



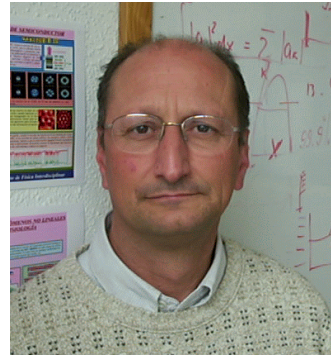
Growth Laws and Stable Droplets in Nonlinear Optics



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Phys. Rev. Lett. 87, 194101 (2001)
IEEE J.Quantum Electr. (2002)



Vectorial Kerr Resonator

$$\partial_t E_{\pm} = -(1 + i\eta\theta)E_{\pm} + i\nabla^2 E_{\pm} + E_0 + \frac{1}{4}i\eta \left[|E_{\pm}|^2 + \beta|E_{\mp}|^2 \right] E_{\pm}$$

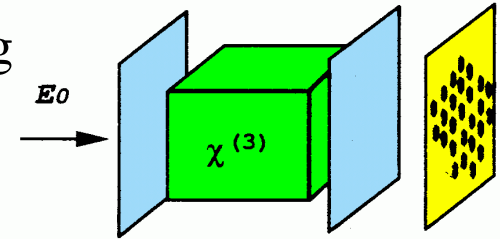
E_{\pm} : circularly polarized components

E_0 : \hat{x} -linearly polarized input

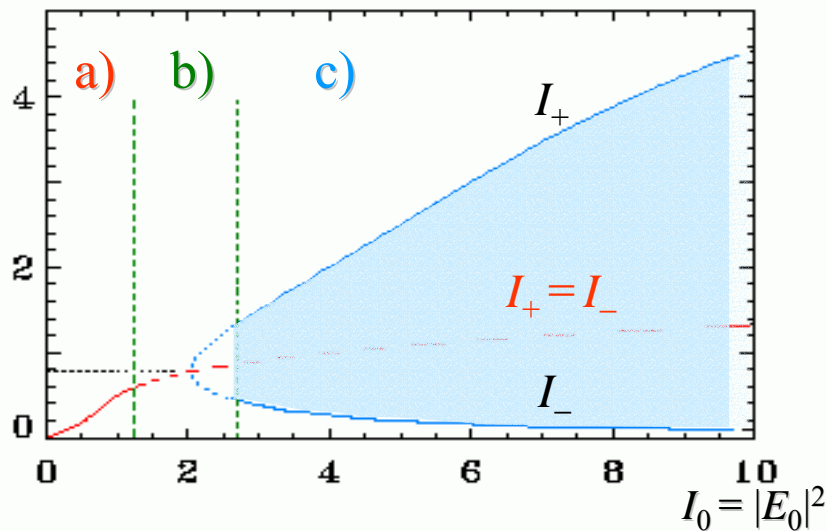
$\eta = -1$: self-defocusing

$\theta = 1$: detuning

$\beta = 7$: coupling



Homogeneous solutions:



- a) Symmetric homogeneous solution
- b) Stripe patterns \hat{y} -polarized
- c) Bistability between homogeneous elliptically polarized solutions

Stable Ising walls

Geddes et al. *Opt. Comm.* 111, 623 (1994)

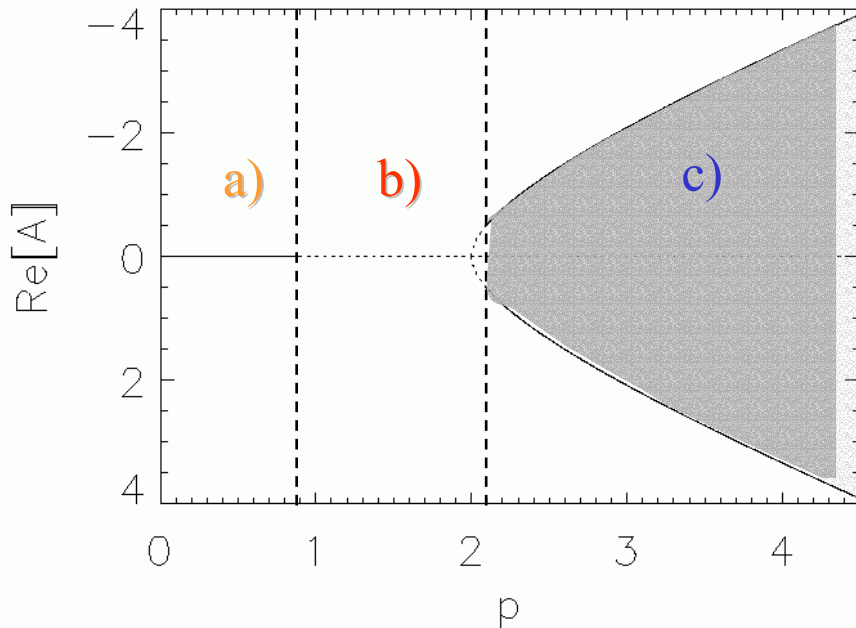
Hoyuelos et al. *Phys. Rev. E* 58, 2992 (1998)

$$\partial_t A = (\mu + i\nu)A + (1 + i\alpha)\nabla^2 A - (1 + i\beta)|A|^2 A + pA^*$$

$\alpha \sim \nu \sim p \gg \mu, \beta \rightarrow$ Pattern forming transition

P. Coulet and K. Emilsson, Physica D, 61, 119 (1992)

Homogeneous solutions:



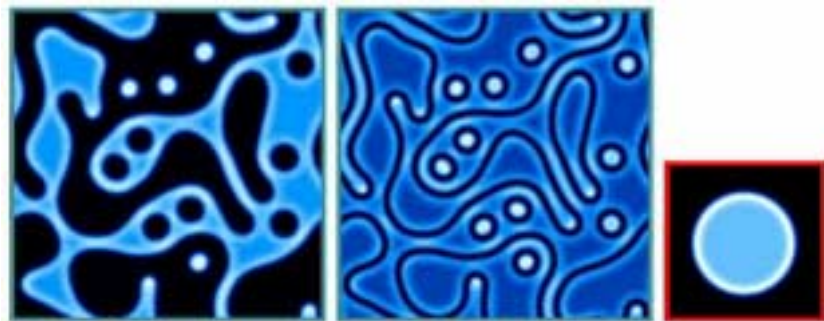
- a) Zero solution
- b) Stripe and hexagonal Patterns
- c) Bistability between homogeneous solutions (frequency locked solutions).

Stable Ising walls.

$$\alpha = \nu = 2, \mu = \beta = 0$$

Dynamics of Polarization Domain Walls in 2D

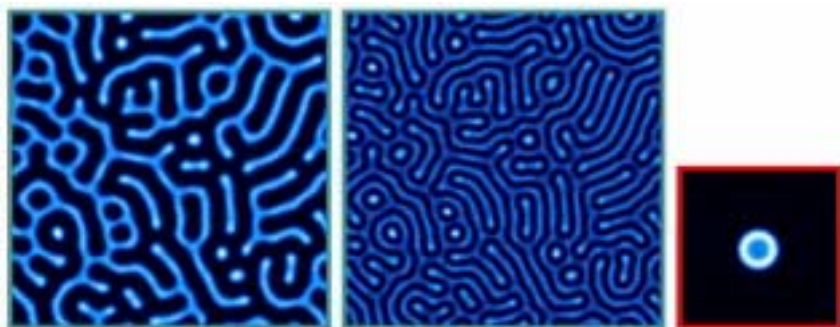
Localized Structures



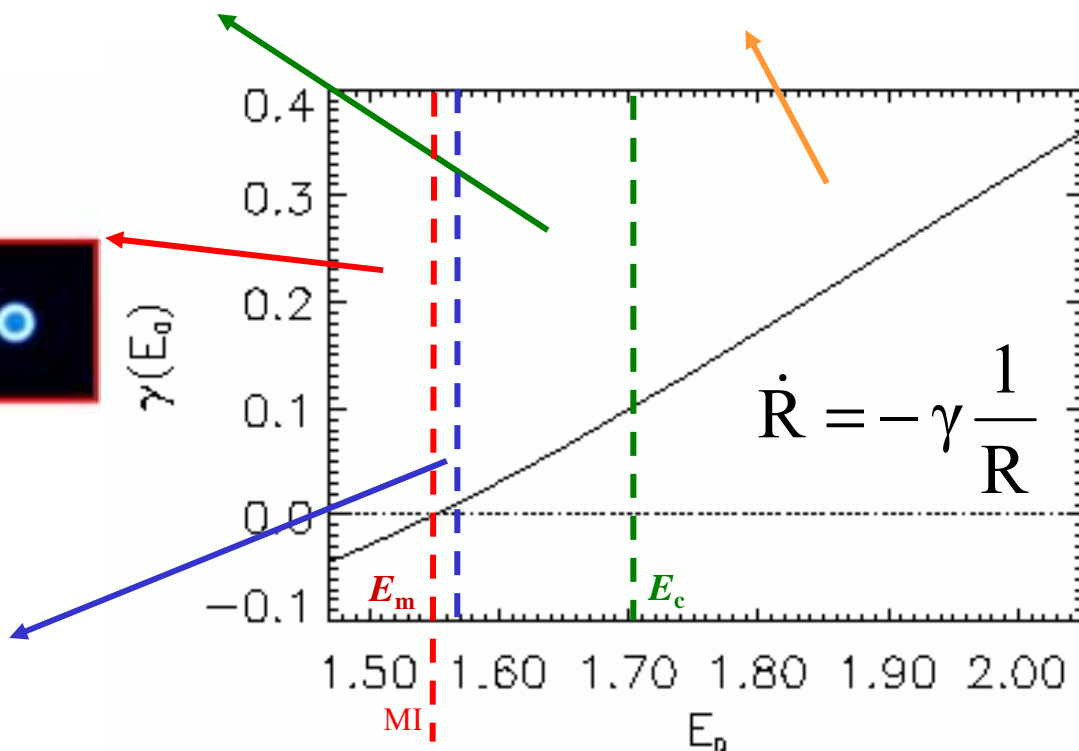
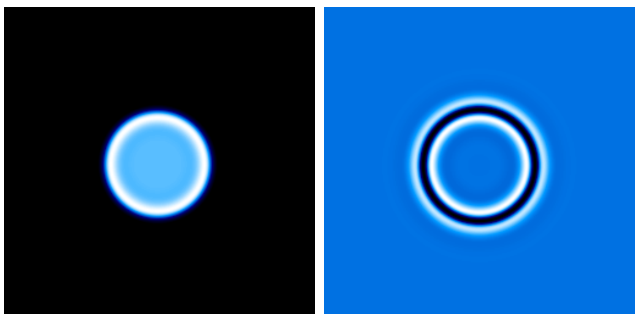
Domain Coarsening



Labyrinthine Patterns

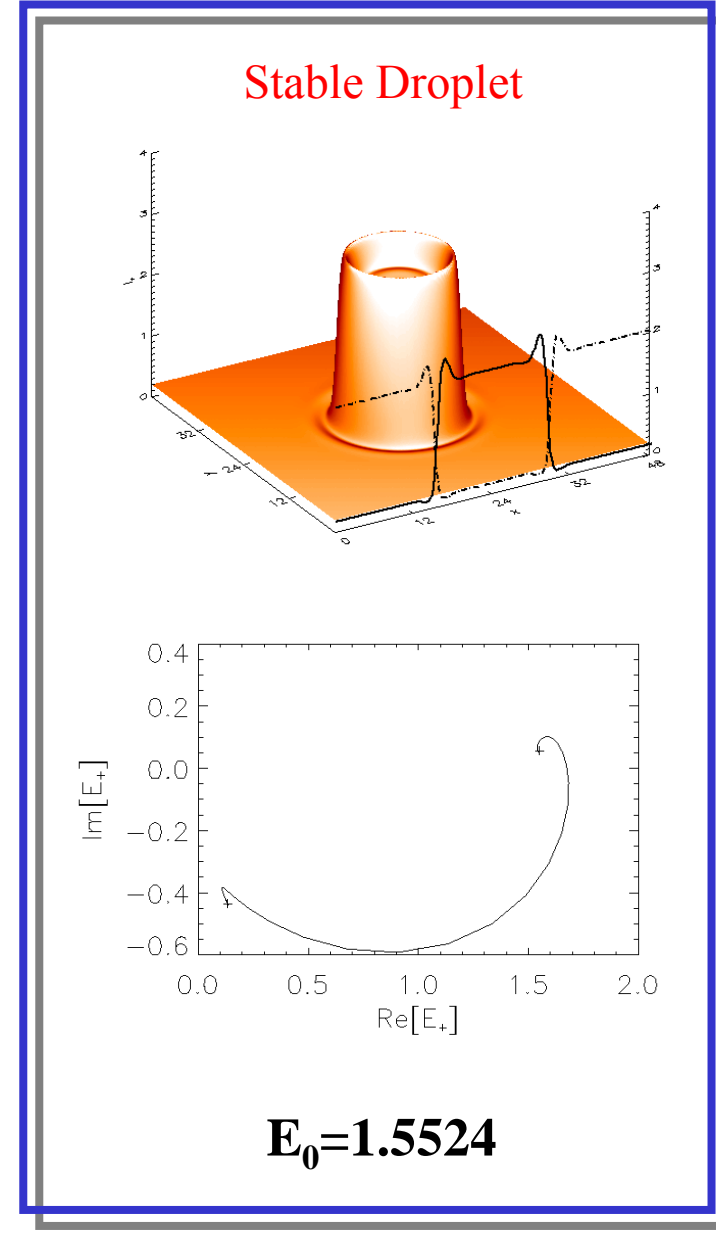
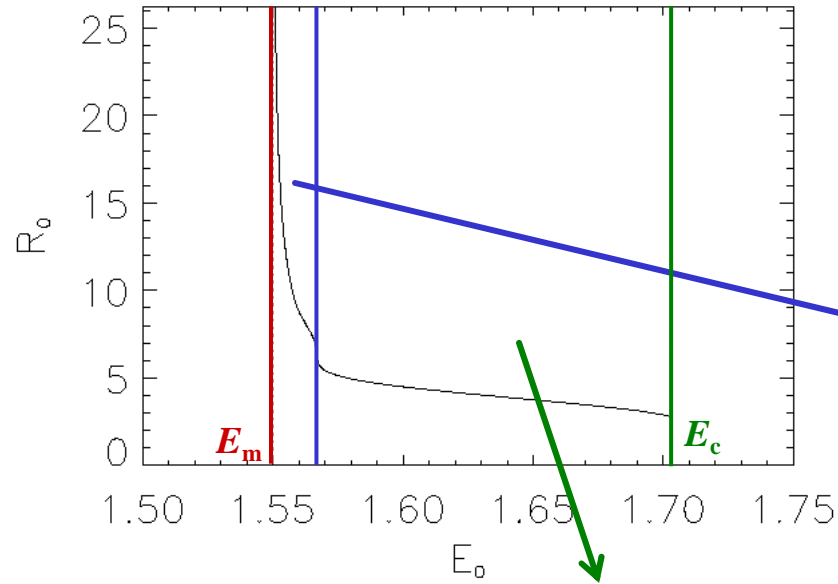


Stable Droplets

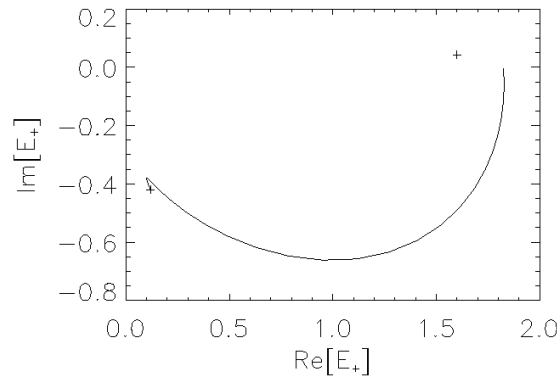
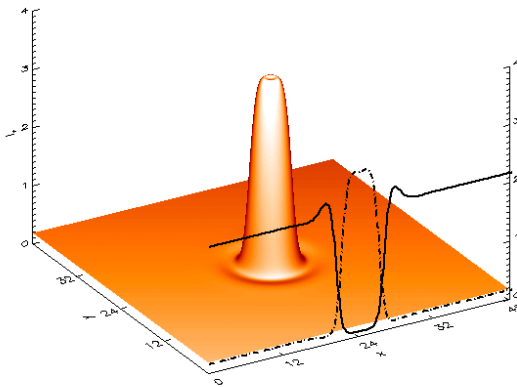


Localized Structures vs Stable droplets

Vectorial Kerr



Localized Structure



$E_0 = 1.6$

$$\partial_t \vec{\psi}(\vec{x}) = D \nabla^2 \vec{\psi}(\vec{x}) + \vec{W}(\vec{\psi}(\vec{x}), p)$$

PCGLE

$$\vec{\psi} = \begin{pmatrix} \text{Re}[A] \\ \text{Im}[A] \end{pmatrix} \quad D = \begin{pmatrix} 1 & -\alpha \\ \alpha & 1 \end{pmatrix}$$

Vectorial Kerr

$$\vec{\psi} = \begin{pmatrix} \text{Re}[E_+] \\ \text{Im}[E_+] \\ \text{Re}[E_-] \\ \text{Im}[E_-] \end{pmatrix} \quad D = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

- $\vec{\psi}$ is a real vector field
- $\vec{W}(\vec{\psi})$ is a local nonlinear function of the fields.
- There exists a discrete symmetry \mathbf{Z} such that there are two equivalent stable homogeneous solutions (PCGLE $\mathbf{Z}: A \rightarrow -A$; **Vectorial Kerr** $\mathbf{Z}: E_{\pm} \rightarrow E_{\mp}$).
- Stable d=1 l sing walls $\vec{\psi}_0(x, p)$.

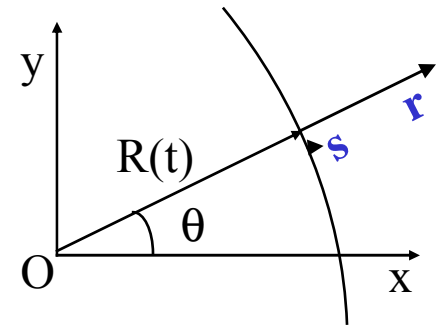
Local Coordinates \mathbf{r}, \mathbf{s}

$$\partial_t \rightarrow -v \partial_r$$

$$(v \equiv \dot{R})$$

$$\nabla^2 \rightarrow \partial_r^2 + \frac{\kappa}{1+r\kappa} \partial_r + \frac{\kappa^2}{(1+r\kappa)^2} \partial_{\theta}^2$$

$$\left(\kappa \equiv \frac{1}{R} \right)$$



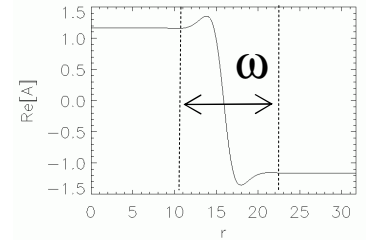
E. Meron, Phys. Rep. 218 (1992)

$$D\partial_r^2 \vec{\psi}(r) + \left(v + \frac{\kappa}{1+r\kappa} D\right) \partial_r \vec{\psi}(r) + \vec{W}(\vec{\psi}(r), p) = 0$$

Linearizing around the 1D profile: $\vec{\psi}(r) = \vec{\psi}_0 + \vec{\psi}_1$

$$M \vec{\psi}_1 = -\left(v + \kappa D\right) \partial_r \vec{\psi}_0, \quad M \equiv D\partial_r^2 + \left. \frac{\delta \vec{W}}{\delta \vec{\psi}} \right|_{\vec{\psi}_0}$$

Gently curved fronts
 $\kappa\omega \ll 1$



\vec{e}_0 Goldstone Mode: $\vec{e}_0 \equiv \partial_r \vec{\psi}_0$, $M\vec{e}_0 = 0$; $M^+ \vec{a}_0 = 0$

Solvability condition:

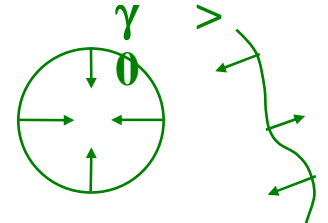
$$v = -\gamma(p)\kappa$$

$$\gamma(p) \equiv \frac{\int_{-\infty}^{\infty} \vec{a}_0 \cdot D\vec{e}_0 dr}{\int_{-\infty}^{\infty} \vec{a}_0 \cdot \vec{e}_0 dr}$$

Circular domain: $\dot{R} = -\gamma \frac{1}{R}$

• $D=dI \Rightarrow \gamma = d > 0$, $v = -d\kappa$ $\gamma > 0$
 $\vec{\psi}_1 \propto \vec{e}_0$, no deformation

Droplet shrinking

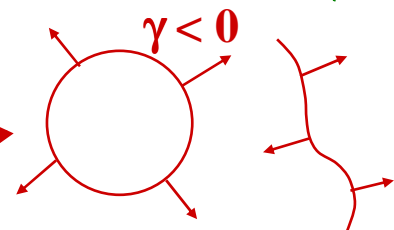


• $D=dI+C \Rightarrow \gamma = \gamma(p)$, $v = -\gamma\kappa$ $\left\{ \begin{array}{l} \gamma > 0 \\ \gamma = 0 \\ \gamma < 0 \end{array} \right.$
 $\vec{\psi}_1 \not\propto \vec{e}_0$, deformation $\propto \kappa$, C

Droplet shrinking

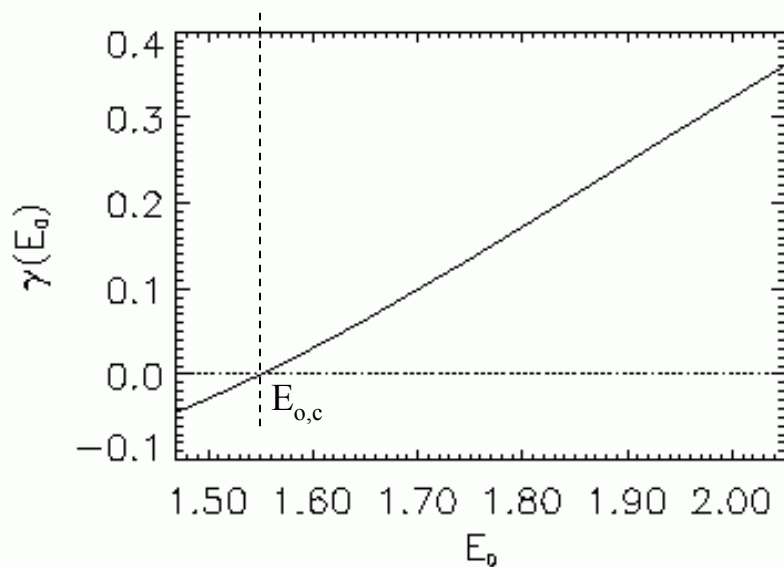
Modulational instability

Droplet growth



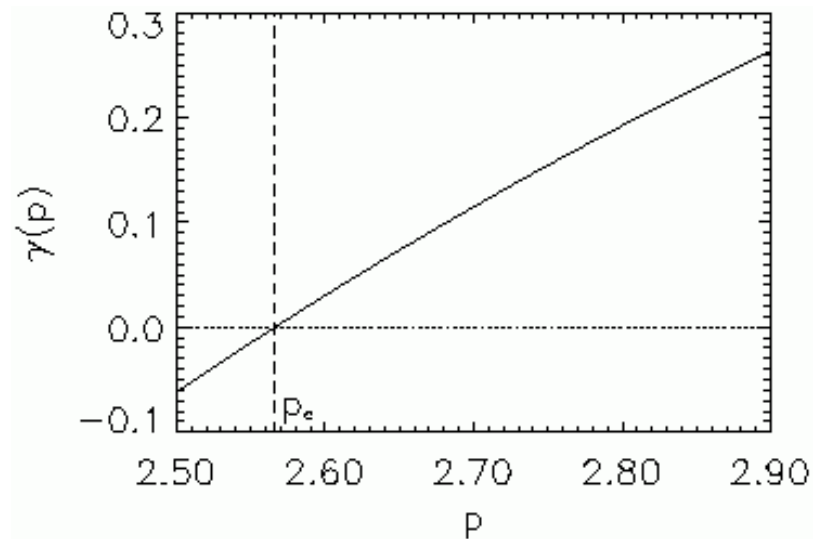
$$V = -\gamma K$$

Vectorial Kerr



$$E_{0,c} = 1.5498$$

PCGLE



$$p_c = 2.5663$$

Radial symmetry $\rightarrow D\partial_r^2\vec{\psi}(r) + \left(-\frac{\dot{\kappa}}{\kappa^2} + \frac{\kappa}{1+r\kappa}D\right)\partial_r\vec{\psi}(r) + \vec{W}(\vec{\psi}(r), p) = 0$

$$p = p_c + \varepsilon p_1, \quad \gamma(p_c) = 0, \quad \varepsilon \ll 1,$$

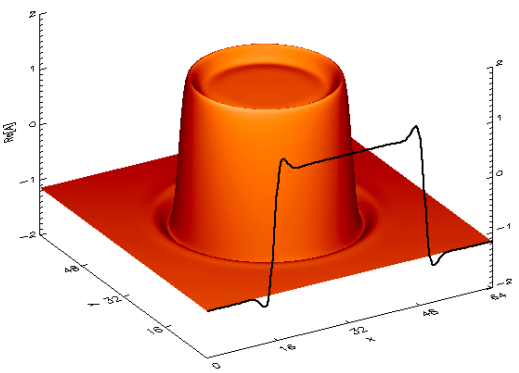
$$\vec{\psi} = \vec{\psi}_0 + \varepsilon^{1/2}\vec{\psi}_1 + \varepsilon\vec{\psi}_2 + \varepsilon^{3/2}\vec{\psi}_3, \quad \Rightarrow \quad O(\varepsilon^{3/2})$$

$$\kappa = \varepsilon^{1/2}\kappa_1, \quad \partial_t = \varepsilon^2\partial_T$$

$$\frac{\partial_T \kappa_1}{\kappa_1^2} = c_1 p_1 \kappa_1 + c_3 \kappa_1^3$$

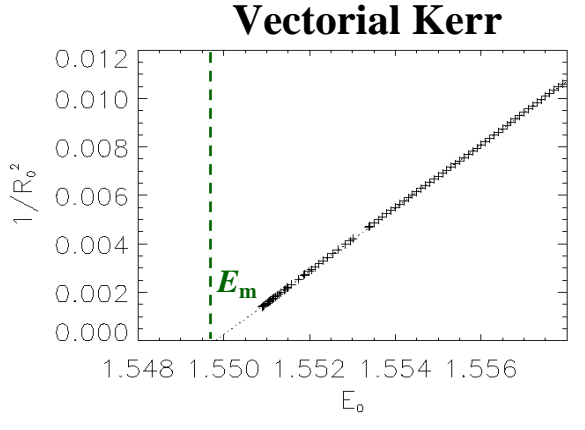
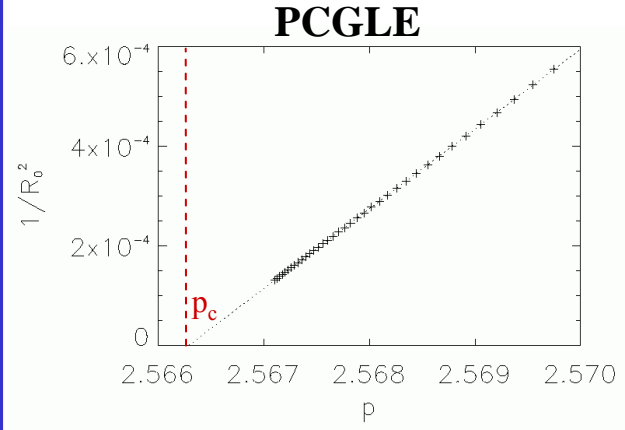
$$c_1 > 0, c_3 < 0$$

$$\dot{R} = -c_1(p - p_c)\frac{1}{R} - c_3\frac{1}{R^3}$$



Stable Droplets

$$R_0 = \frac{1}{\sqrt{p - p_c}} \sqrt{\frac{-c_3}{c_1}}$$





IMEDEA



Domain Growth

Thermodynamic Systems

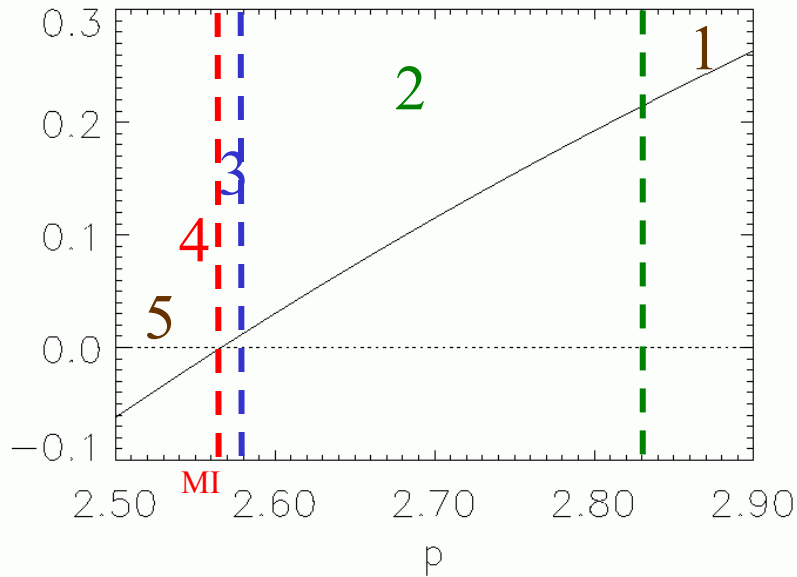
- Well established asymptotic growth laws and mechanisms: $R(t) \sim t, t^{1/2}, t^{1/3}$
- Selfsimilar evolution: Dynamical scaling $C(\mathbf{r}, t) \sim C(\mathbf{r}/R(t))$
- Nonconserved dynamics: $t^{1/2}$ robust: Point defects (Z_N , *Kaski et al. 1985*), 3d vortices (XY, *Mondello et al. 1992*), chiral walls (anisotropic XY, *Tutu et al. 1997*).

Nonequilibrium Systems:

- Pattern formation: Swift-Hohenberg $d=2$, $R \sim t^{1/5}$ (*Cross and Meiron 1995*)
- Hamiltonian vs. dissipative dynamics
- Nonpotential front motion and spiral dynamics (*Gallego et al. 1998, 1999*)
- Localized structures.
- Unstable domain walls.

Optical Systems:

- Asymptotic growth law?, Dominant mechanism?, Dynamical scaling?
Tlidi and Mandel (1998), Oppo et al. (1998), Peschel et al. (1998)



$$1, 5 \rightarrow \dot{R} = -\gamma \frac{1}{R}, \quad R \approx t^{1/2} \quad \text{shrinking, growth}$$

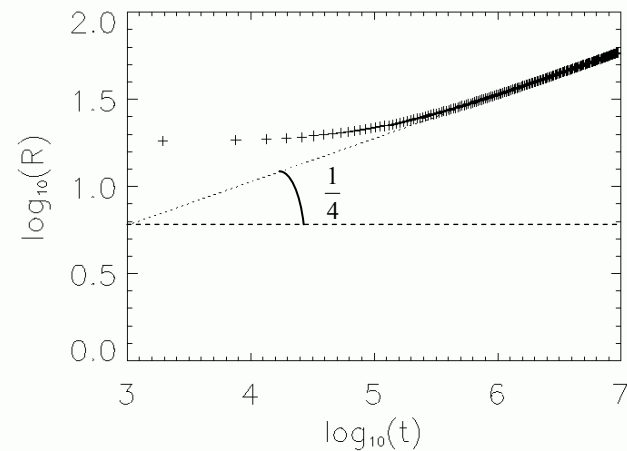
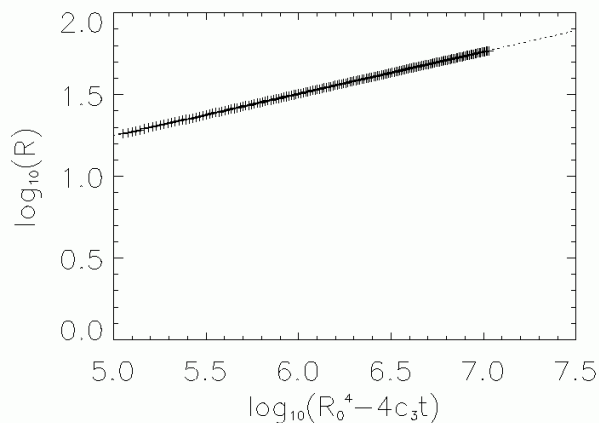
$$2 \rightarrow \dot{R} = -\gamma \frac{1}{R}, \quad R \approx t^{-1/2} \quad \text{until LS forms}$$

$$3 \rightarrow \dot{R} = -\gamma \left(\frac{1}{R} - \frac{R_0^2}{R^3} \right) \quad \text{no power law}$$

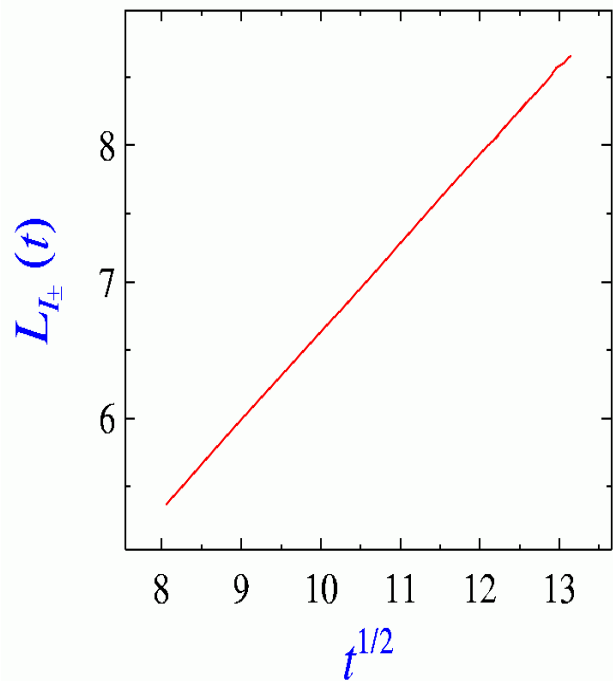
$$4 \rightarrow \dot{R} = -c_3 \frac{1}{R^3},$$

$$R \approx t^{1/4}$$

modulational instability, $\gamma = 0$

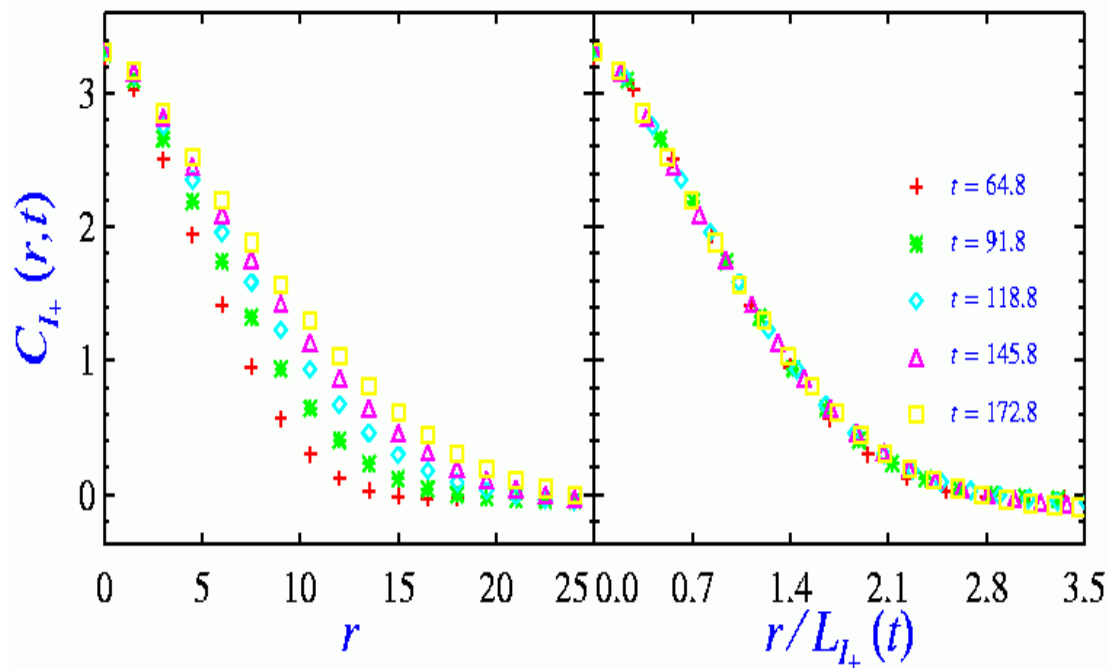


Growth law



Curvature driven

Dynamical scaling

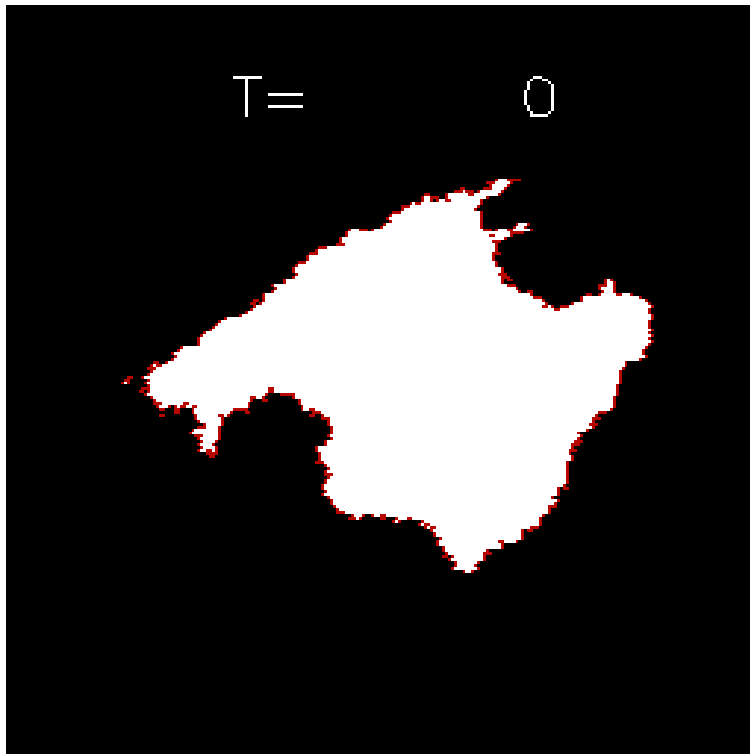


Normal front velocity: $v = -c_1(p - p_c)\kappa - c_2\kappa^2\partial_\theta^2\kappa - c_3\kappa^3$ $c_1 > 0, c_2, c_3 < 0$

← Shrinking term $O(\kappa)$
← Reduction of curvature differences $O(\kappa^2)$
← Exploding term $O(\kappa^3)$

At the Modulational Instability $p = p_c$:

$$v = -c_2\kappa^2\partial_\theta^2\kappa - c_3\kappa^3$$



D. Gomila et al,

Phys. Rev. Lett. **87**, 194101 (2001);

IEEE J. Quantum Electr. (2002)