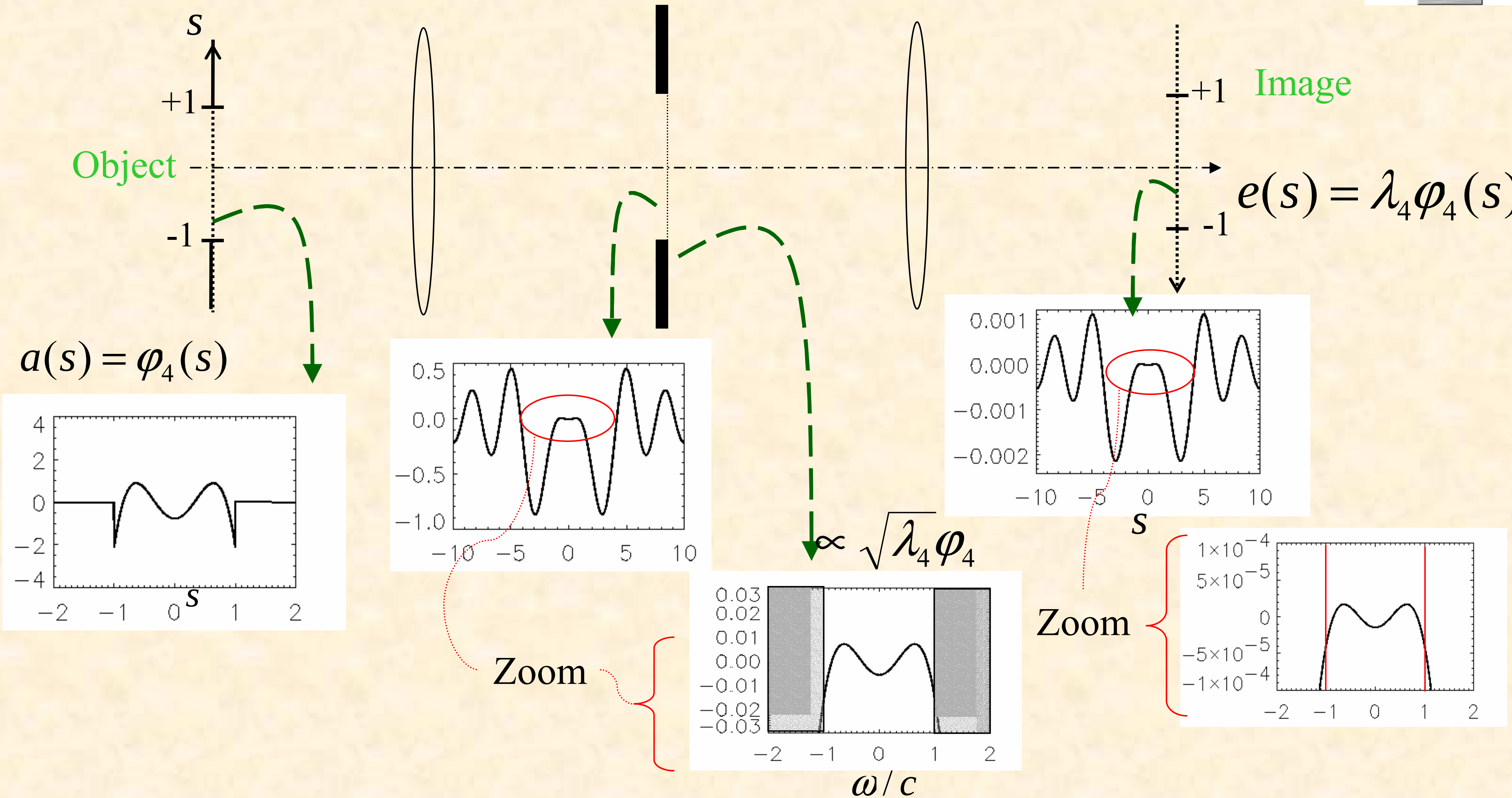
Diffraction limited optical device



The prolate functions retain their identity / imaging transformation.

Diffraction => Attenuation

Reconstruction scheme.

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$$a(s) = \sum_k a_k \varphi_k(s)$$

$$e(s) = \sum_k e_k \psi_k(s)$$

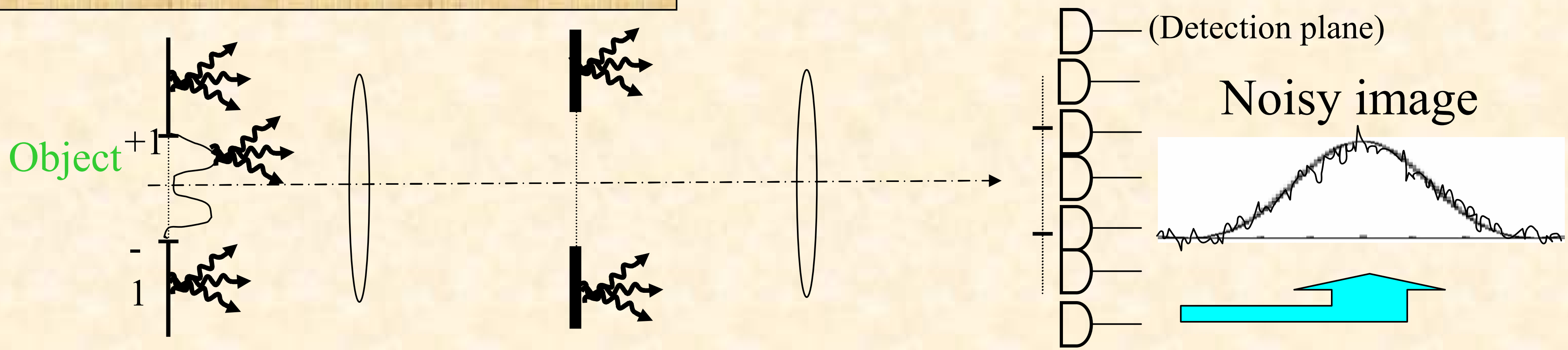
$$a_k = \frac{e_k}{\sqrt{\lambda_k}}$$

For a noise-free image, the object reconstruction can be performed ***exactly***

$$\varphi_k(s) = \sqrt{\lambda_k}^{\,-1/2}\,\psi_k(s)$$

$$\int_{-1}^{+1} \varphi_k(x)\varphi_l(x)\,dx = \delta_{kl}$$

$$\int_{-\infty}^{+\infty} \psi_k(x)\psi_l(x)\,dx = \delta_{kl}$$



SOURCES OF NOISE

- Noise added by detection device
- Quant.fluct.of illumination light
- Vacuum fluct.from “dark” parts

Feasibility of the reconstruction procedure strongly limited by the noise in the image.

Reconstruction scheme.

Efficiency

Taking <u>a few modes</u> in the expansion: $a_{rec}(s) = a_0\varphi_0(s) + a_2\varphi_2(s) + a_4\varphi_4(s)$

S=0.6 (small Shannon number)

Object (2 slit object)

a(s)

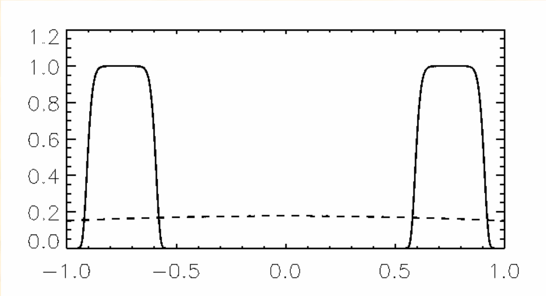
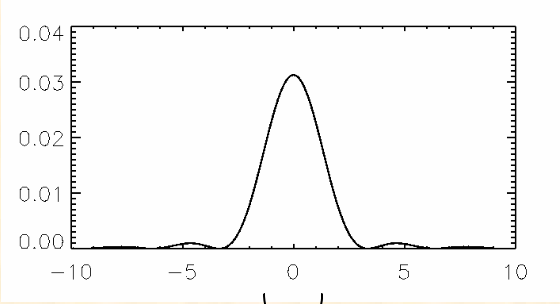


Image: **dominated by diffraction**

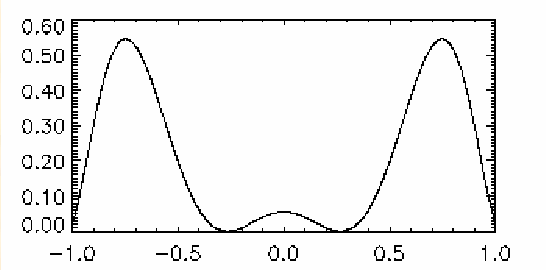
e(s)



Object support

Reconstructed Object

$a_{rec}(s)$



∝ Resolution length achieved with truncated reconstruction procedure

Few modes are sufficient to obtain significant superresolution!

=> Gain in resolution by a factor 10 !

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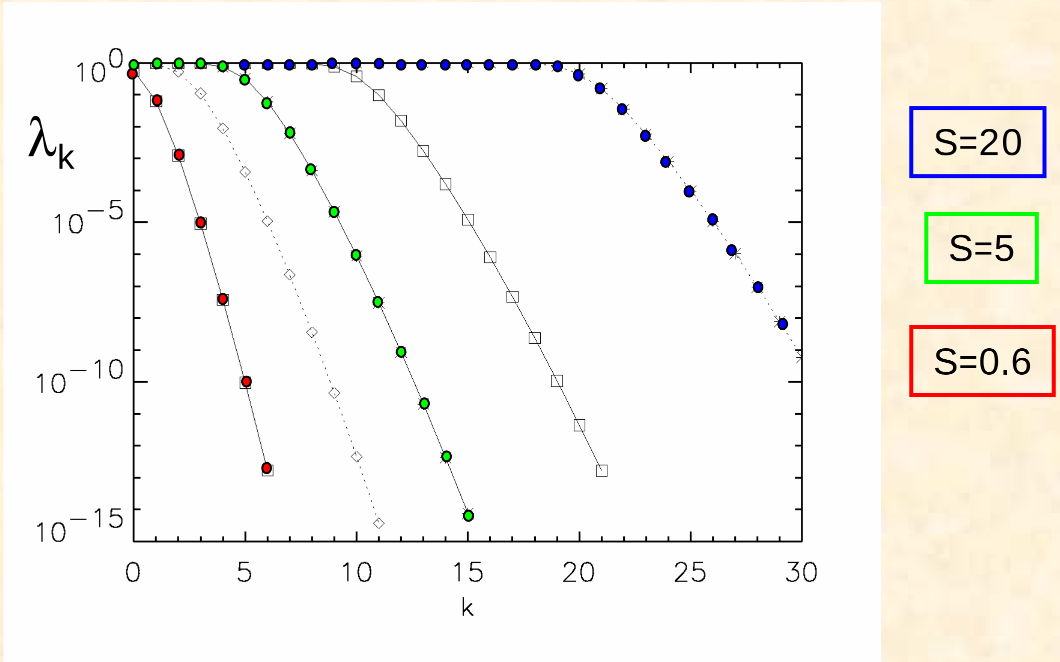
Difficulties (part I)

The eigenvalues λ_k decrease extremely rapidly to 0 for k>S.



- Difficulty to determine the prolate functions of high order.
- At large k, any small error in e_k (noise in the measurement) will be tremendously amplified by the reconstruction procedure.

$$a_k = \frac{e_k}{\sqrt{\lambda_k}}$$

This limits the number of functions in the expansion that can be used in practise.

Difficulties (part II)

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Determination of the expansion coefficients

$$a(s) = \sum_k a_k \varphi_k(s)$$

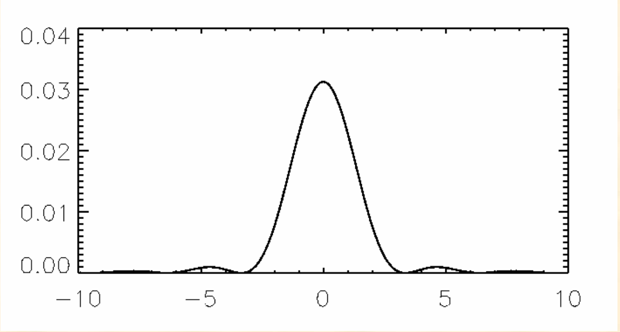
$$a_k = \frac{1}{\sqrt{\lambda_k}} \int_{-\infty}^{+\infty} ds' e(s') \Psi_k(s') \approx \frac{1}{\sqrt{\lambda_k}} \int_{-x_{max}}^{+x_{max}} ds' e(s') \Psi_k(s')$$

acquisition over a finite portion of the image plane

Error on the determination of a_k: $$\Delta a_k \approx \frac{2}{\sqrt{\lambda_k}} \int_{x_{max}}^{+\infty} ds' e(s') \Psi_k(s') \propto \frac{2}{\sqrt{\lambda_k}} \frac{1}{x_{max}}$$

Accurate determination of a_k only if $x_{max} >> \frac{1}{\sqrt{\lambda_k}}$

Intensity distribution in the image plane



Previous example S=0.6

$\lambda_4 \approx 3.7.10^{-8}$

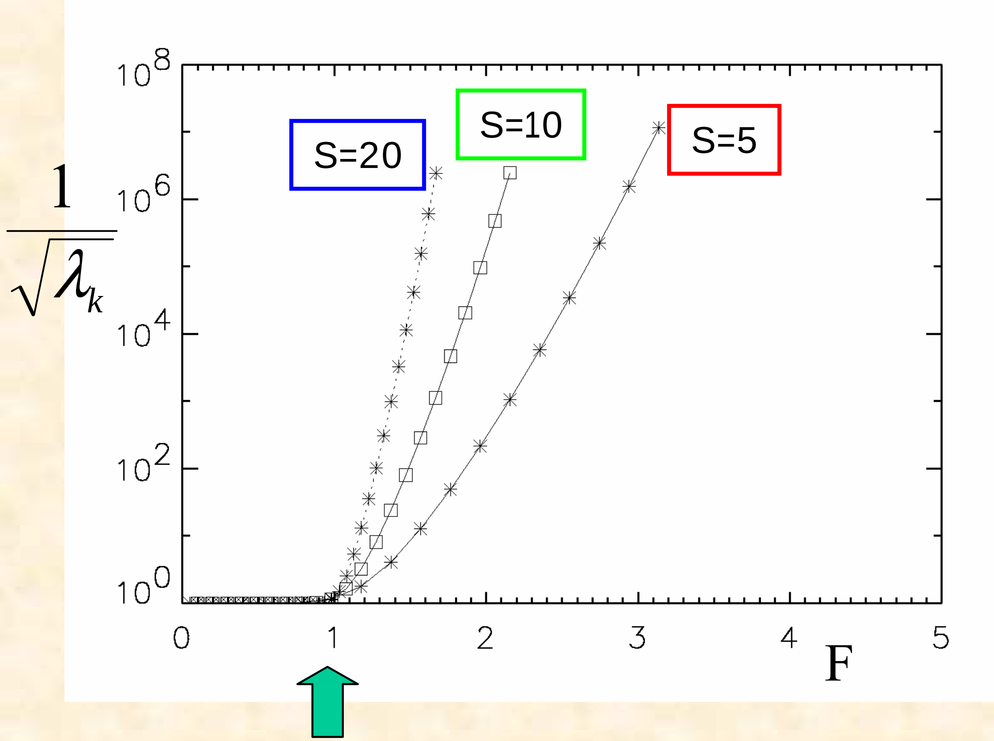
$1/\sqrt{\lambda_4} \approx 5000$

	exact	$x_{max} = 10$	$x_{max} = 10^3$	$x_{max} = 10^5$
a_4	-0.43	-179	-2.43	-0.454

Difficulties (part II)

What about large Shannon numbers?

$$x_{\max} >> \frac{1}{\sqrt{\lambda_k}}$$



Rayleigh criterium

F=Superresolution factor =l_{Rayleigh} /“new” resol. length

Superresolution? Yes, but only if one is able to collect the field over a HUGE portion of the image plane (!).

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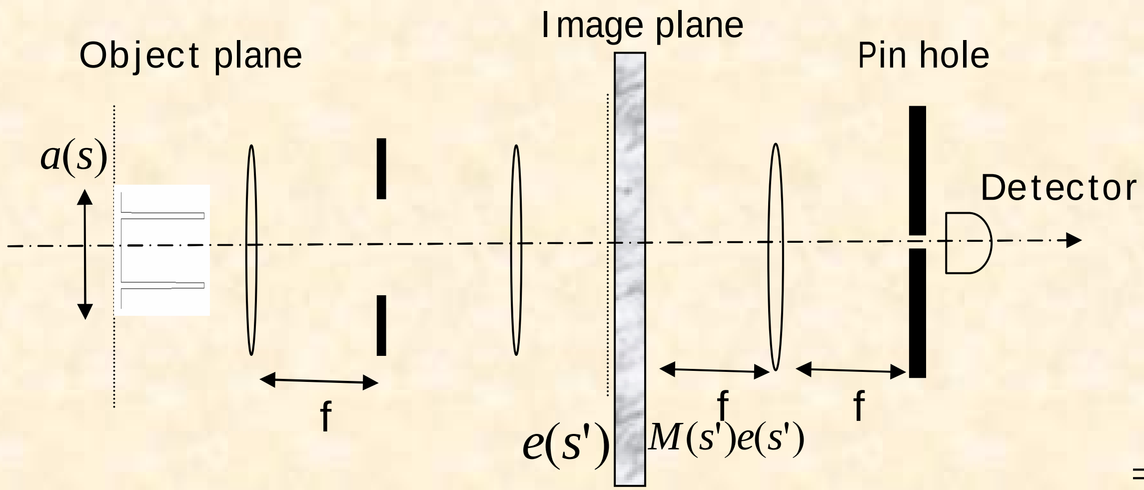
Alternative 1: Avoid pixellisation of image plane   Superresolution with optical masks

Principle :

$$a_{rec}(s_0) = \underbrace{\sum_{k=0}^{k_{\max}} \frac{1}{\sqrt{\lambda_k}} \varphi_k(s_0) \int_{-\infty}^{+\infty} ds' \psi_k(s') e(s')}_{\text{reconstruction formula}} = \int_{-\infty}^{+\infty} ds' e(s') \underbrace{\left\{ \sum_{k=0}^{k_{\max}} \frac{1}{\sqrt{\lambda_k}} \varphi_k(s_0) \psi_k(s') \right\}}_{M(s')}$$

$$\lim_{k_{\max}\to\infty} a_{rec}(s_0) = a(s_0)$$



Optical mask with amplit. transmit. $t = M(s')$

Multiplication and spatial integration can be done all-optically!

M. Bertero et all., Inverse Prob. **8**, 1-23 (1992)

Retrieval of the object at a given point

=> Scanning to retrieve the whole object

ADVANTAGES: all optical processing, a single detector

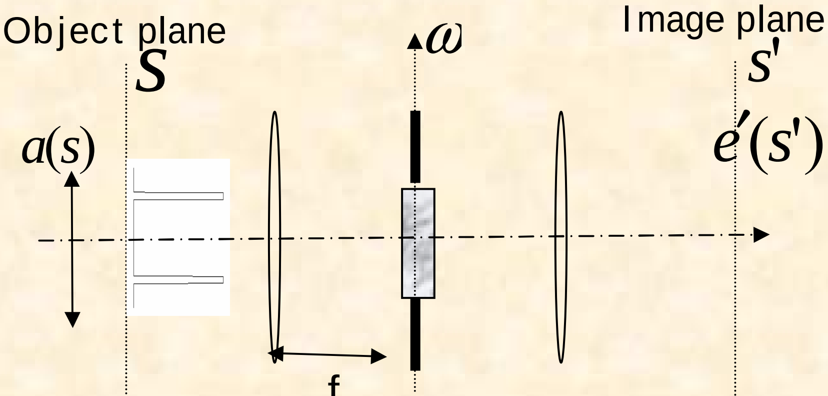
PROBLEM: Necessity of masks with unrealistic dimensions!!!

Alternative 2 Use of an intrapupil mask

Real time image extrapolation

B.R. Frieden, Opt. Acta 16, 795 (1969)

Principle: One can find a $P(\omega)$ such that the image **exactly coincides** with the object.



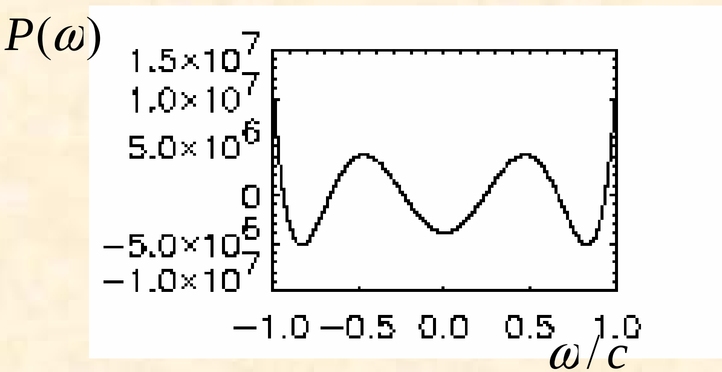
Optical mask with amplit. transmit. t=P(ω)

Example: $$P(\omega)=\sqrt{\frac{2\pi}{c}}\sum_{k=0}^{k_{\max}}\frac{(-1)^{k/2}}{\sqrt{\lambda_k}}\varphi_k(0)\varphi_k\left(\frac{\omega}{c}\right)$$

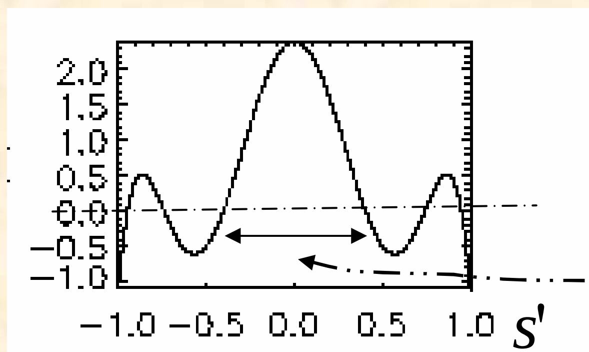
Then $$e'(s)=a(s)$$

for objects confined to [-X/2,+X/2]

Optical mask



Transfer function (Response to a Dirac peak)



Resol.length for this scheme

ADVANTAGES: equivalent to an extension of the pupil outwards in space

PROBLEM: Tremendous signal attenuation!!!

CONCLUSION AND OUTLOOK

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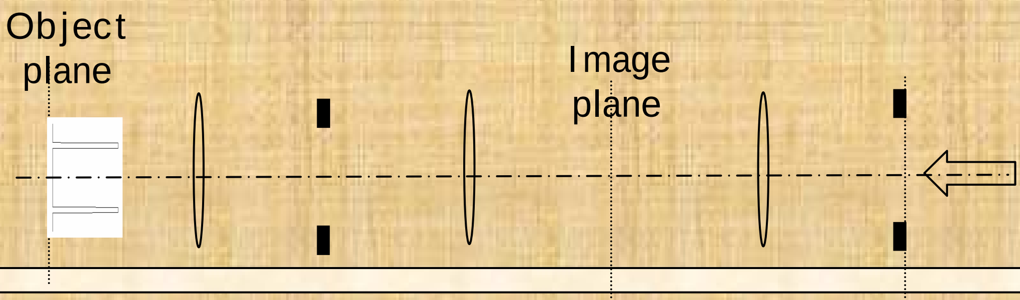
1)Well defined mathematical framework for the retrieval of diffraction distorted objects

2)Practical implementation:
* real space (image detection + processing) FAILS
* all-optically (using masks) FAILS

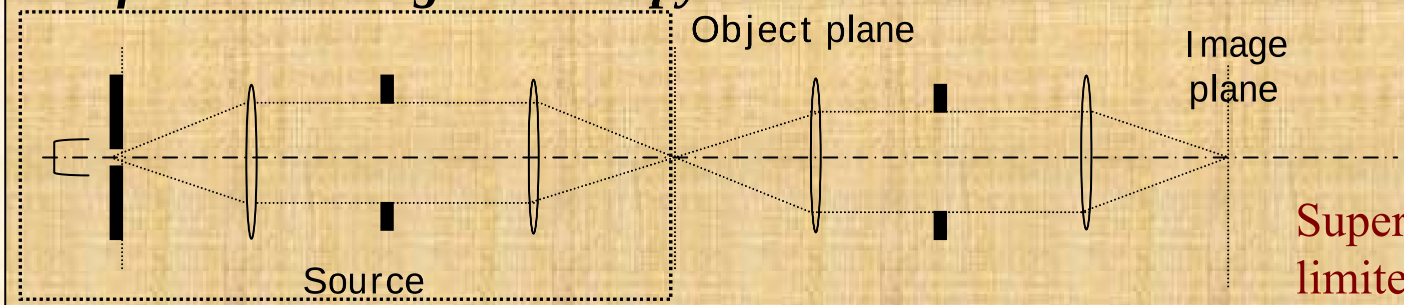
But...

Work in Fourier Space?



Truncated Fourier spectrum of object

Confocal scanning microscopy



It works!

Superresolution demonstrated but limited to 2 times the Rayleigh limit.

Object illumination by a focused beam

J.G. Walker, E.R. Pike et al, J.Opt. Soc. Am. A **10**, 59 (1993)