POLARIZATION COUPLING AND TRANSVERSE PATTERNS IN TYPE-II OPTICAL PARAMETRIC OSCILLATORS

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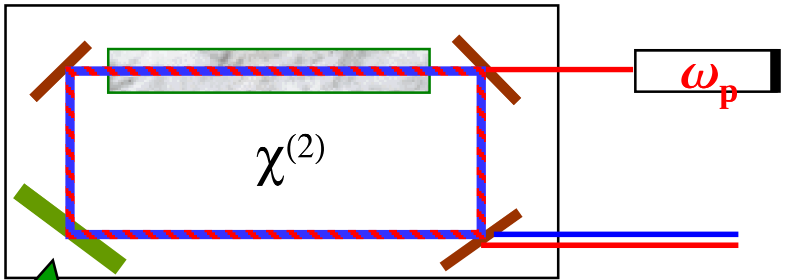
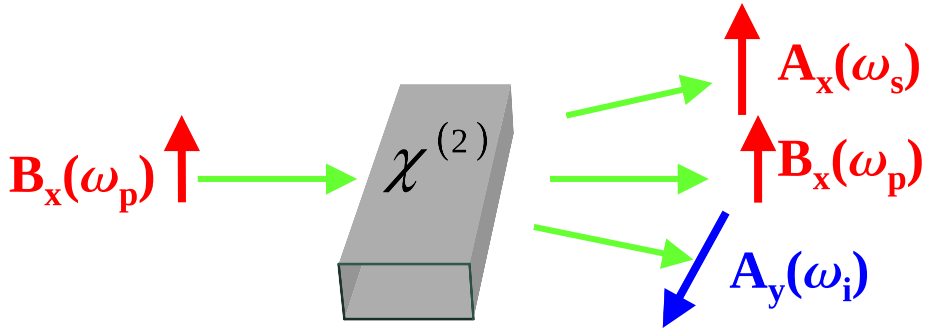
Opt. Lett. 25, 1454 (2000);
Phys. Rev. E, 64, 056231-1/15 (2001);
Phys. Rev. E (2002)

Polarization Coupling in Type-II OPO





Mirror: Birefringent (δ) / Dichroic (p) ; φ: angle between crystal and mirror axis

$$\partial_t B_x = \gamma'_x\left[-\left(1+i\Delta'_x\right)B_x + E_0 + ia'_x\nabla^2 B_x + 2iK_0 A_x A_y + c'_x B_y\right]$$

$$\partial_t B_y = \gamma'_y\left[-\left(1+i\Delta'_y\right)B_y + ia'_y\nabla^2 B_y + c'_y B_x\right]$$

$$\partial_t A_x = \gamma_x\left[-\left(1+i\Delta_x\right)A_x + ia_x\nabla^2 A_x + iK_0 A_y^* B_x + c_x A_y\right]$$

$$\partial_t A_y = \gamma_y\left[-\left(1+i\Delta_y\right)A_y + ia_y\nabla^2 A_y + iK_0 A_x^* B_x + c_y A_x\right]$$

$$c_{x,y} = \frac{(p+i\delta)\sin(2\varphi)}{T \pm p\cos(2\varphi)}$$

$$c_x = -c_y^* = \varepsilon_0 e^{i\theta}$$

Intracavity $\lambda/4$ plate

c = 0 : $A_x \rightarrow A_x\, e^{i\varphi},\; A_y \rightarrow A_y\, e^{-i\varphi} \longrightarrow$ **No relative phase preferred**

$$\Delta_e = (\gamma_x \Delta_x + \gamma_y \Delta_y)/(\gamma_x + \gamma_y)$$

- $\Delta_e > 0$: **Homogeneous solutions selected**
- $\Delta_e < 0$: **Phase patterns (Travelling Waves)**

c ≠ 0 : **Phase locked solutions.**

E. Manson, and N. Wong Opt. Lett. 23, 1733 (1998)
C. Fabre, E. Manson, and N. Wong Opt. Comm. 170, 299 (1999)

Transverse effects? : Phase Walls, Threshold lowering, Standing Waves

Type-II Optical Parametric Oscillators: homogeneous solutions ($\Delta_e > 0$)

A) No linear polarization coupling: $c = 0$

- *Threshold of instability of $A_x = A_y = 0$* →

$$|F_c|^2 = |E_0 K_0 / (1+i\,\Delta_{x'})|^2 = 1 + \Delta_e^2$$

$$q_0 = 0$$

$$\omega_0 = \gamma_x \gamma_y (\Delta_x - \Delta_y) / (\gamma_x + \gamma_y)$$

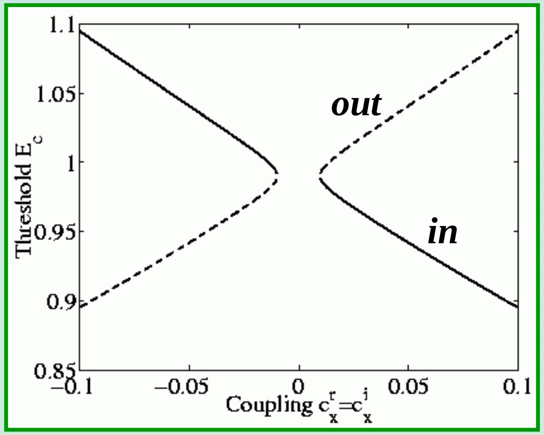
B) Linear polarization coupling: $c \neq 0$ $(\Delta_{x,y} > 0)$ $c_x = c_y = c_r + i\,c_i$ $(\varphi = 45°)$

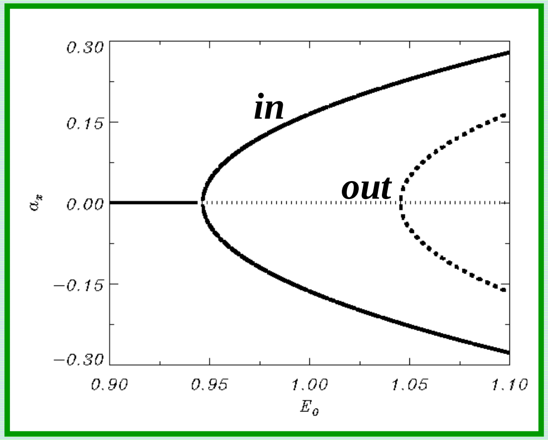
Stationary uniform phase locked solution $A_x = a_x \exp(i\,\psi_x)$, $A_y = a_y \exp(i\,\psi_y)$

- $|E| > E_c$
- $4\ (c_r + c_i \Delta_x)\ (c_r + c_i \Delta_y) \geq (\Delta_y - \Delta_x)^2$

- $\sin(\psi_y - \psi_x) = (\Delta_x - \Delta_y) / [\,4\,(c_r + c_i \Delta_x)(c_r + c_i \Delta_y)]^{1/2}$
- $a_y^2 = \Gamma\, a_x^2$ $\Gamma = (c_r + c_i \Delta_x) / (c_r + c_i \Delta_y)$

THRESHOLD





- *Well within the locked regime: "In" ($\psi_y - \psi_x \cong 0$) and "Out" ($\psi_y - \psi_x \cong \pi$) phase solutions of different threshold*
- *Two equivalent "in" homogeneous solutions (+/-) $A^+_{x,y} = -A^-_{x,y}$*
- *Domain walls between A^+ y A^-*

Polarization of Phase Locked States and Domain Walls

A) Homogeneous phase locked solutions

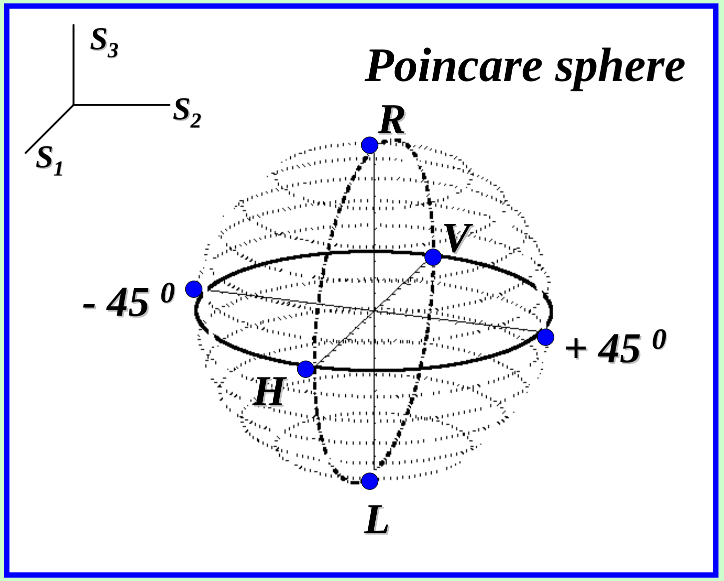
Polarization state determined by locked value of $\psi_y - \psi_x$

Stokes parameters

$S_1 = (1-\Gamma) / (1+\Gamma)$

$S_2 = [2\Gamma^{1/2} / (1+\Gamma)] \cos(\psi_y - \psi_x)$

$S_3 = [-2\Gamma^{1/2} / (1+\Gamma)] \sin(\psi_y - \psi_x)$



$\Gamma = (c_r + c_i \Delta_x) / (c_r + c_i \Delta_y)$

$a_y^2 = \Gamma a_x^2$

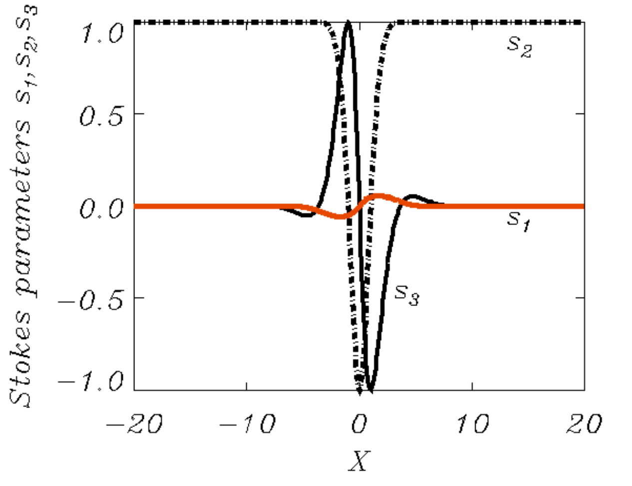
B) Phase polarization Bloch domain walls

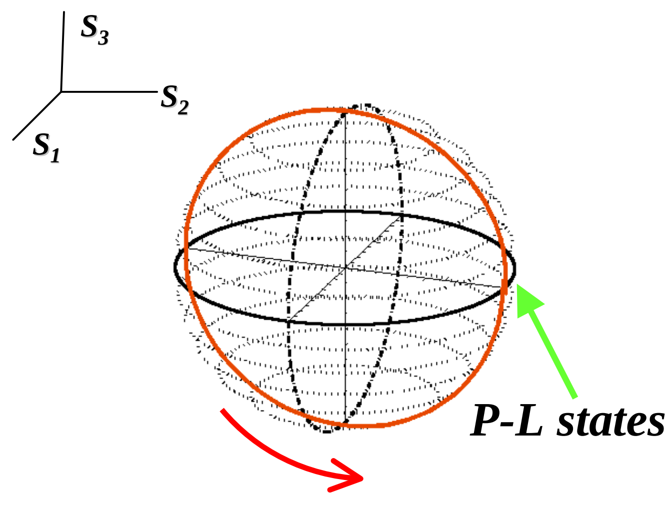
Deeply in the P-L regime

$\Delta_x = \Delta_y, \Gamma = 1, \quad \psi_y = \psi_x$

$(S_1, S_2, S_3) = (0, 1, 0)$

LPS at: $\theta = \pi/4 \quad (-3\pi/4)$

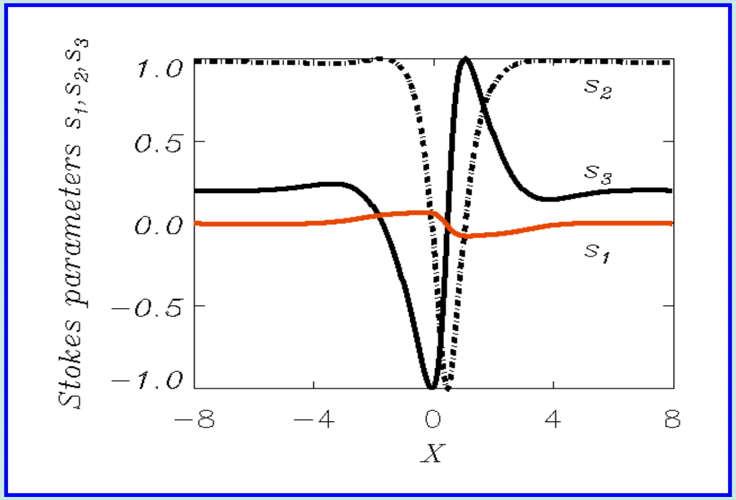


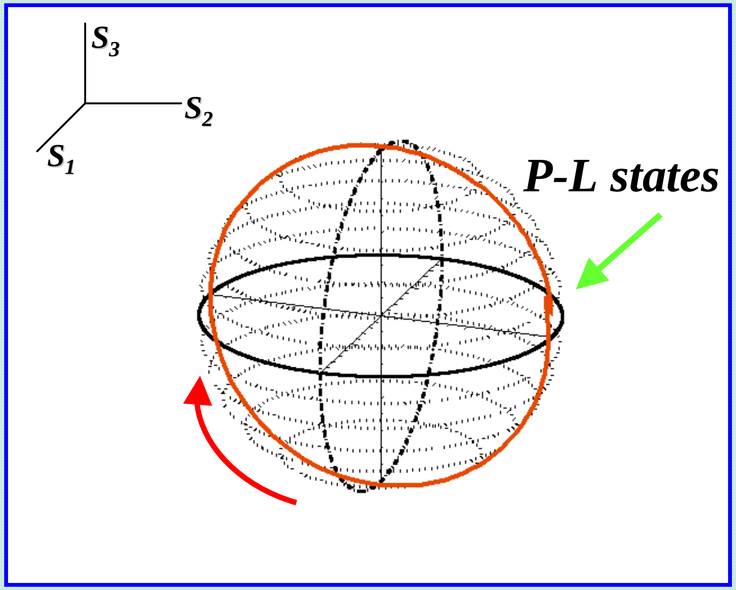


Polarization of Domain Walls

- *Phase Locked state*

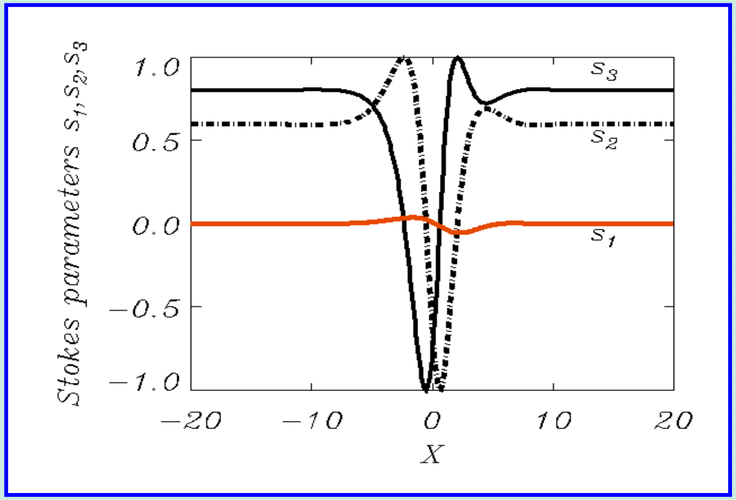
$\Delta_x \neq \Delta_y$, $\Gamma = 1$, $\sin(\psi_y - \psi_x) = 0.2$

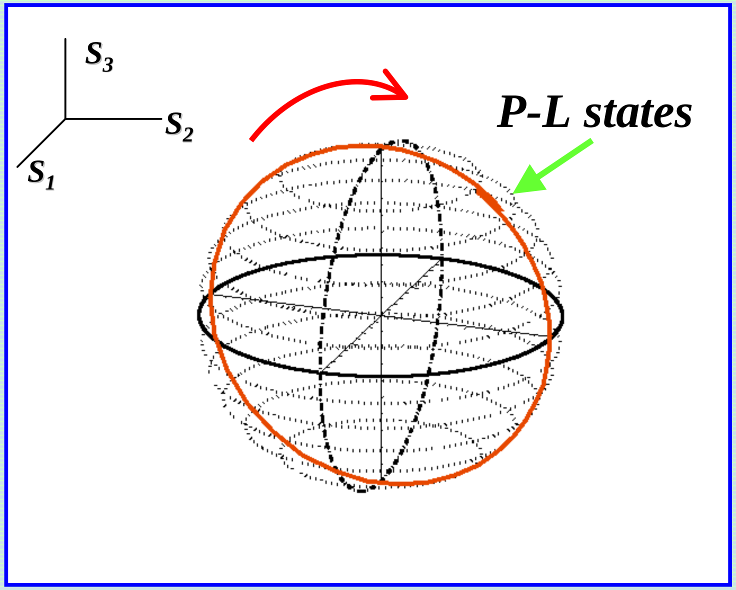




- *Near the P-L transition*

$\Delta_x \neq \Delta_y$, $\Gamma = 1$, $\sin(\psi_y - \psi_x) = 0.8$

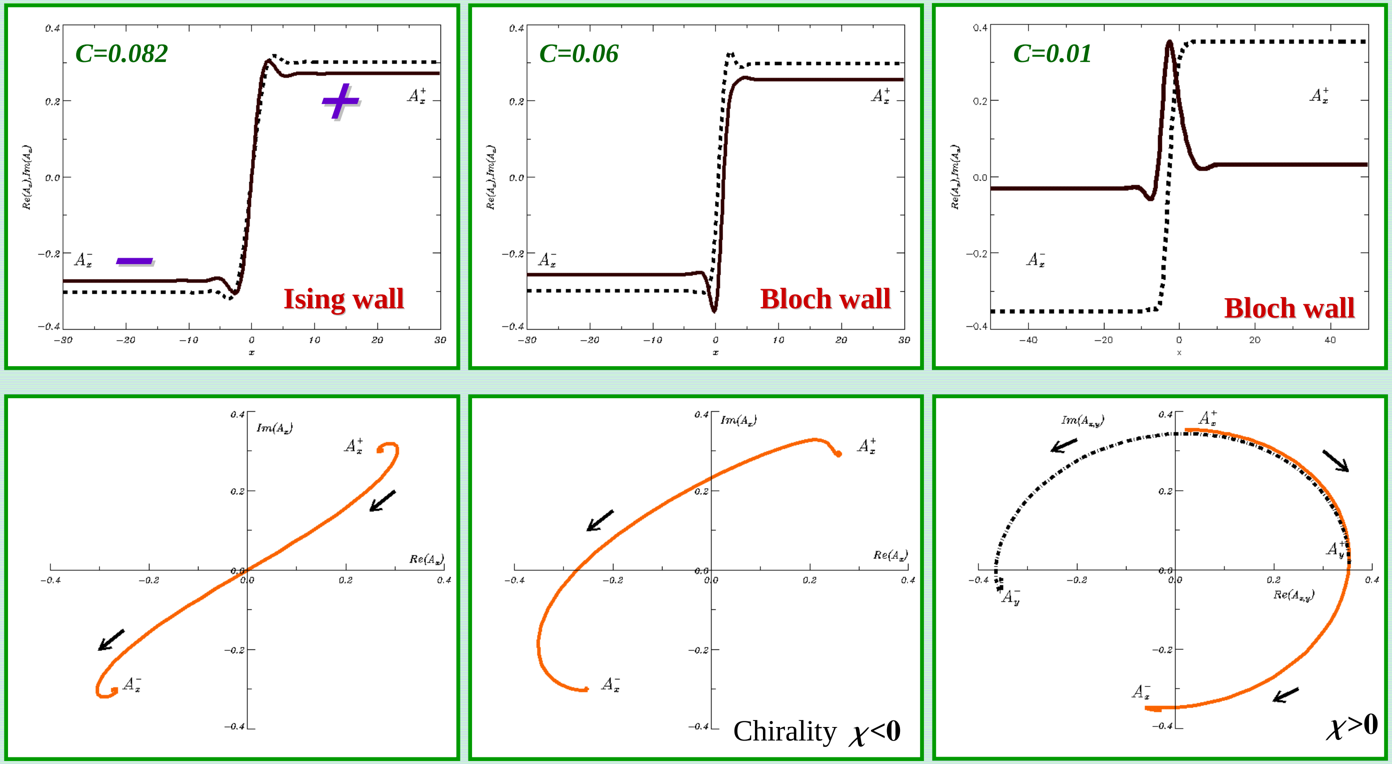




d= 1 Domain Walls

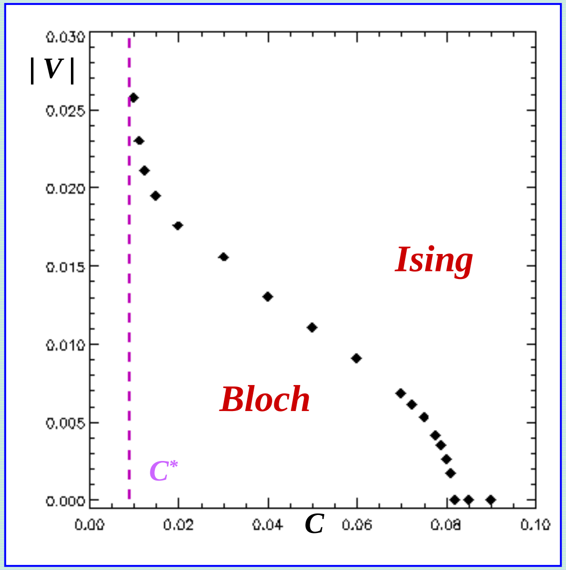
G. Izús, M. Santagiustina, and M. San Miguel, Opt. Lett. 23, 1167 (2000)
G. Izús, M. San Miguel, and M. Santagiustina, submitted (2001)

Bloch-Ising transition controlled by polarization coupling

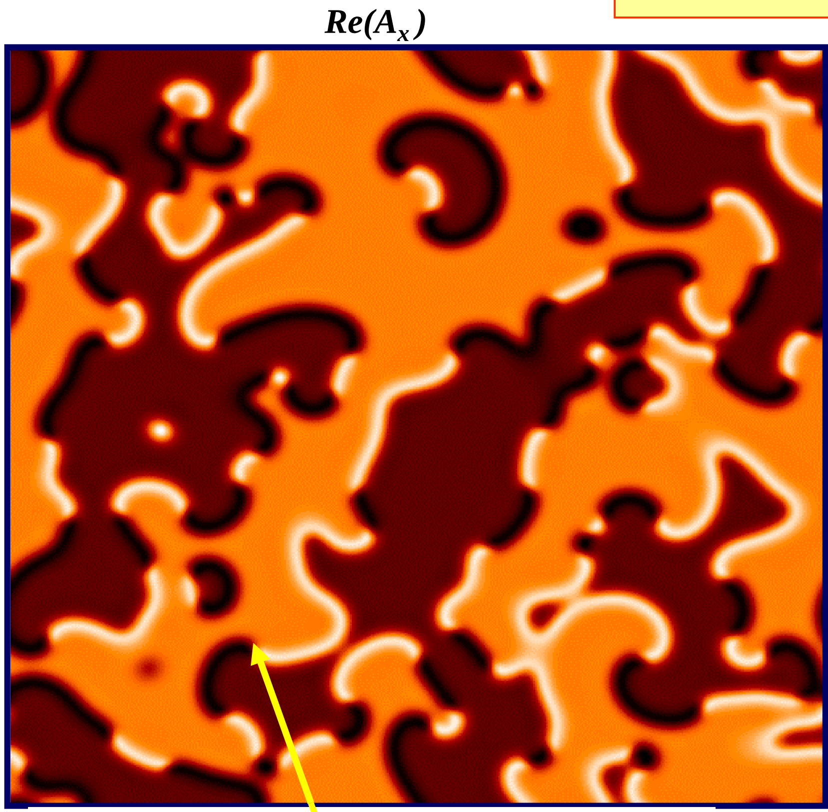


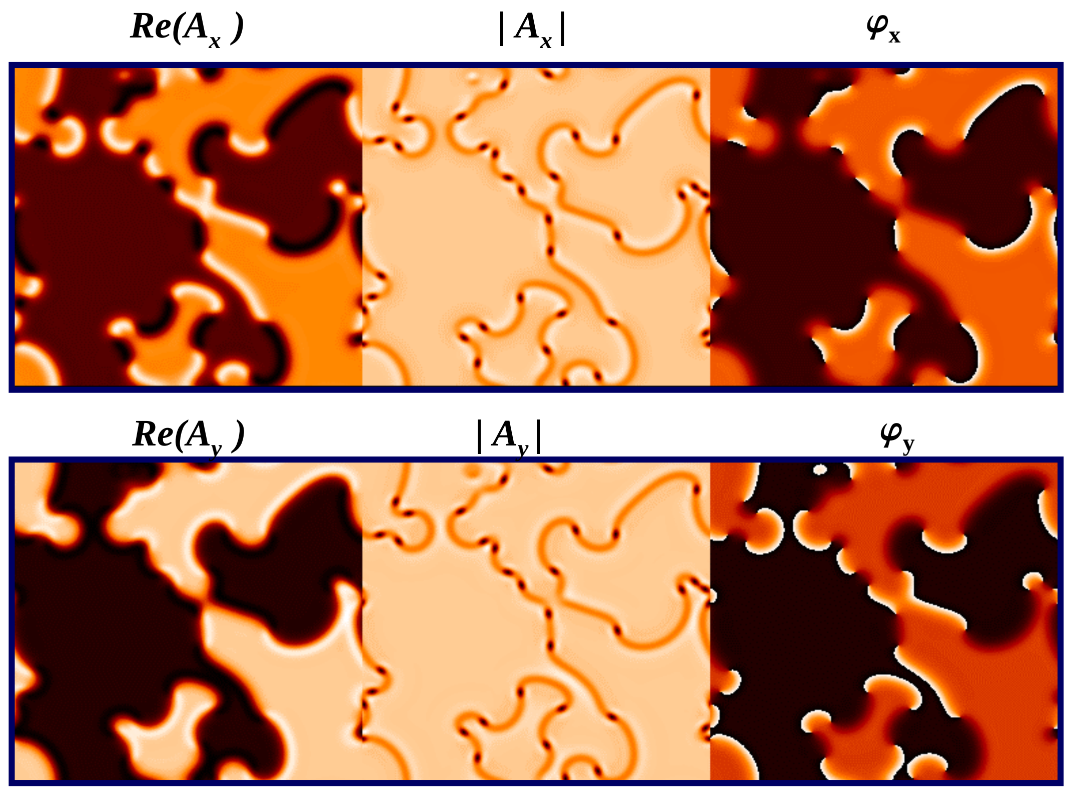
Two dynamical regimes

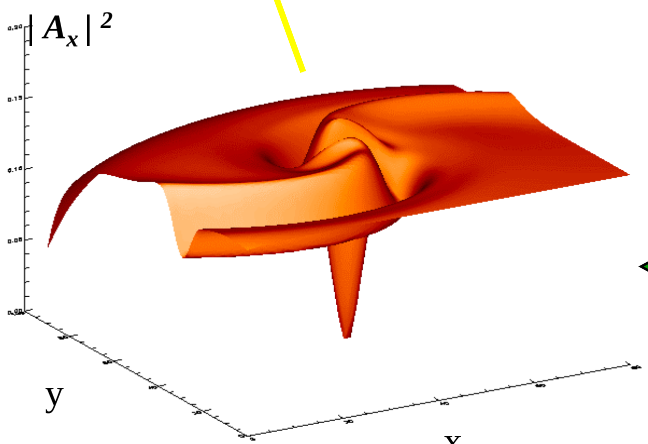
- $\gamma_x\ \Delta_x = \gamma_y\ \Delta_y$ static Bloch wall
- $\gamma_x\ \Delta_x \neq \gamma_y\ \Delta_y$ Bloch walls move in opposite directions for opposite χ



$d=2$ BLOCH WALLS





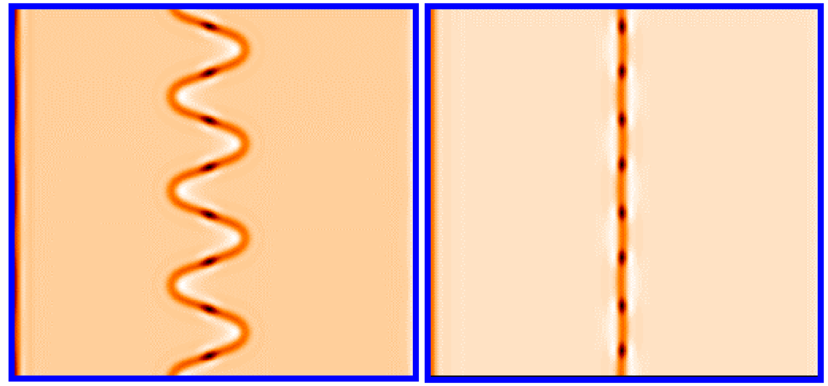


Defect: Point of change of chirality on the wall

BLOCH WALL DYNAMICS (I)

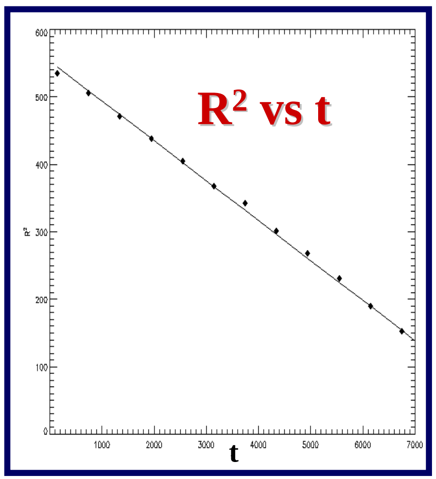
$\gamma_x \Delta_x = \gamma_y \Delta_y$ ***Flat wall stable***

$|A_x|$



t=0 t=2000

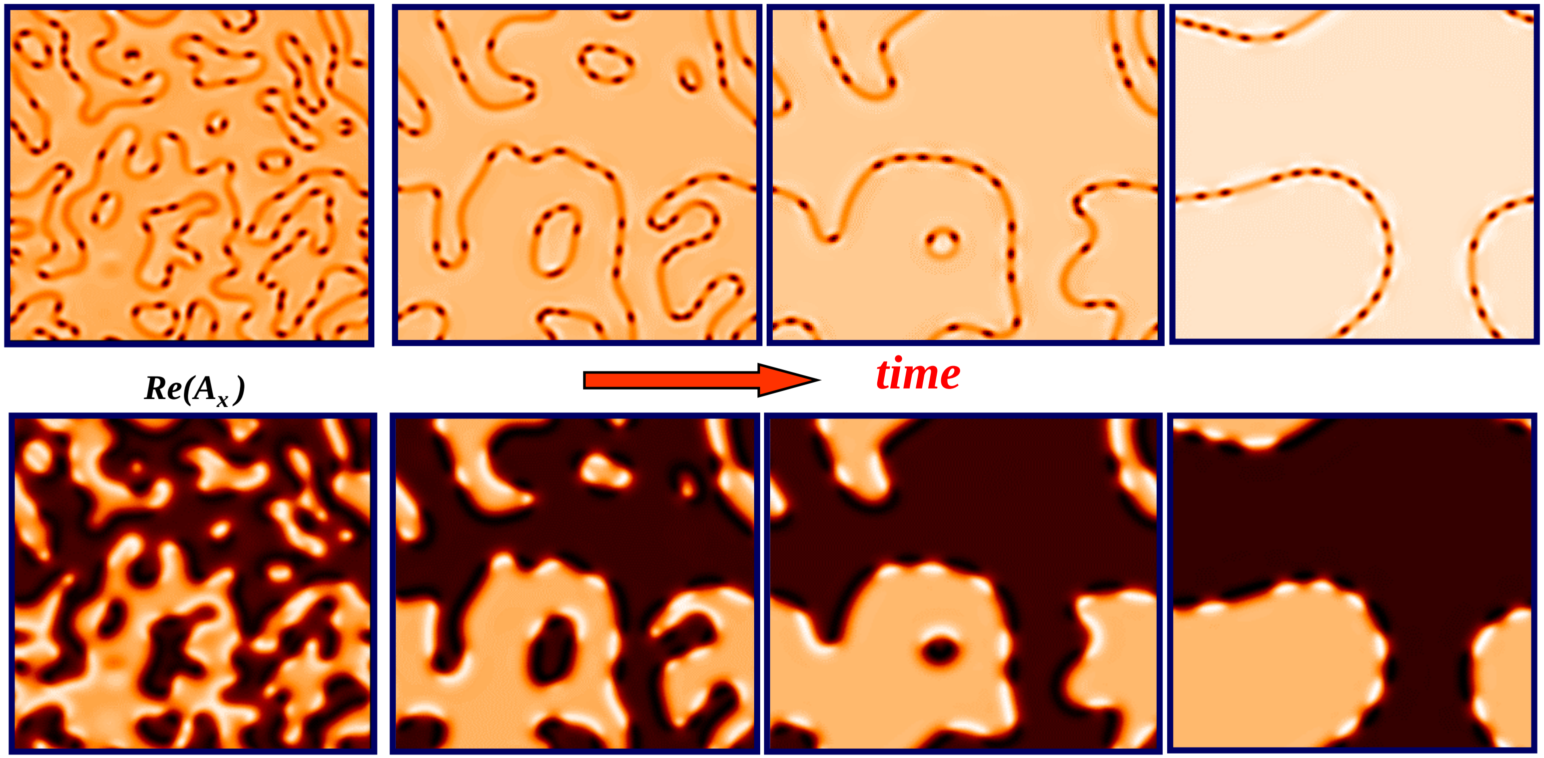
DomainGrowth controlled by curvature



$|A_x|$

$Re(A_x)$

time



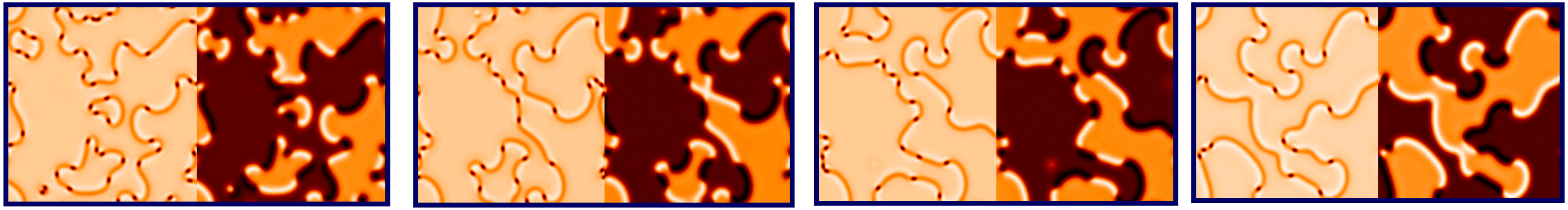
t=100 t=350 t=750 t=2900

BLOCH WALL DYNAMICS (II)

$\gamma_x \Delta_x \neq \gamma_y \Delta_y$ *Walls with different chirality move in opposite directions*

Persistent Dynamics

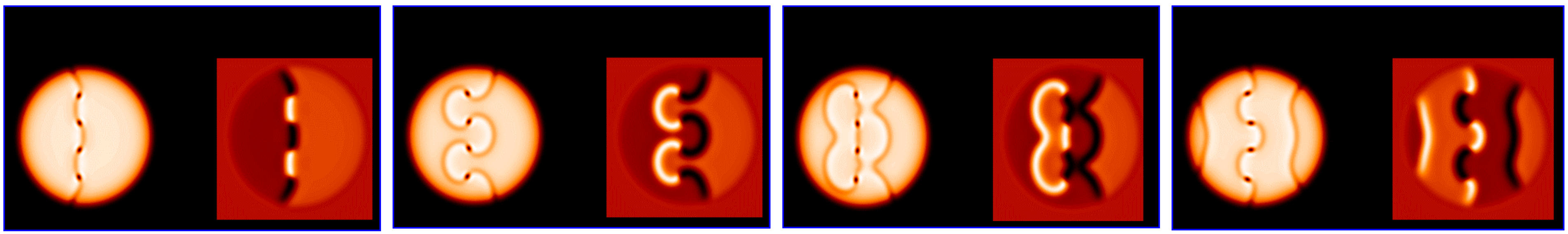
$|A_x|$ $Re(A_x)$



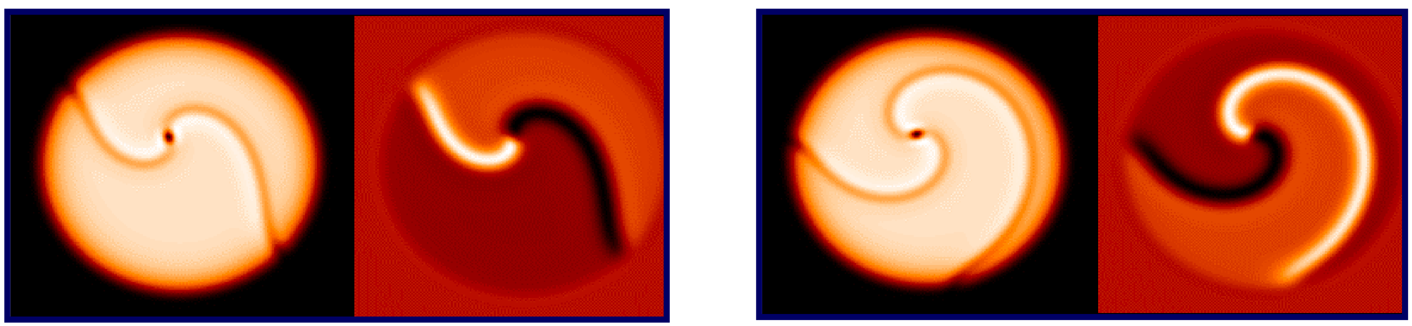
t=500 t=800 t=1100 t=2200

Front emission from array of defects

$|A_x|$ $Re(A_x)$

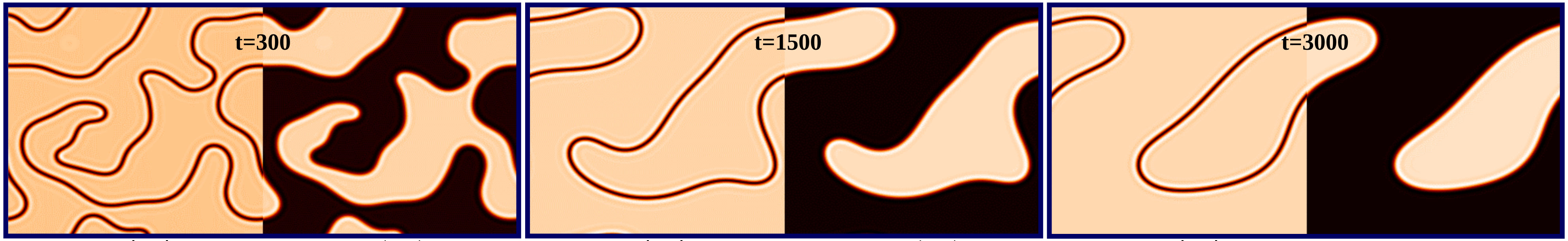


Two armed rotating spiral centered in defect



ISING WALL DYNAMICS

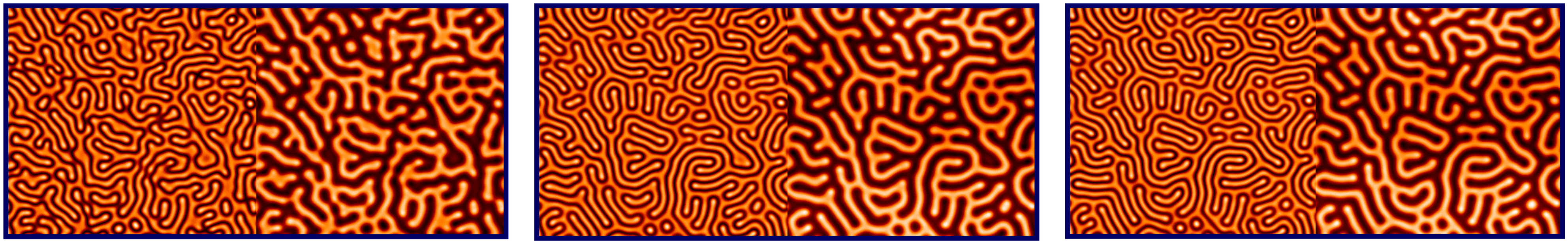
Coarsening near the Bloch-Ising transition



$|A_x|$ $Re(A_x)$ $|A_x|$ $Re(A_x)$ $|A_x|$ $Re(A_x)$

Labyrinthine Patterns

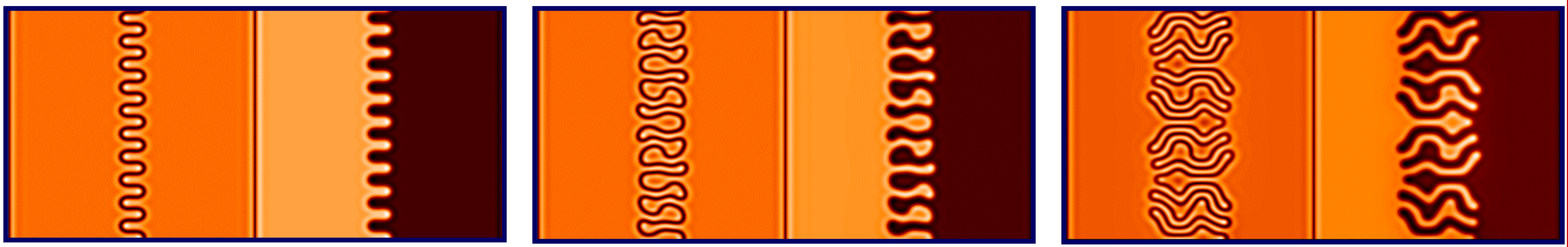
$|A_x|$ $Re(A_x)$ $|A_x|$ $Re(A_x)$



t=75 t=750 t=1500

Modulational Instability

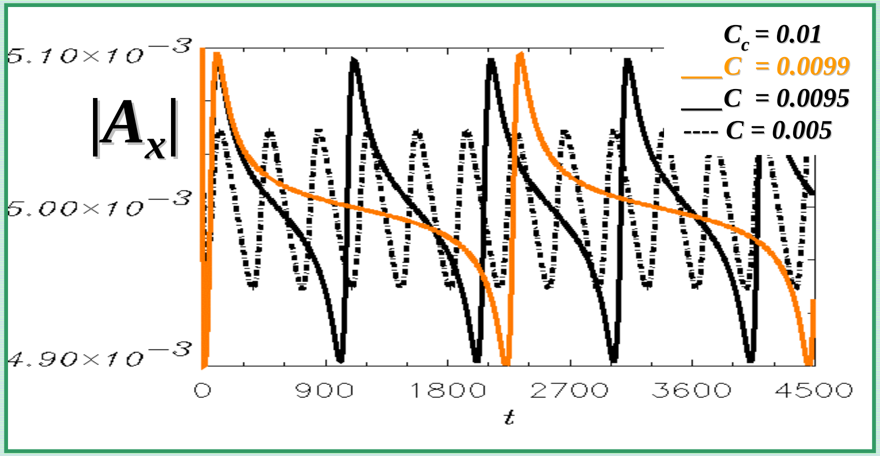
time



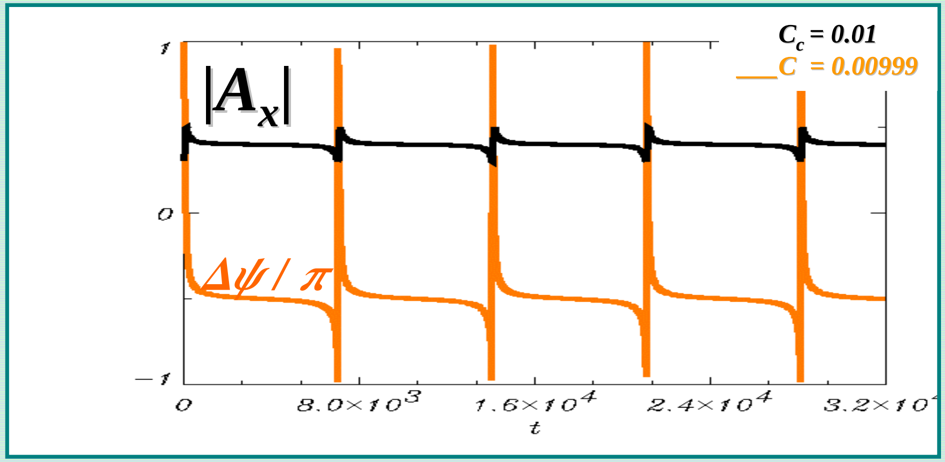
Time Oscillatory Bloch Domain Walls

Outside the phase-locked regime

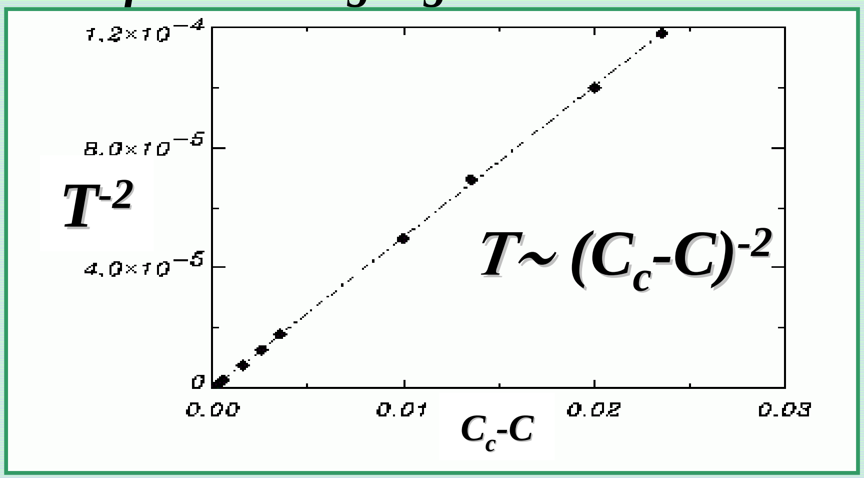
•*Time oscillatory uniform state*



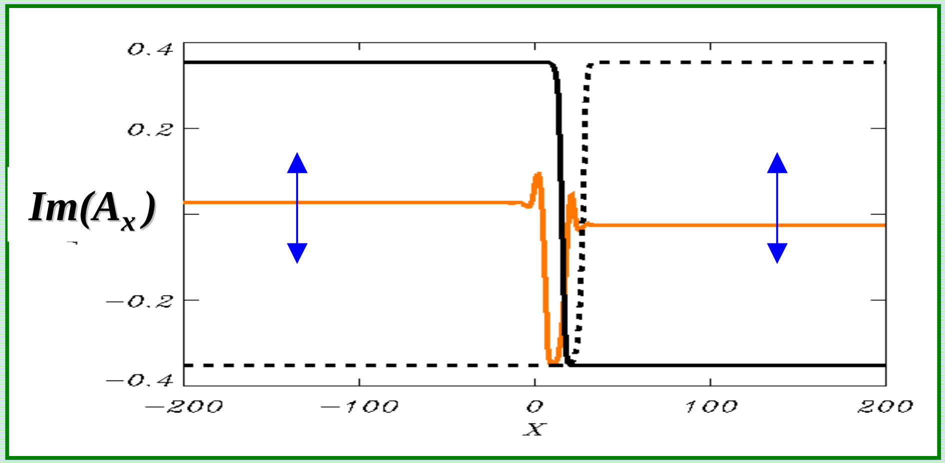
•*Relative phase $\Delta\psi$ oscillates in time*



•*The period of time oscillation diverges in the limit of the locking regime*



•*Oscillatory Bloch domain walls: Dynamics ?*



Type-II Optical Parametric Oscillators: Pattern Formation ($\Delta_e < 0$)

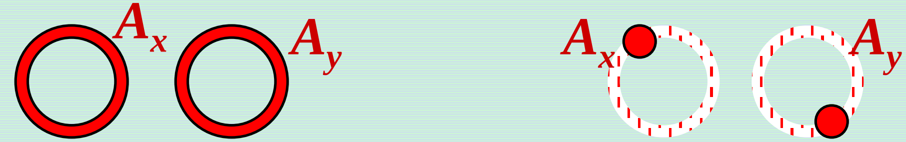
A) No linear polarization coupling: c = 0

- *Threshold of instability of $A_x = A_y = 0$*

$$|F_c|^2 = 1 \;,\; q_0^2 = -\Delta_e / (\gamma_x a_x + \gamma_y a_y),$$
$$\omega_0 = \gamma_x \gamma_y [\Delta_x - \Delta_y + q_0^2 (a_x - a_y)] / (\gamma_x + \gamma_y)$$

Far Field Solutions

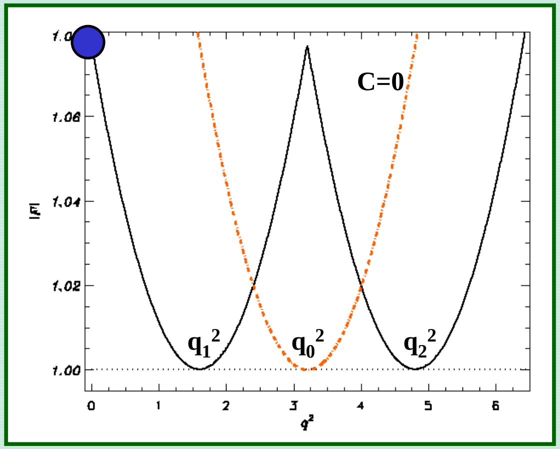
Below Threshold — *Above Threshold*

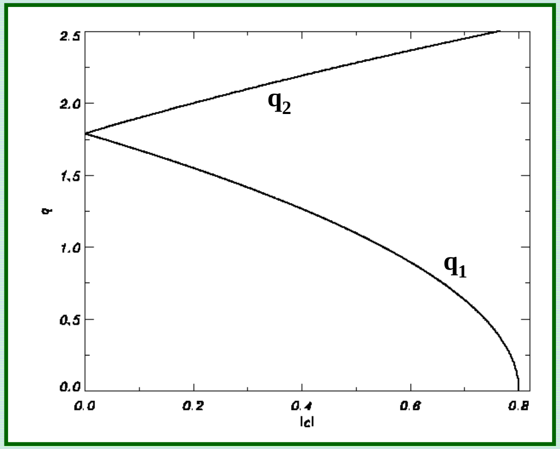


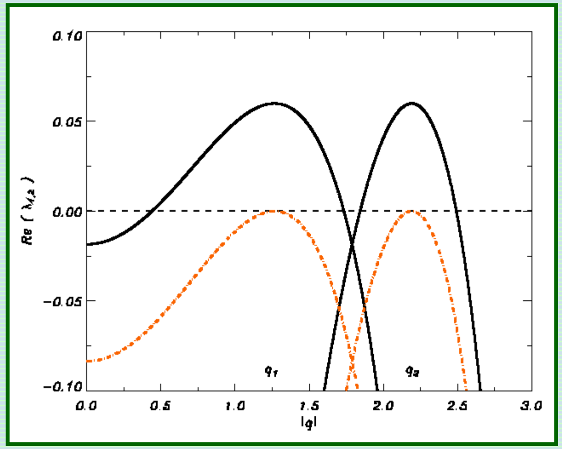
B) Linear polarization coupling $c_x = -c_y^* = c$ (real)

THRESHOLD for symmetric coefficients $\Delta_x = \Delta_y$, $\gamma_x = \gamma_y$, $a_x = a_y$

$$|F_c|^2 = 1 \;,\; \omega_0 = 0$$
$$q^2_{1,2} = (-\Delta \pm c) / a$$





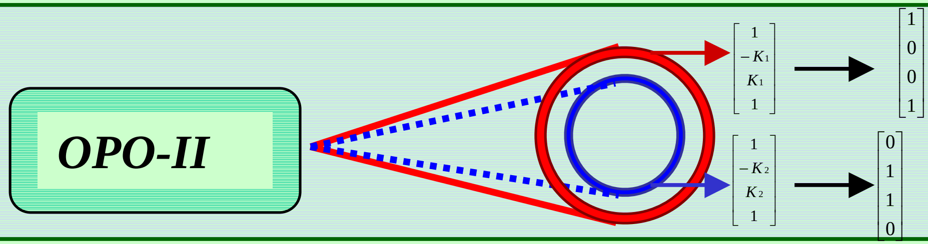


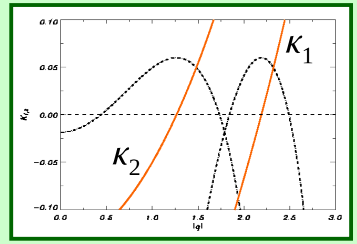
- *Lower threshold for pattern formation: two competing modes with equal growth rate*
- ● *Uniform phase locked solutions (q=0)* (Fabre et. al. Opt. Comm. 170, 299 (1999))

Type-II OPO, $\Delta_e < 0$: Symmetric coefficients

Linear stability analysis

$$\frac{\partial}{\partial t}\begin{bmatrix}\Re(A_x)\\ \Im(A_x)\\ \Re(A_y)\\ \Im(A_y)\end{bmatrix} = \hat{L}(c)\begin{bmatrix}\Re(A_x)\\ \Im(A_x)\\ \Re(A_y)\\ \Im(A_y)\end{bmatrix}$$

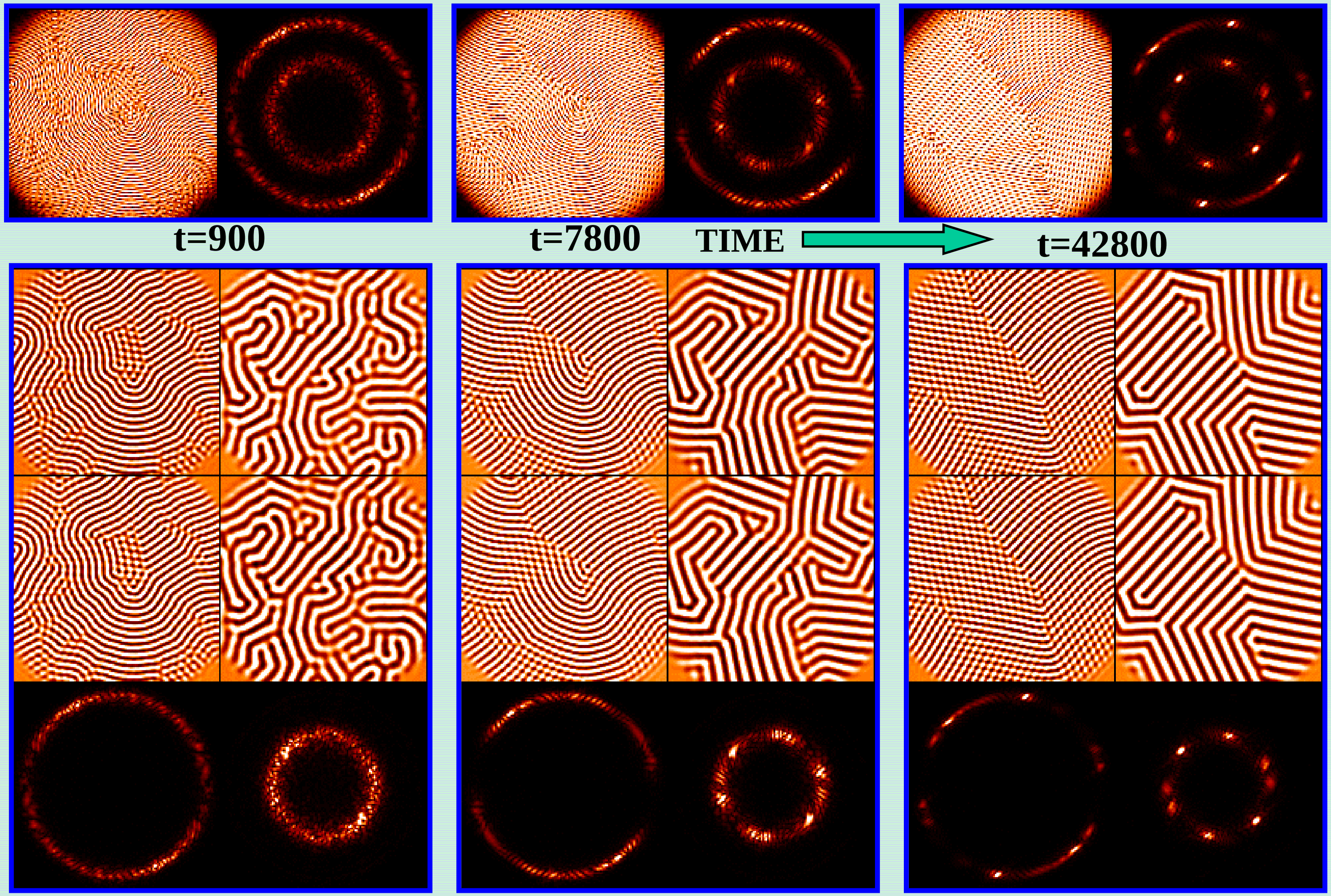




$|A_x|$ / FF

$R_e(A_x)$ / $R_e(A_y)$ →

$I_m(A_y)$ / $I_m(A_x)$ →



Circular Polarized Intensity Patterns

Circular polarized states: $A_{\pm} = (A_X \pm i\, A_Y)\ /\sqrt{2}$

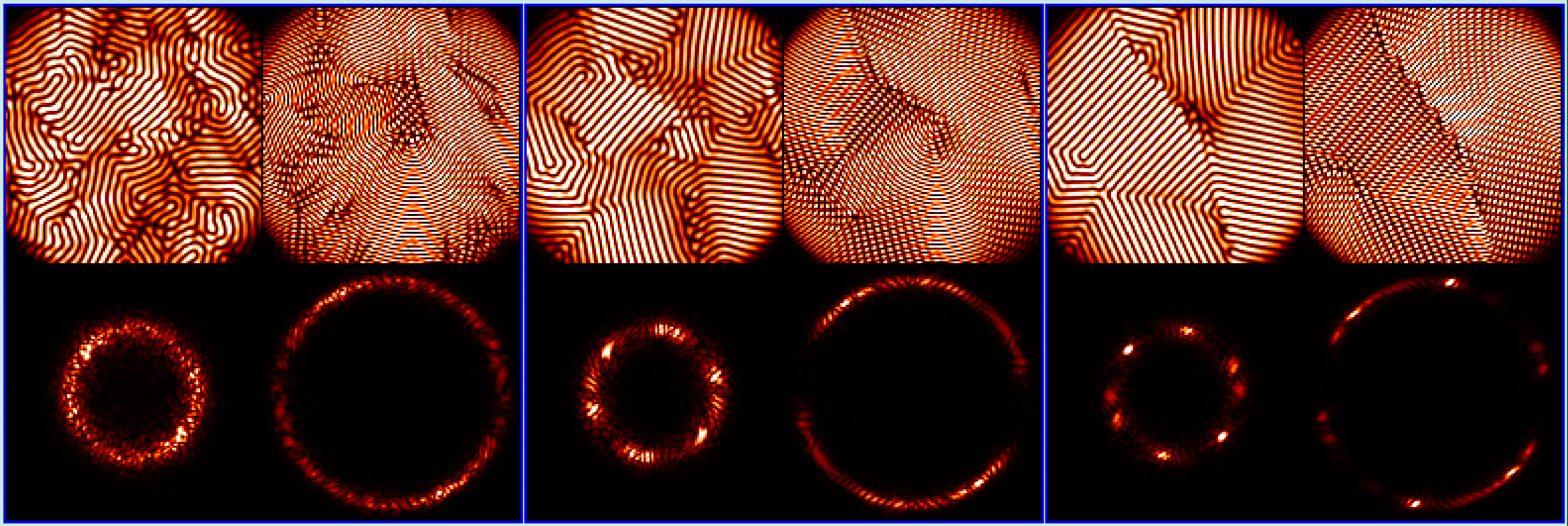
$$A_+ = (2i/\sqrt{2})\ Im\ (A_X)$$

$$A_- = (2/\sqrt{2})\ \ Re\ (A_X)$$

•CP Intensity patterns with different wavelength

t=900 **t=7800** **t=42800**

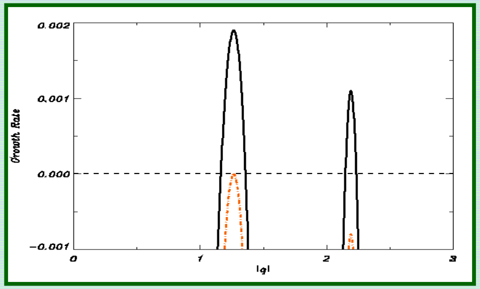
$|A_+|$ $|A_-|$



$FF(A_+)$ $FF(A_-)$

Type-II OPO, $\Delta_e < 0$, $\Delta_x \neq \Delta_y$, $\gamma_x \neq \gamma_y$, $\alpha_x \neq \alpha_y$

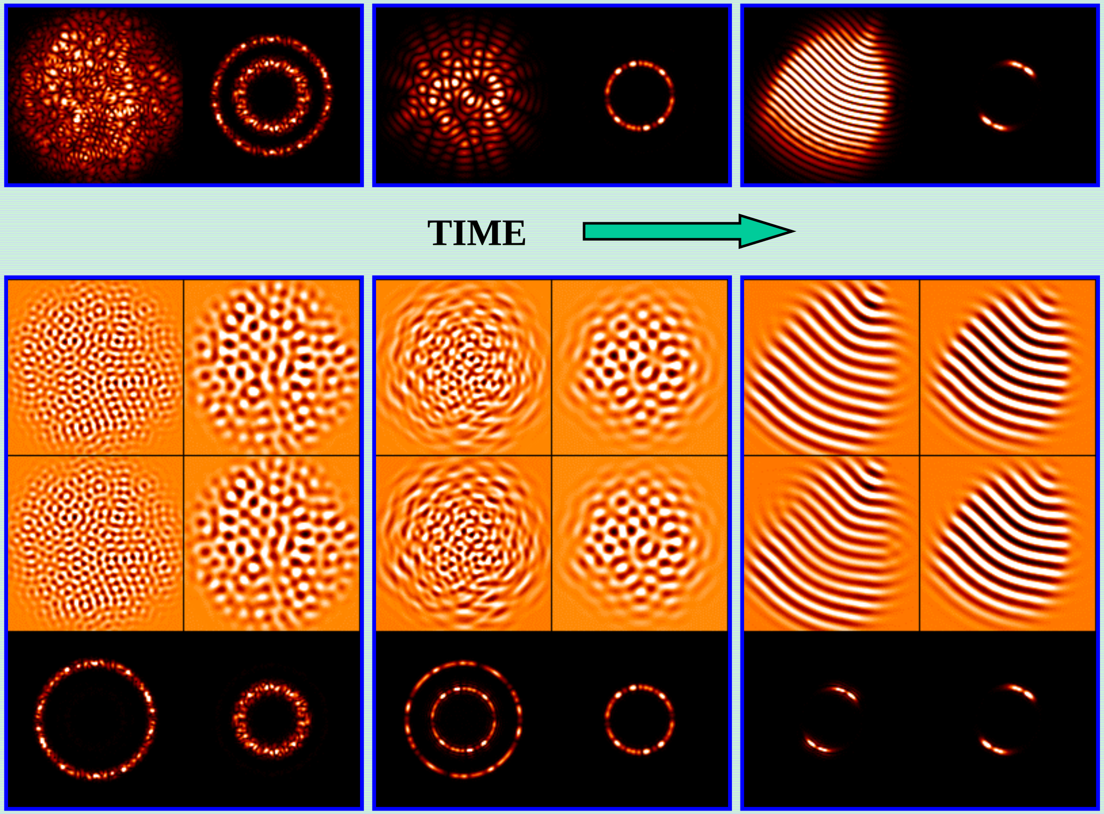
Near threshold



$|A_x|$ / *FF*

$R_e(A_x)$ / $R_e(A_y)$

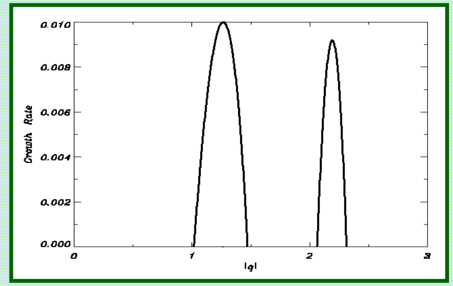
$I_m(A_y)$ / $I_m(A_x)$

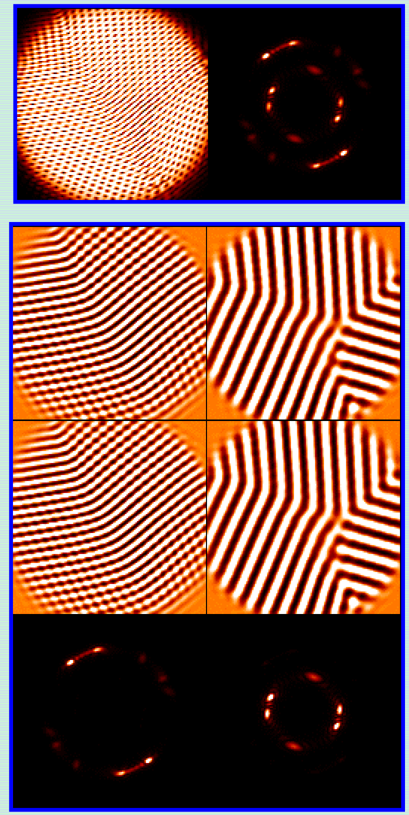


TIME

t=100 **t=3200** **t=17600**

Far from threshold





t=15600

Amplitude Equations

Close to the instability threshold: Amplitude equations for the critical modes

- $A_1 = A_x(q_1)$
- $A_2 = A_y(q_2)$

$$\partial_t A_1 = \mu_1 A_1 - 4K_0^2 A_1 \left(|A_1|^2 + |A_2|^2 \right) - \frac{2K_0^2}{1 + 4i\alpha' \mathbf{q}_1^2} A_1 |A_1|^2$$

$$\partial_t A_2 = \mu_2 A_2 - 4K_0^2 A_2 \left(|A_1|^2 + |A_2|^2 \right) - \frac{2K_0^2}{1 + 4i\alpha' \mathbf{q}_2^2} A_2 |A_2|^2$$

$$\mu_{1,2} = \mu_{1,2}(F)$$

$$\eta_2(c', \Delta', \alpha', \mathbf{q}_{1,2})$$

• For F such that $\mu_1 > \mu_2 / (1+\eta_2)$

Stable steady state

$$|A_1|^2 \neq 0$$
$$|A_2|^2 = 0$$

Only patterns with wavectors q_1 are stable (near threshold)

• For F such that $\mu_1 < \mu_2 / (1+\eta_2)$

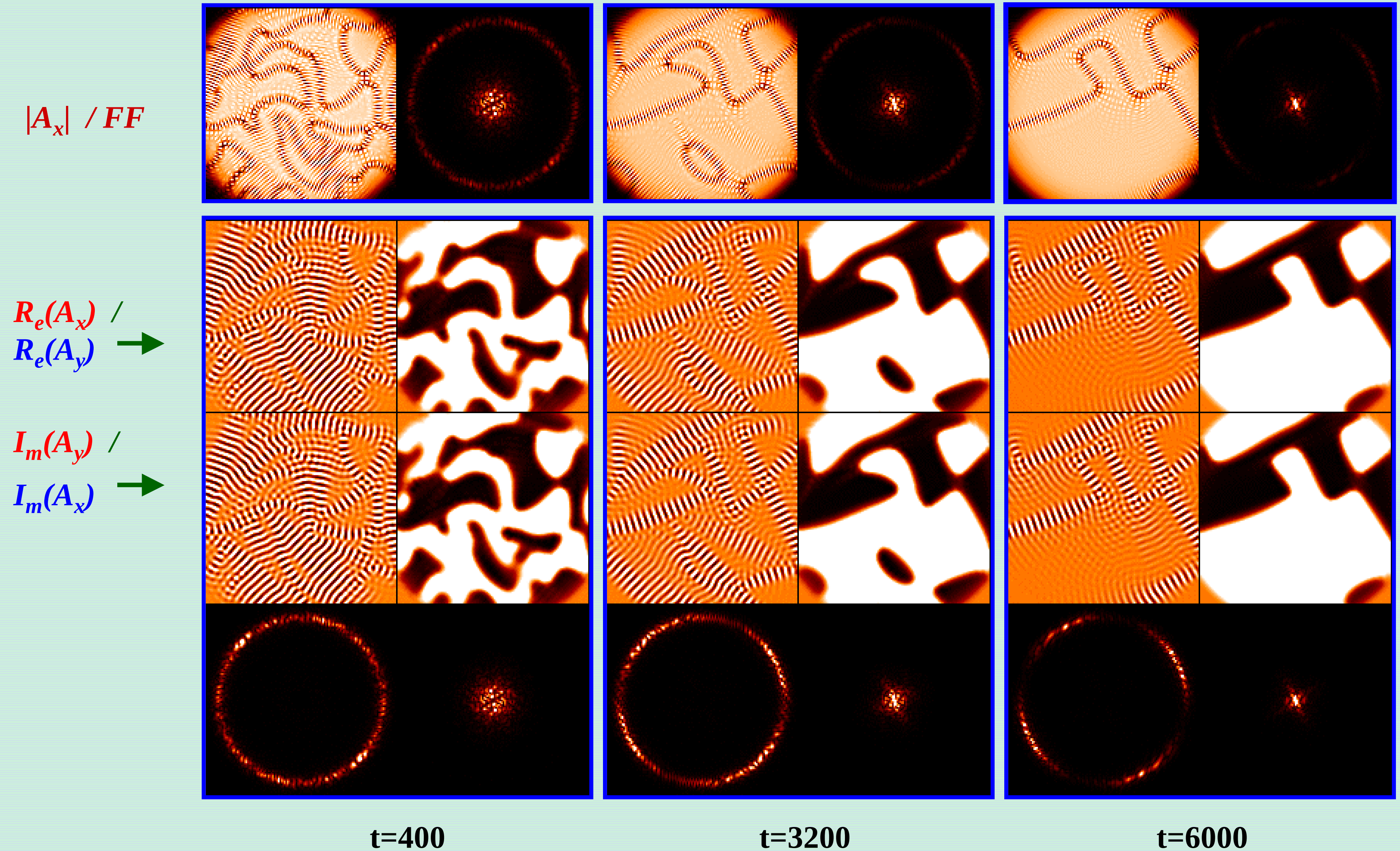
Stable steady state

$$|A_1|^2 \neq |A_2|^2 \neq 0$$

Patterns with both wavectors $q_{1,2}$ are stable (far from threshold)

Type-II OPO, $\Delta_e < 0$, $q = 0$ mode selected

Symmetric coefficients



SUMMARY

Birefringent / Dichroic coupling breaks relative phase invariance in Type II OPO : PHASE LOCKED states

$\Delta_e > 0$:

Inside the phase-locking regime

- **Phase polarization domain walls**
- **Bloch Ising transition controlled by polarization coupling**
- **Core of the Bloch wall of orthogonal linear polarization**
- **Point defects on Bloch walls at points where chirality changes sign**

Outside the phase-locking regime

- **Oscillatory Bloch Domain Walls**

$\Delta_e < 0$:

- **Pattern formation**
- **Standing waves for A_x and A_y**
- **Two competing modes in each linear polarization component**
- **Nonlinear mode selection: - mode coexistence far from threshold**
  - one mode selection close to threshold for asymmetric coefficients