IMEDEA

NOISE-SUSTAINED PATTERNS IN NONLINEAR OPTICS

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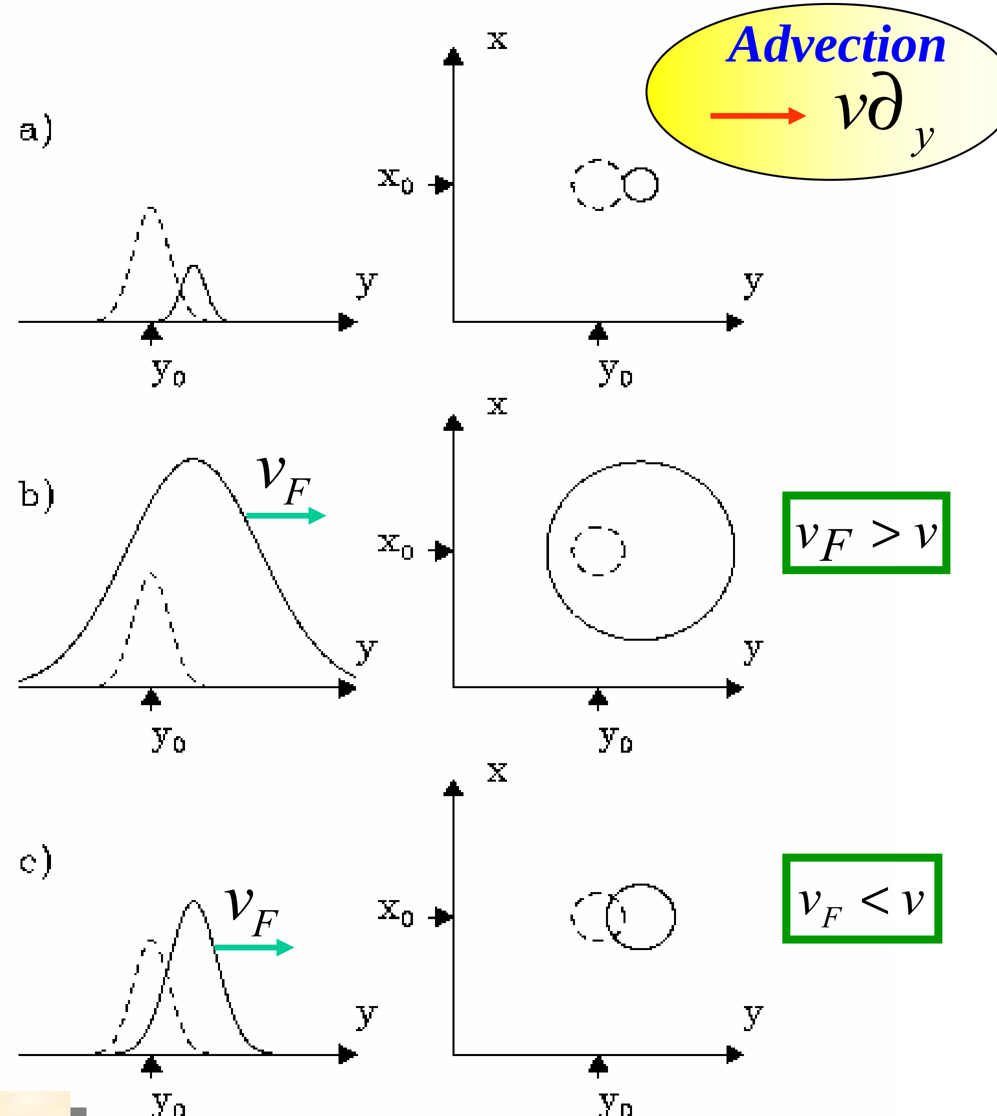
3 REGIMES

•Absolutely Stable:
Perturbation decays

•Absolutely Unstable:
Perturbation grows and spreads faster than the advection velocity

•Convectively Unstable:
Perturbation grows but spreads slower than the advection velocity



Absolute Instability threshold: $v_F = v$

Noise-sustained patterns

Convectively unstable regime
+
noise
⇒ Noise-sustained patterns

Taylor-Couette flow

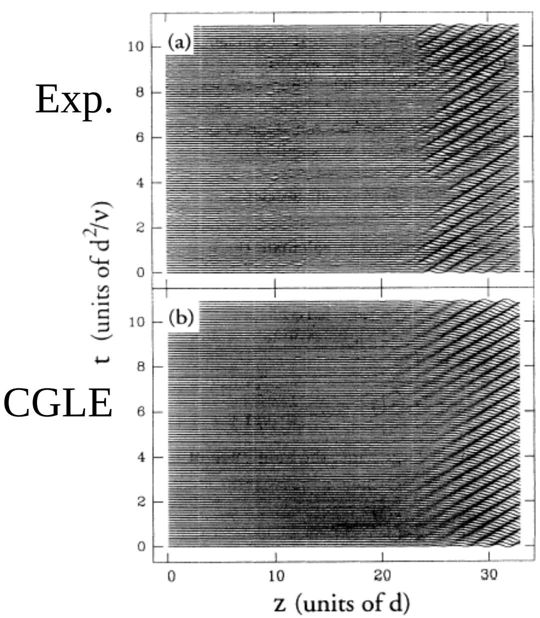


FIG. 7. Space-time plots of noise-sustained structure in the convectively unstable regime. (a) Experimental flow visualization with Kalliroscope for $R=3.0$, $\nu=0.025$ S, and $\bar{\epsilon}=0.040$. (b) Integration of Eq. (5.2) with noise level $F_A=2.1\times10^{-7}$.

Variance of time spectra at a fixed position

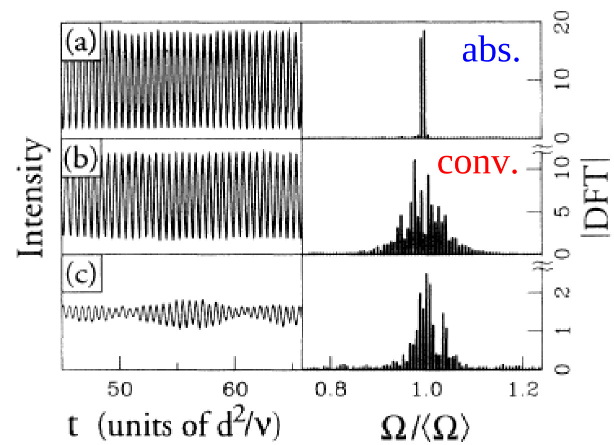


FIG. 10. Time series of reflected light, and their DFT moduli near the fundamental peak, at $z=100$ for $R=3.0$. About $\frac{1}{4}$ of each series is shown. (a) $\epsilon-\epsilon_c=0.0896$. (b) $\epsilon-\epsilon_c=0.0318$. The DFT peak shows considerable broadening. In (c), structure persists at small $\epsilon-\epsilon_c=0.0077$.

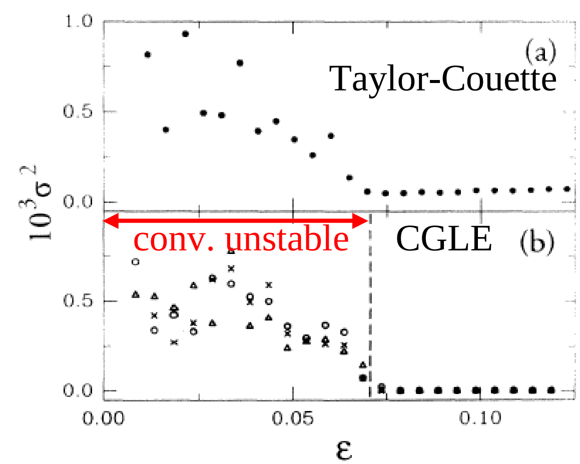


FIG. 11. (a) Normalized second moment of the fundamental peak in the DFT power vs ϵ at $R=3.0$ for experimental time series at $z=100$. The transition at $\epsilon=0.065$ indicates the onset of phase noise. (b) Corresponding results for Eq. (5.2) for inlet noise levels 10^{-6} (+), 10^{-5} (△), and 10^{-4} (○), respectively. The dashed line locates $\epsilon_a^{CGL}=0.071$.

K.L. Babcock, G. Ahlers, D.S. Cannell, PRE, **50**, 3670 (1994).

CONCEPTS IN NOISE SUSTAINED STRUCTURES

PHENOMENON OF NOISE AMPLIFICATION, while noisy precursors are weakly damped critical fluctuations

NOISE NEEDED AT ALL TIMES: A laser requires amplification of spontaneous emission, but when the laser is lasing noise is no longer required to maintain the oscillation

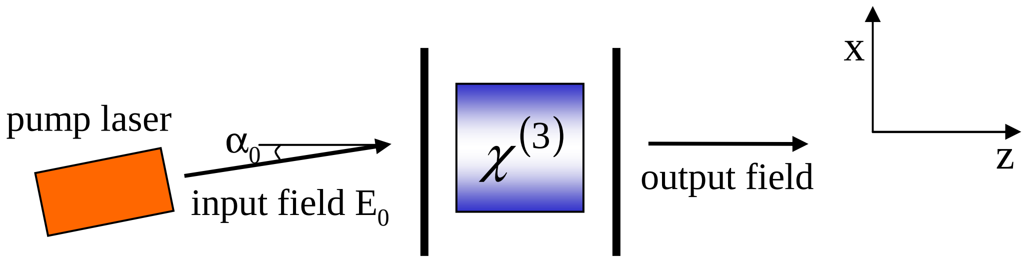
PRECURSOR OF AN ABSOLUTELY UNSTABLE REGIME: An optical amplifier is in a convective regime, but there is no regime of absolute instability

Noise-sustained patterns in optical cavities

Examples

- Fabry-Perot Cavities filled with a cubic (Kerr) nonlinear medium
  Transverse drift due to tilted pump
  - One-dimensional (stripe pattern)
  - Two-dimensional (hexagonal pattern)
- Ring optical parametric oscillators (quadratic nonlinearity)
  Transverse walk-off due to birefringence
  - Type I (one polarization)
  - Type II (two polarizations)

1d self-focusing Kerr cavities



$$E(x,z,t) = \underbrace{A(x,t)}_{\text{field envelope}}\, e^{i(k_0 z - w_0 t)}$$

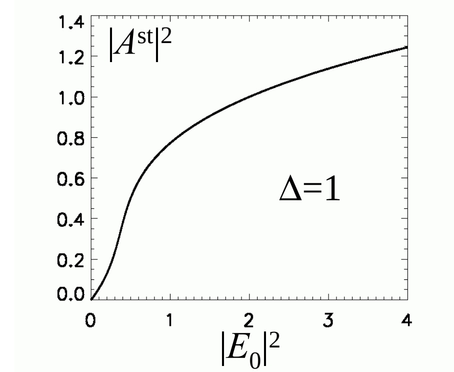
Time evolution of field envelope

$$\partial_t A - 2\alpha_0 \partial_x A = -(1+i\Delta)A + ia\partial_x^2 A + E_0 + i|A|^2 A + \sqrt{\varepsilon}\xi(t)$$

α_0: tilt angle a: diffraction strength E_0: pump

Δ: cavity detuning ξ_i: Gaussian white noise

Homogeneous solution: $$E_0 = A^{\mathrm{st}}\left[1 + i(\Delta - |A^{st}|^2)\right]$$



Linear stability analysis: $$A(x,t) = A^{\mathrm{st}} + \delta A(x,t)$$

$$\delta A(x,t) = \sum_q b_q(t) e^{iqx} \qquad b_q(t) \propto e^{\lambda(q)t}$$

$$\lambda_{\pm}(q) = 2i\alpha_0 q - 1 \pm \sqrt{|A^{st}|^4 - (2|A^{st}|^2 - \Delta - aq^2)^2}$$

Instability threshold: $$\mathrm{Re}(\lambda_1) = 0 \Rightarrow \begin{cases} \Delta < 2: \ |A|^2 = 1 & |q_c| = \sqrt{2-\Delta}/a \quad \text{pattern} \\ \Delta > 2: \ |A|^2 = \frac{2\Delta - \sqrt{\Delta^2 - 3}}{3} & |q_c| = 0 \quad \text{homogeneous} \end{cases}$$

Convective stability analysis

Linearized asymptotic behaviour of a generic perturbation ψ of steady state:

$$\psi(x,t) = \iint_{-\infty}^{+\infty} dq\, \tilde{\psi}(q,0) \exp\left[iqx + \lambda(q)t\right]$$

$\tilde{\psi}(q,0)$: initial perturbation in Fourier space
$\lambda(q)$: eigenvalue with largest real part

$\mathrm{Re}\left[\lambda(q)\right] < 0,\ q \in R, \Rightarrow \psi(x,t) \to 0 \text{ as } t \to \infty$ — Absolutely stable

$\mathrm{Re}\left[\lambda(q)\right] > 0 \begin{cases} |\psi(x,t)| \to \infty \text{ for arbitrary } x & \text{Absolutely unstable} \\ |\psi(x_0,t)| \to 0 \text{ and } |\psi(x_0+vt,t)| \to \infty \text{ for some } v & \text{Convectively unstable} \end{cases}$

To evaluate the integral:
- Analytically continue $\lambda(q)$ over complex plane k in the form $\lambda(k/i)$
- Deform integration contour to pass through saddle point k^s along direction of stepest descent.

$$\partial_k \lambda(k)\big|_{k^s} = 0, \quad \mathrm{Re}\left[\partial_k^2 \lambda(k)\big|_{k^s}\right] \geq 0$$

The system is

Absolutely unstable for: $\mathrm{Re}\left[\lambda(q)\right] > 0, \quad \mathrm{Re}\left[\lambda(k^s)\right] > 0$

Convectively unstable for: $\mathrm{Re}\left[\lambda(q)\right] > 0, \quad \mathrm{Re}\left[\lambda(k^s)\right] < 0$

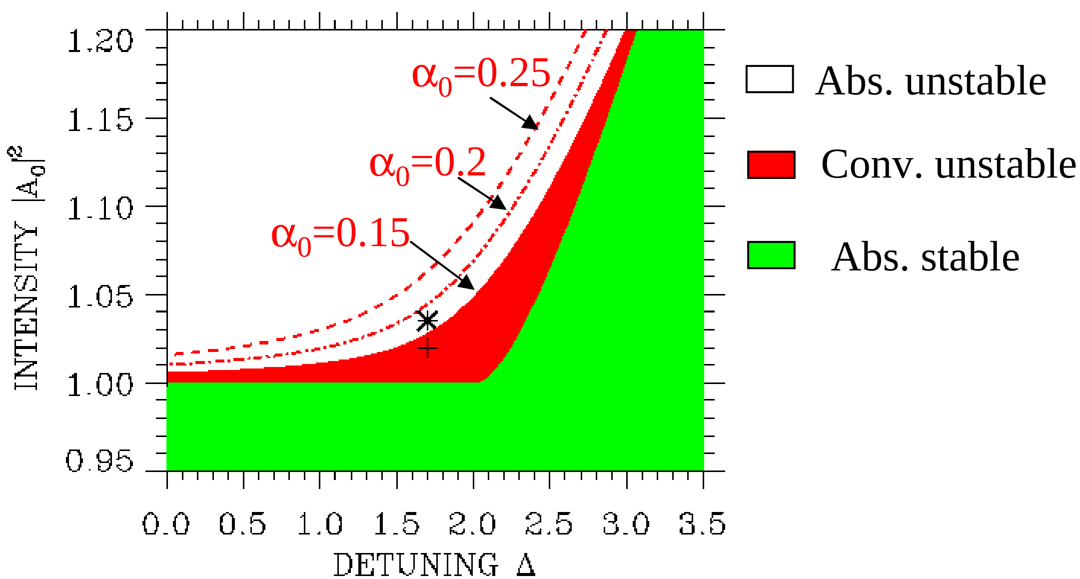
Deissler, J. Stat. Phys., **40**, 376 (85); **54**, 1459 (89); Physica D **56**, 303 (92);
Hall & Heckrotte, Phys. Rev. **166**, 120 (68); T.H. Stix, *Waves in plasmas,* AIP (92);
Infield & Rowlands, *Nonlinear waves, solitons and chaos,* Cambridge (90)

1d self-focusing Kerr cavities

Analytic continuation of $\lambda_+(q)$ over complex plane: $\lambda(k) = 2\alpha_0 k - 1 + \sqrt{|A^{st}|^4 - (2|A^{st}|^2 - \Delta + ak^2)^2}$

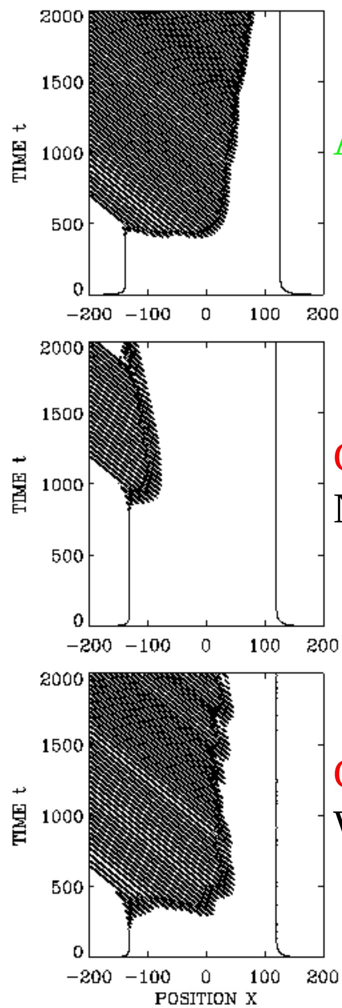
Saddle: $$\partial_k \lambda(k)\Big|_{k^s} = 0 \Rightarrow \alpha_0 - \frac{2a(2|A^{st}|^2 - \Delta + ak^2)k^s}{\sqrt{|A^{st}|^4 - (2|A^{st}|^2 - \Delta + ak^2)^2}} = 0$$

Absolute instability threshold: $\mathrm{Re}\left[\lambda(k^s)\right] = 0$



1d self-focusing Kerr cavities

$|A(x,t)|^2$

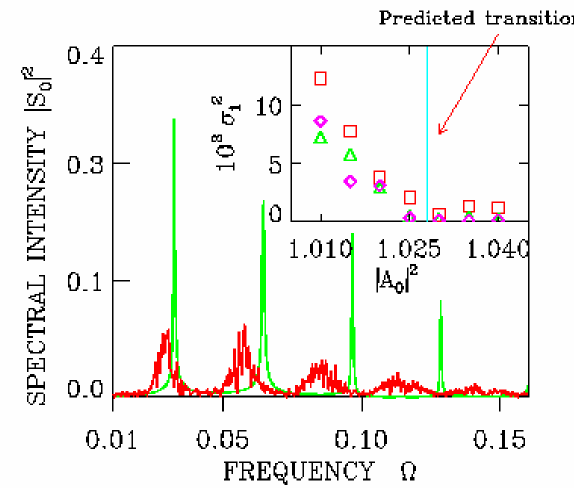


Absolutely unstable

Convectively unstable
No noise

Convectively unstable
With noise

Variance of time spectra at a fixed position



σ^2_i: i-harmonic variance:
□ i=1 △ i=2 ◇ i=3

Spectral narrowing identifies transition from noise-sustained to deterministic pattern

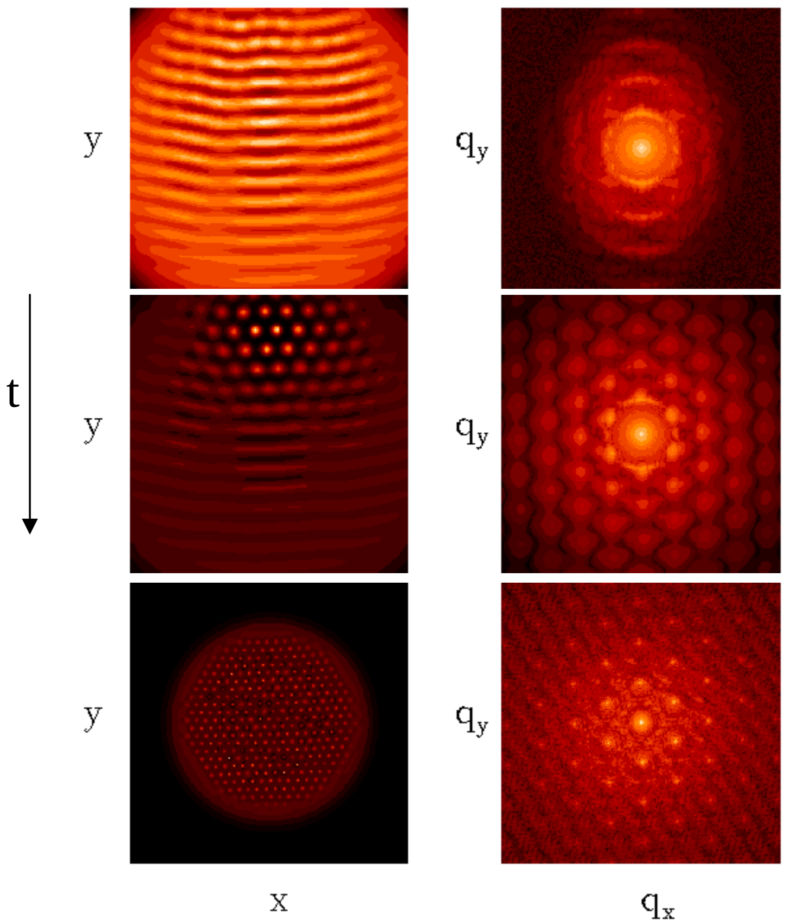
M. Santagiustina, P. Colet, M. San Miguel & D. Walgraef, Phys. Rev. Lett. 79, 3633 (1997)

2d self-focusing Kerr cavities

Absolutely unstable:

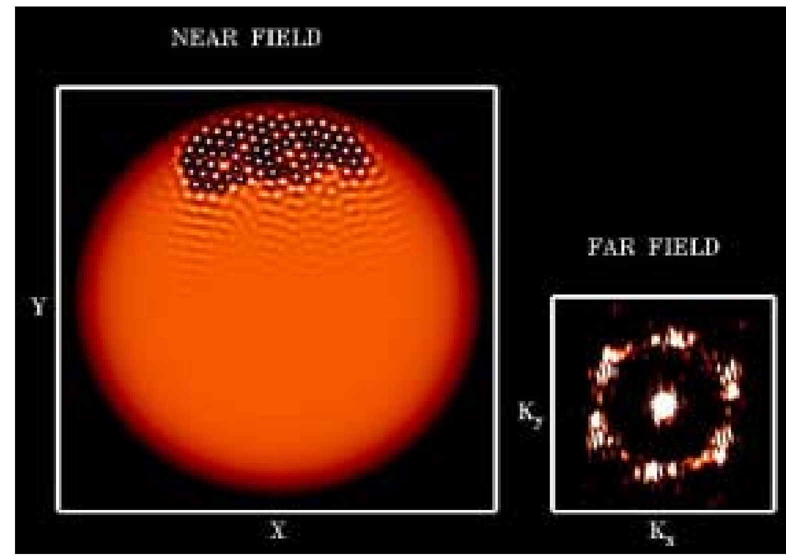
Stability analysis shows first unstable modes are stripes perpendicular to tilt.

Stripes not stable in nonlinear regime hexagons.



Convectively unstable:

Noise generates disordered hexagons preceded by unstable noisy stripes.



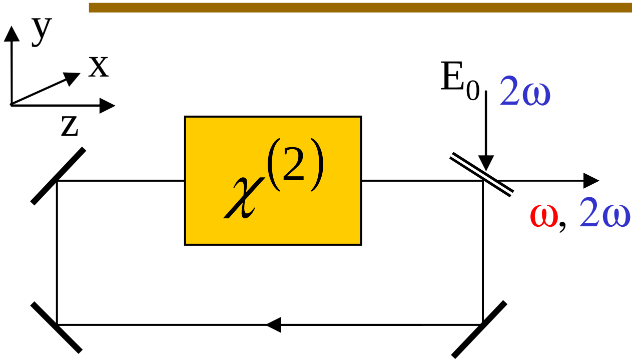
NOISE SUSTAINED STRUCTURE:

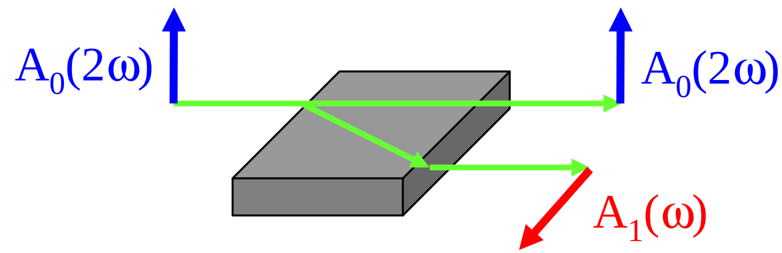
MACROSOCPIC AND SPATIALLY STRUCTURED

MANIFESTATION OF QUANTUM NOISE

M. Santagiustina, et al, JEOS B, Quantum and Semiclassical Optics 1, 191(1999)

Type I degenerate optical parametric oscillator





pump(2ω): A_0 $\quad \partial_t A_0 = \gamma_0\left[-(1+i\Delta_0)A_0 + E_0 + ia_0\nabla^2 A_0 + 2iK_0 A_1^2\right] + \sqrt{\varepsilon_0}\xi_0(\vec{x},t)$

signal(ω): A_1 $\quad \partial_t A_1 = \gamma_1\left[-(1+i\Delta_1)A_1 + ia_1\nabla^2 A_1 + iK_0 A_1^* A_0 + \rho\partial_y A_1\right] + \sqrt{\varepsilon_1}\xi_1(\vec{x},t)$

γ_i: cavity decay rates $\quad a_i$: diffraction strength $\quad E_0$: pump

Δ_i: cavity detunings $\quad \xi_i$: Gaussian white noise $\quad K_0$: nonlinear coef.

Walk-off → convection term

Homogeneous solution: $\quad A_0^{st} = E_0/(1+i\Delta_0), \quad A_1^{st} = 0 \qquad F \equiv K_0\left|A_0^{st}\right|$

$A_j(\vec{x},t) = A_j^{st} + \delta A_j(\vec{x},t)$

$\delta A_j(\vec{x},t) = \sum_{\vec{q}} b_{j\vec{q}}(t)e^{i\vec{q}\vec{x}} \quad b_{j\vec{q}}(t) \propto e^{\lambda_j(\vec{q})t}$

$$\begin{cases} \lambda_0(\vec{q}) = -\gamma_0(1+i\Delta_0 + ia_0 q^2) \quad \mathrm{Re}(\lambda_0) < 0 \\ \lambda_{1\pm}(\vec{q}) = \gamma_1\left[-1 + i\rho q_y \pm \sqrt{F^2 - (\Delta_1 + a_1 q^2)}\right] \end{cases}$$

Instability threshold: $\quad \mathrm{Re}(\lambda_{1+}) = 0 \Rightarrow F^2 = 1$

$$\begin{cases} \Delta_1 < 0: \quad |\vec{q}_c| = \sqrt{-\Delta_1/a_1} \quad \text{pattern} \\ \Delta_1 > 0: \quad |\vec{q}_c| = 0 \quad \text{homogeneous} \end{cases}$$

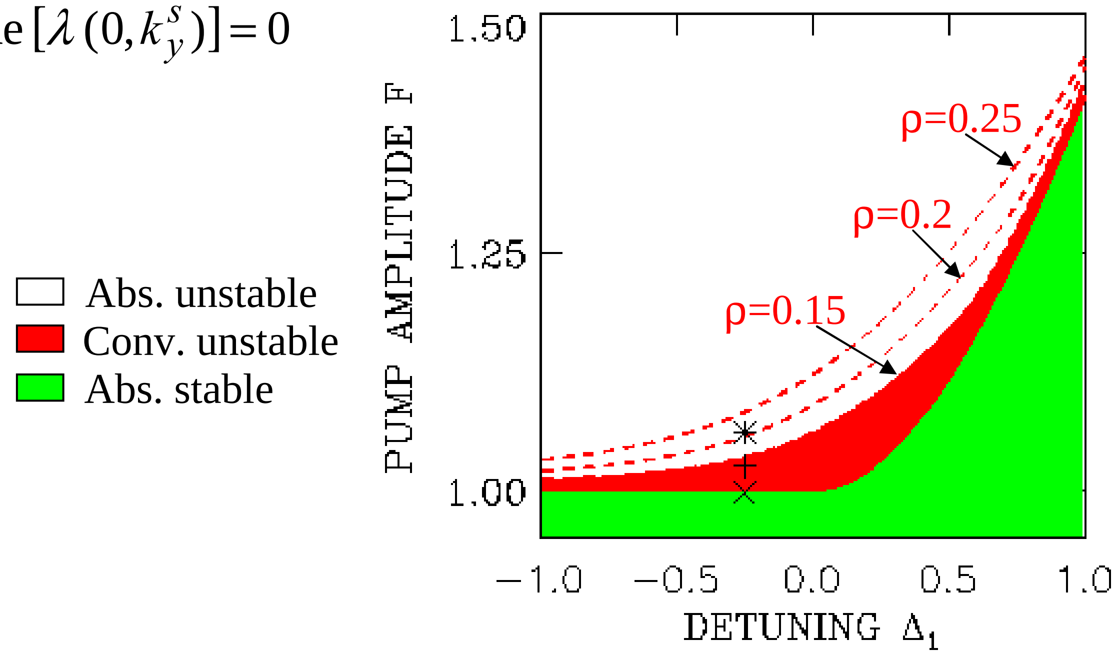
Convective stability analysis

Analytic continuation of $\lambda_{1+}(\vec{q})$ over complex plane: $\lambda(\vec{k}) = \gamma_1\left[-1+\rho k_y + \sqrt{F^2 - a_1(q_c^2+k^2)^2}\right]$

Saddle: $\nabla_{\vec{k}}\lambda(k_x,k_y)|_{k_x^s,k_y^s} = 0 \Rightarrow \begin{cases} k_x^s = 0 \Rightarrow q_x = 0 \\ \rho - \dfrac{2a_1^2(q_c^2+{k_y^s}^2)k_y^s}{\sqrt{F^2-a_1(q_c^2+{k_y^s}^2)^2}} = 0 \end{cases}$

pattern orientation perpendicular to drift

Absolute instability threshold: $\mathrm{Re}[\lambda(0,k_y^s)] = 0$



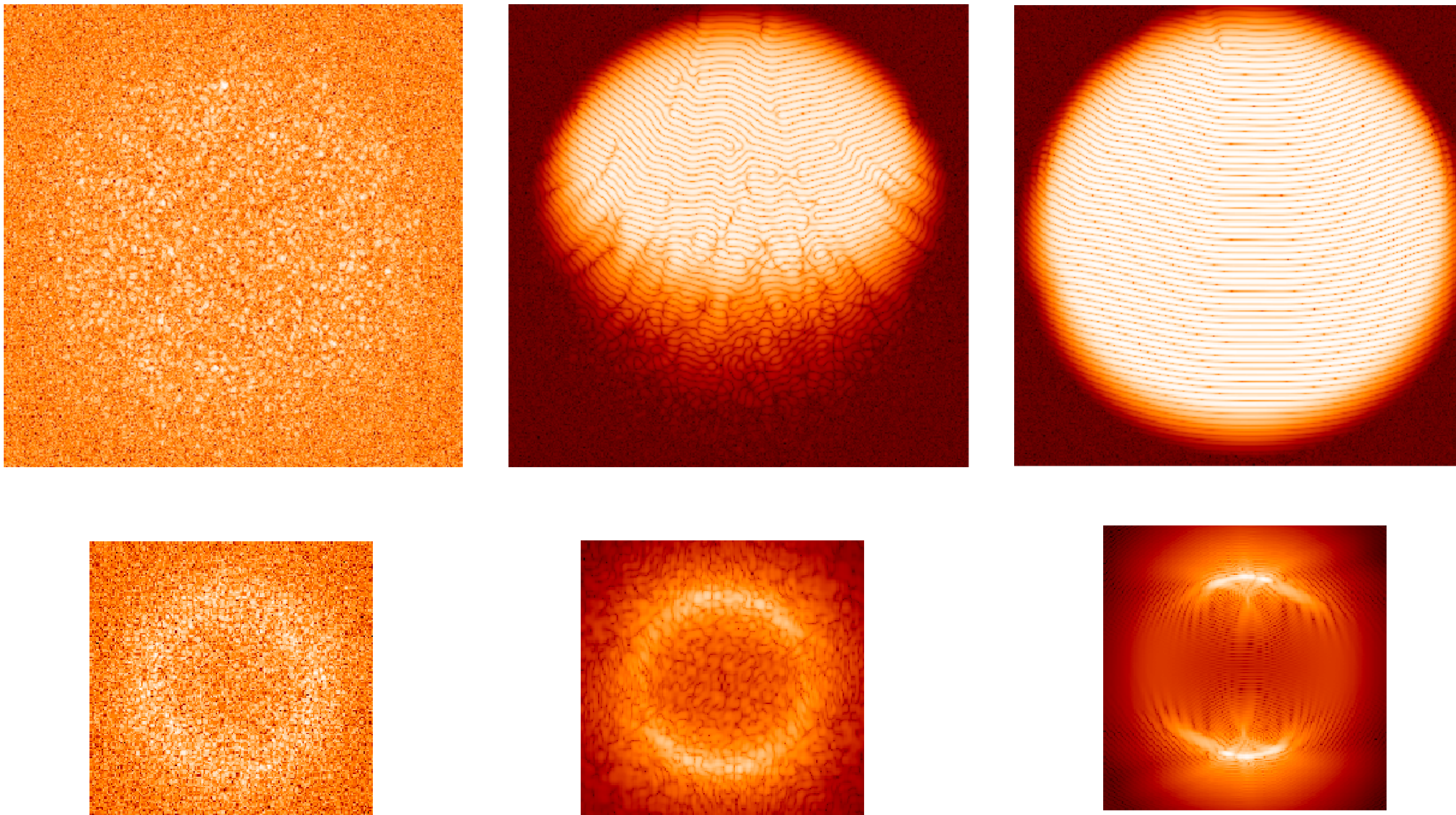
M. Santagiustina, P. Colet, M. San Miguel and D. Walgraef, Phys. Rev. E **58**, 3843 (1998) ;
H. Ward, M.N. Ouarzazi, M. Taki, P. Glorieux, Eur. Phys. J. D **3**, 275 (1998);
M. Taki, M.N. Ouarzazi, H. Ward, P. Glorieux, J. Opt. Soc. Am. B **17**, 997 (2000).

Type I degenerate optical parametric oscillator

absolutely stable | convectively unstable | absolutely unstable

$|A_1(x,y)|^2$

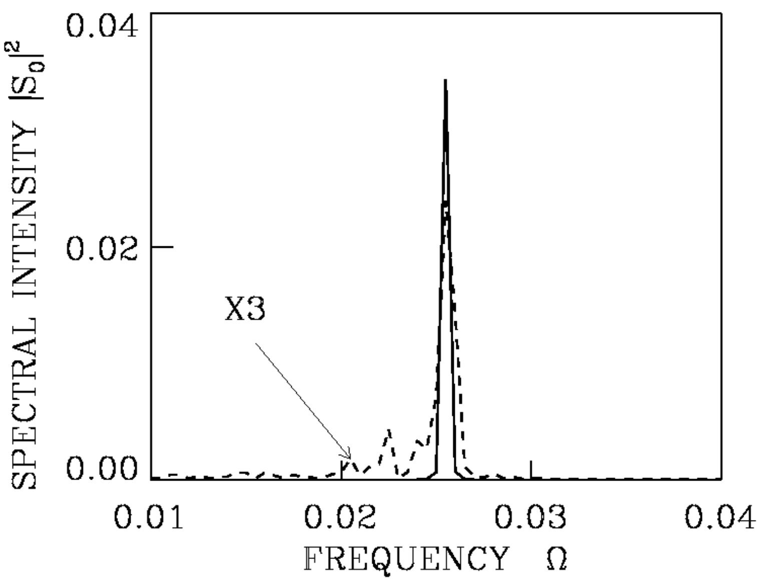
$|A_1(q_x,q_y)|^2$

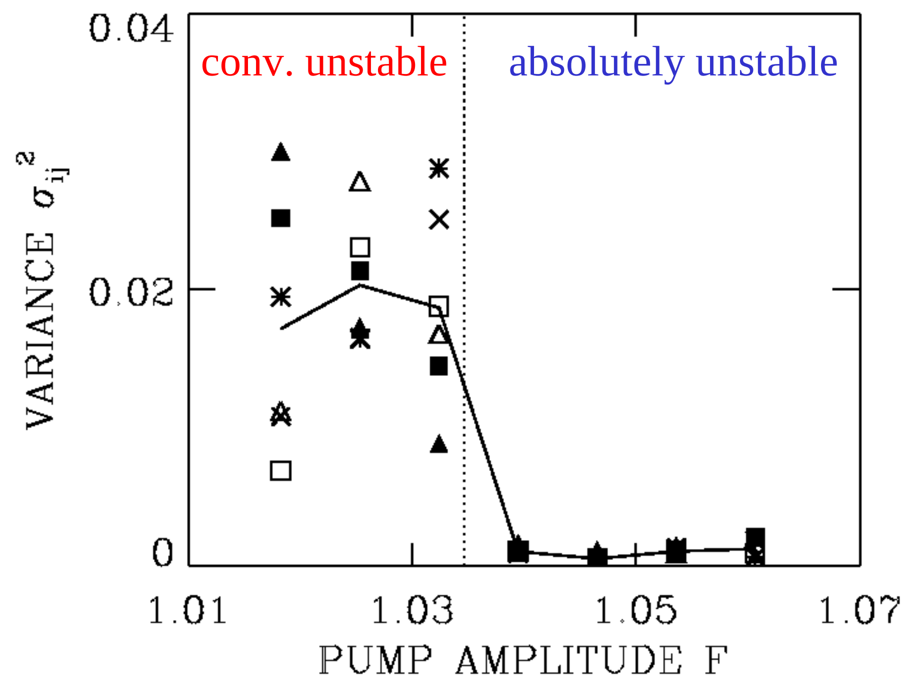


M. Santagiustina, P. Colet, M. San Miguel and D. Walgraef, PRE 58, 3843 (1998)

Type I degenerate optical parametric oscillator

Variances of spectra at different spatial positions.





F=1.0465

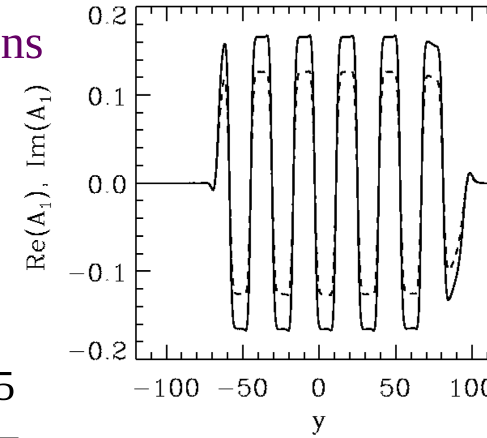
F=1.025

Large spread due to relatively short integration time.

M. Santagiustina, P. Colet, M. San Miguel and D. Walgraef, PRE **58**, 3843 (1998)

Type I DOPO. Wavevector selection

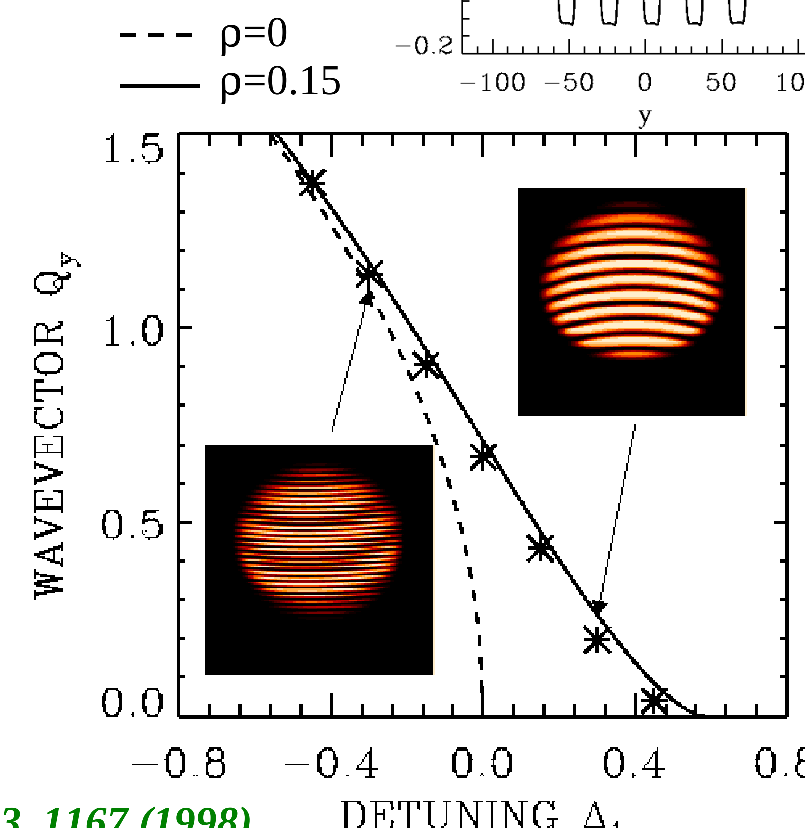
Periodic array of dark solitons



Selected wavevector Q_y:

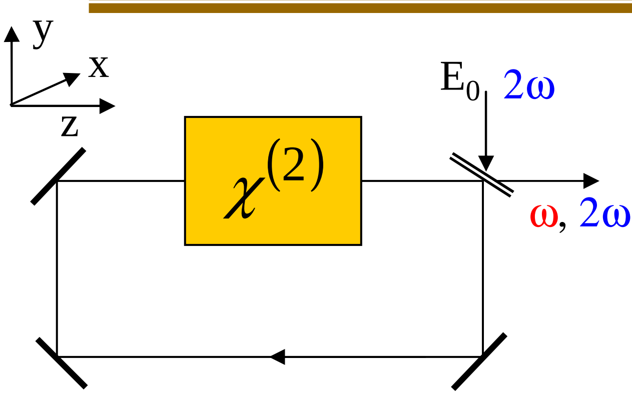
- The linearly unstable wavevector
  - -oscillates at frequency $\mathrm{Im}[\lambda(\vec{k}^s)]$
  - -drifts at velocity ρ.
- Imposing conservation of field node flux from linearly unstable regime to the stable one:

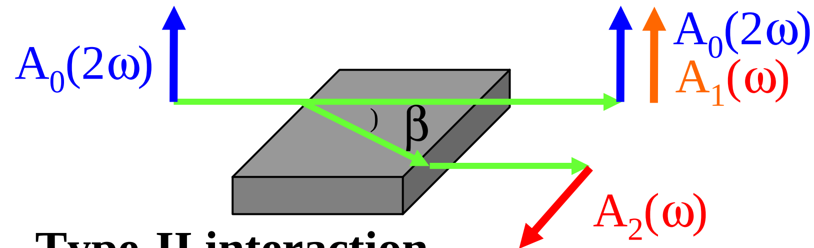
$$Q_y = \frac{\mathrm{Im}[\lambda(\vec{k}^s)]}{\rho}$$



M. Santagiustina, M. San Miguel and P. Colet, Opt. Lett. 23, 1167 (1998) .

Type II Optical Parametric Oscillator





Type-II interaction

Polarization plays a fundamental role.

pump(2ω), ordinary: A_0
$$\partial_t A_0 = \gamma_0\left[-(1+i\Delta_0)A_0 + E_0 + ia_0\nabla^2 A_0 + 2iK_0 A_1 A_2\right] + \sqrt{\varepsilon_0}\,\xi_0(\mathbf{r},t)$$

signal(ω), ordinary: A_1
$$\partial_t A_1 = \gamma_1\left[-(1+i\Delta_1)A_1 + ia_1\nabla^2 A_1 + iK_0 A_2^* A_0\right] + \sqrt{\varepsilon_1}\,\xi_1(\mathbf{r},t)$$

idler(ω), extraordinary: A_2
$$\partial_t A_2 = \gamma_2\left[-(1+i\Delta_2)A_2 + ia_2\nabla^2 A_2 + iK_0 A_1^* A_0 + \rho_2\partial_y A_2\right] + \sqrt{\varepsilon_2}\,\xi_2(\mathbf{r},t)$$

Walk-off → convection term

γ_i: cavity decay rates a_i: diffraction strength E_0: pump
Δ_i: cavity detunings ξ_i : Gaussian white noise K_0: nonlinear coef.

Type II OPO: Instabilities of the solution $A_1 = A_2 = 0$

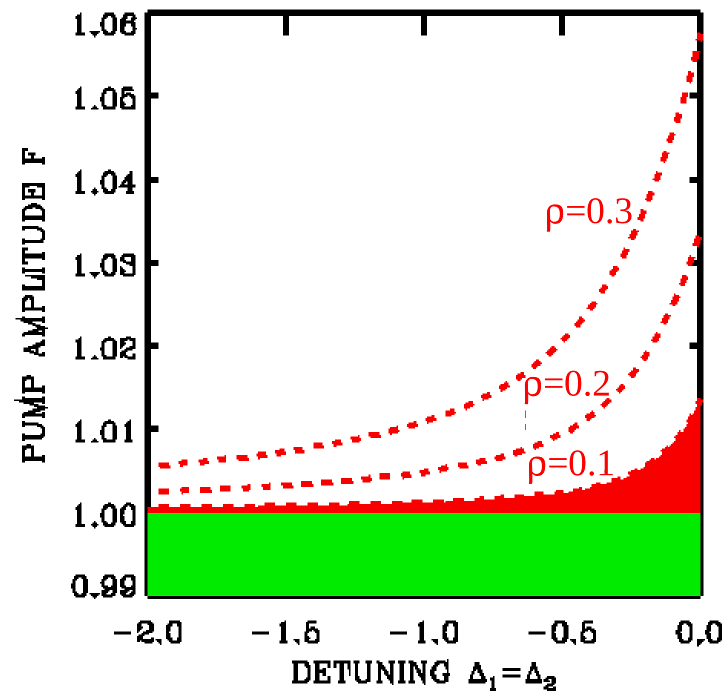
• For small E_0, $A_{1,2} = 0$ is a stable solution

• For $\Delta_{1,2} < 0$, zero solution unstable at:

$$F = |K_0 E_0 / (1 + i\,\Delta_0)| = 1.$$

Instability takes place at finite wavevector for A_1 and A_2

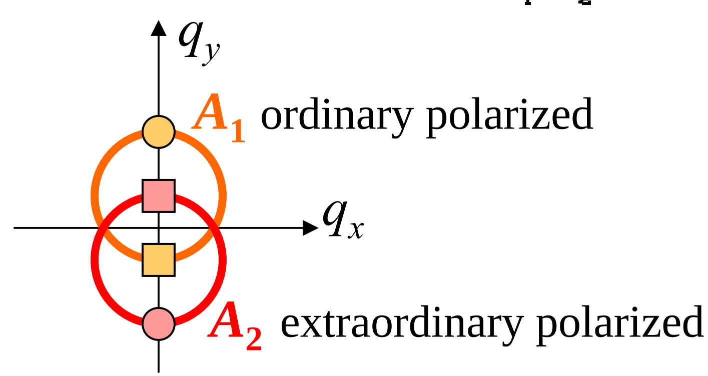
Close to threshold, instability is convective.



• Larger pump: absolute instability.

One transverse $\boldsymbol{q}$ mode, parallel to the walk-off direction, is selected for A_1 and the opposite $-\boldsymbol{q}$ for A_2.

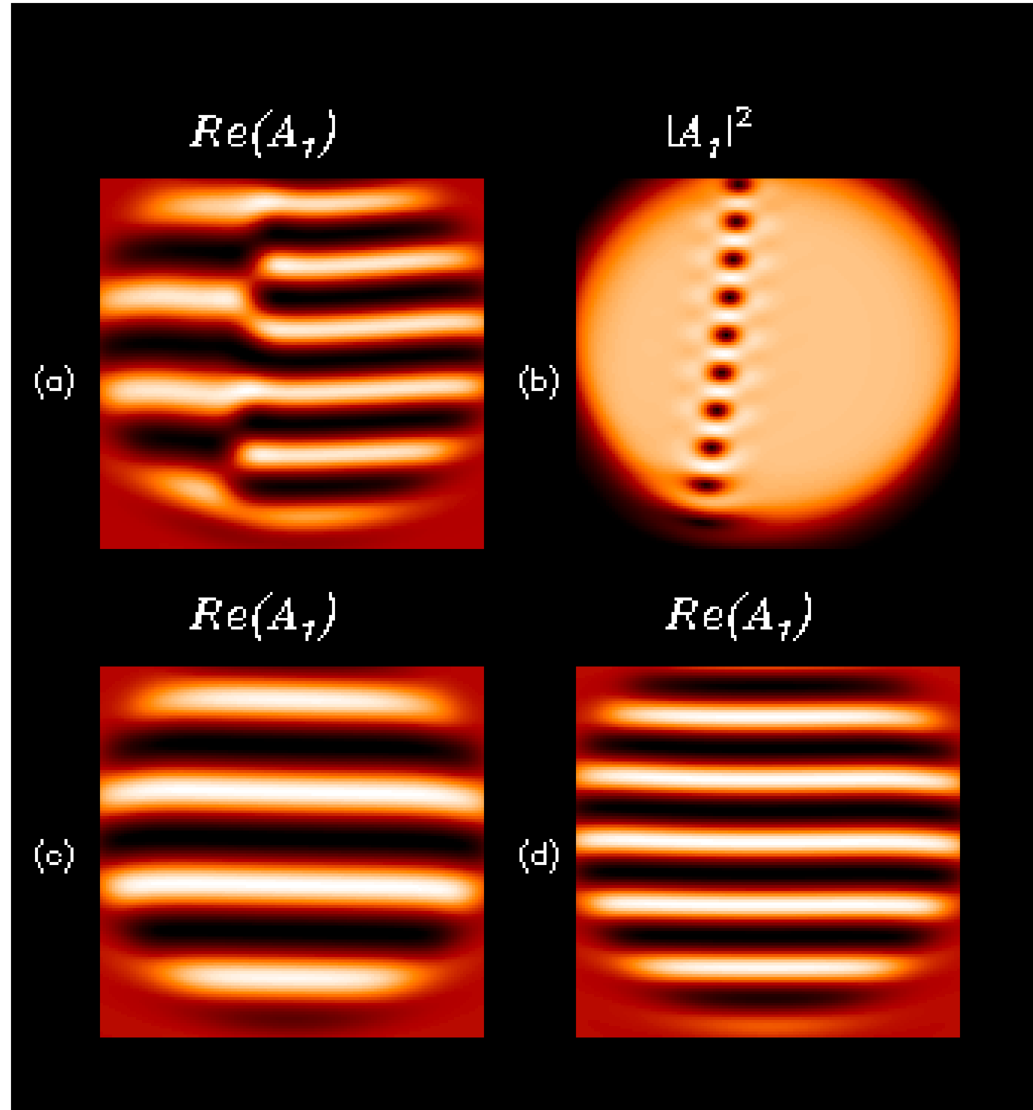
Two equally probable selections: (□, □) and (○, ○) .



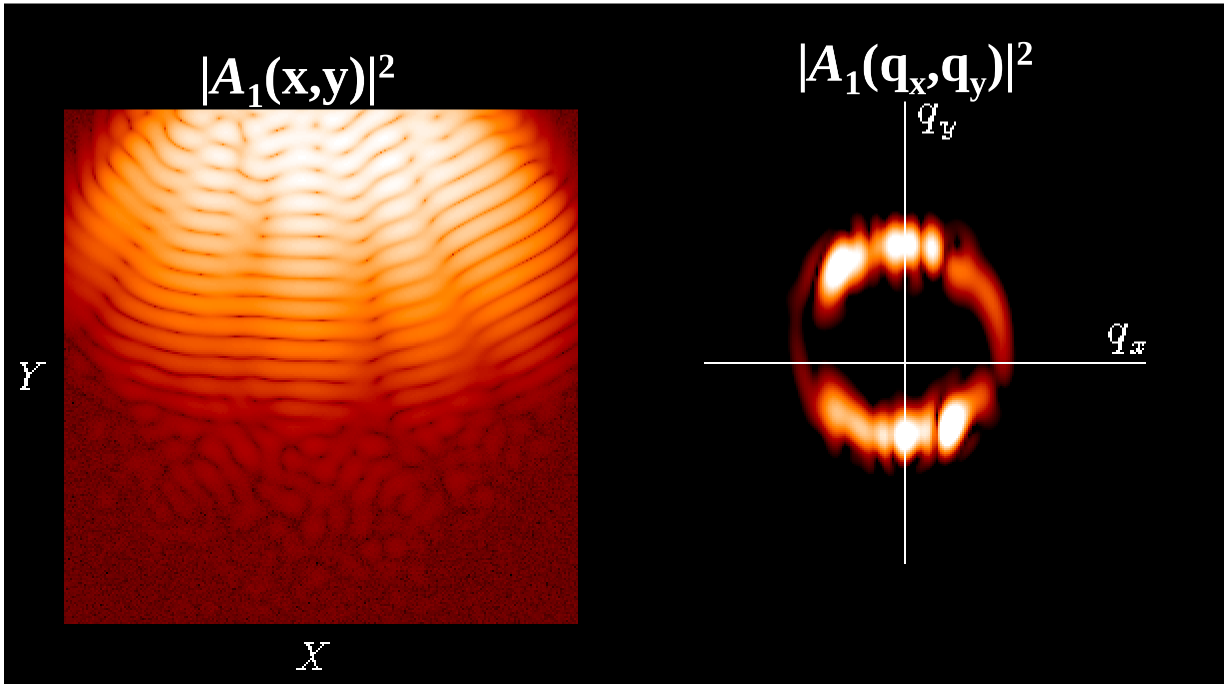
Type II OPO: Absolutely unstable regime

Transient regime
• Competiton between two travelling waves of different wavevector (modes ○ and □)
• Intensity constant in each travelling wave domain.
• defects appear at domain boundaries

Final state
• One travelling wave takes over.
•Selection is random.
•No intensity patterns, only phase stripes



Type II OPO: Convectively unstable regime



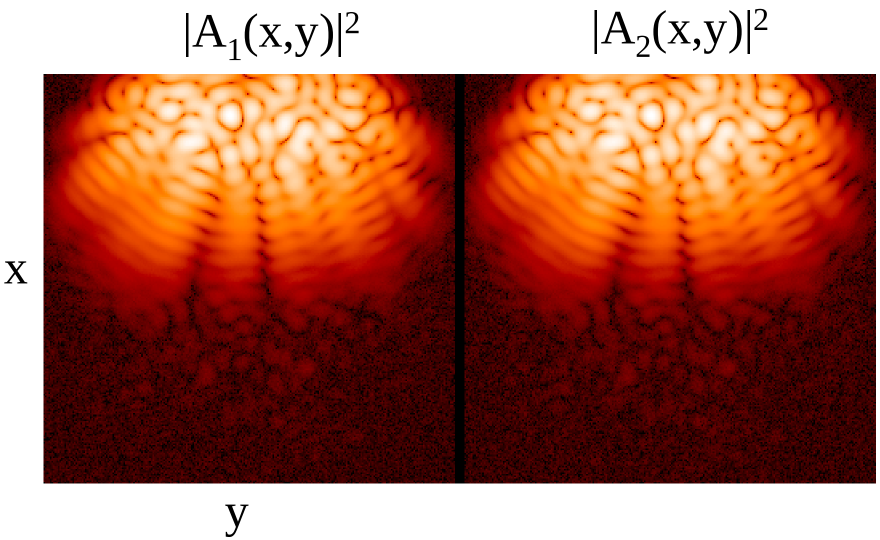
In the convective regime, all spatial modes are sustained by noise, partially restoring symmetry

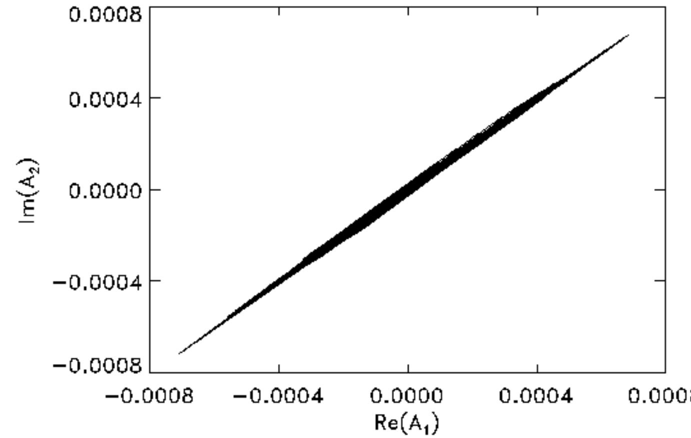
Intensity stripes of random direction appear at bottom of pattern.
They orient themselves while they are amplified and drift

Interference of the modes sustained by noise generates an intensity pattern

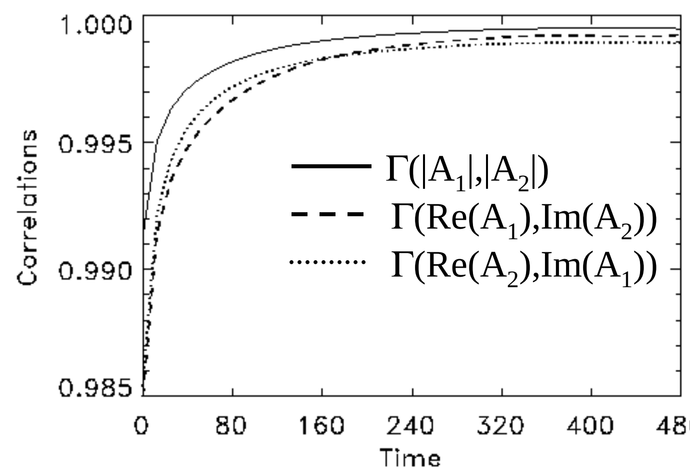
G. Izus, M. Santagiustina, M. San Miguel and P. Colet, JOSA B **16**, 1592-1596 (1999) .

Type II OPO: Synchronization of noise-sustained patterns





$$\Gamma(P,Q) = \frac{\iint (P - \langle P \rangle)(Q - \langle Q \rangle)\,dxdy}{\sqrt{\sigma(P)}\sqrt{\sigma(Q)}}$$



Despite the fact that patterns in A_1 and A_2 are stochastic, macroscopic size structures driven by independent noises, their correlation grows with time and finally leads to synchronization of noise-sustained structures.