IMEDEA

WHEN IS NOISE IMPORTANT?

	LASER	PATTERNS	QSTRUCT
NEAR (BELOW) INSTABILITY	**Spontaneous Emission**	**Noisy Precursors**	**Quantum Images Q.Correlations**
Unstable points: **AMPLIFIED NOISE**	**Laser Switch-on**	*Convective instability* **Noise Sustained Patterns**	**Nonlinear Q. Noise**
MARGINAL MODES: *Symmetry Restoring*	**Laser Linewidth**	**No Long Range Order**	**Complentarity** *Conjugate variables*

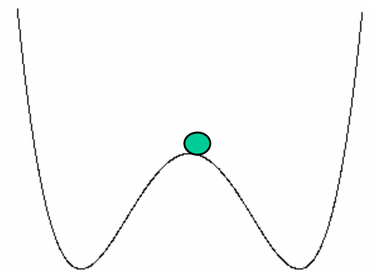
NOISE INDUCED TRANSITIONS

STOCHASTIC RESONANCE

ENTANGLEMENT?

IMEDEA

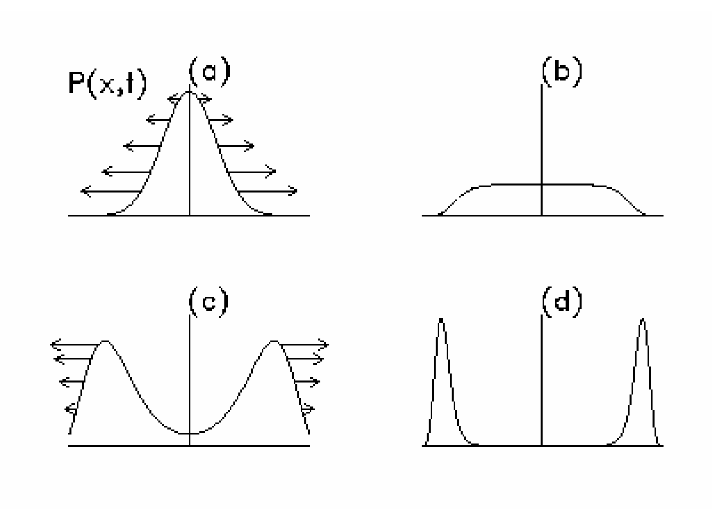
NOISE AMPLIFICATION: Decay of an unstable state



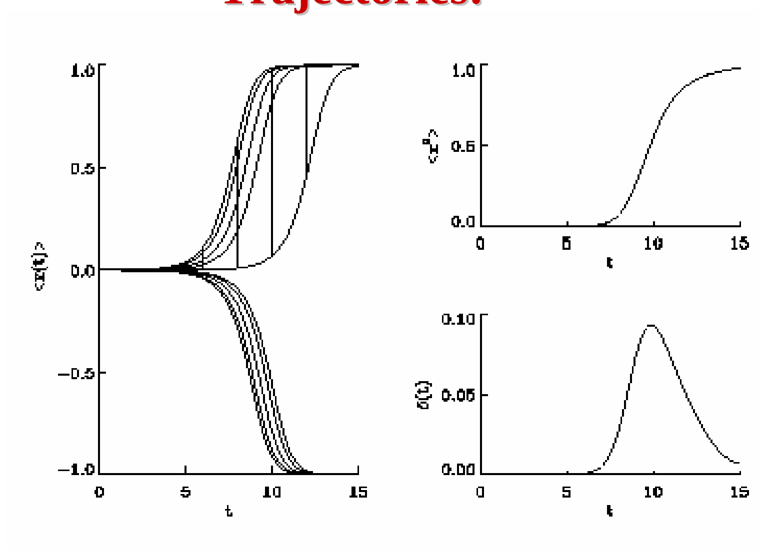
$$d_t x = ax - x^3 + \sqrt{\varepsilon}\,\xi(t)$$

$$a > 0, \quad x(0) = 0$$

Probability distribution:

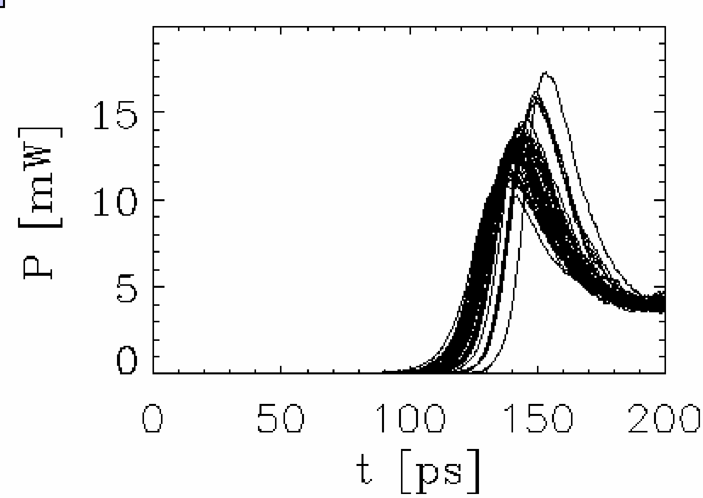


Trajectories:



LASER SWITCH-ON

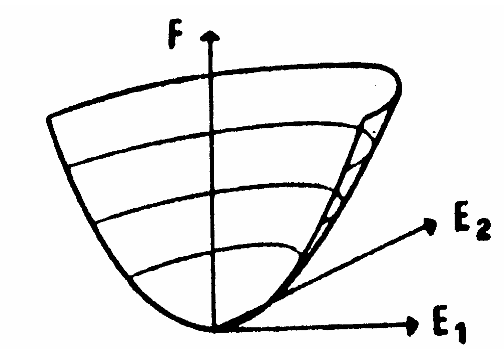
- **Amplification of spontaneous emission noise**
- **Jitter**

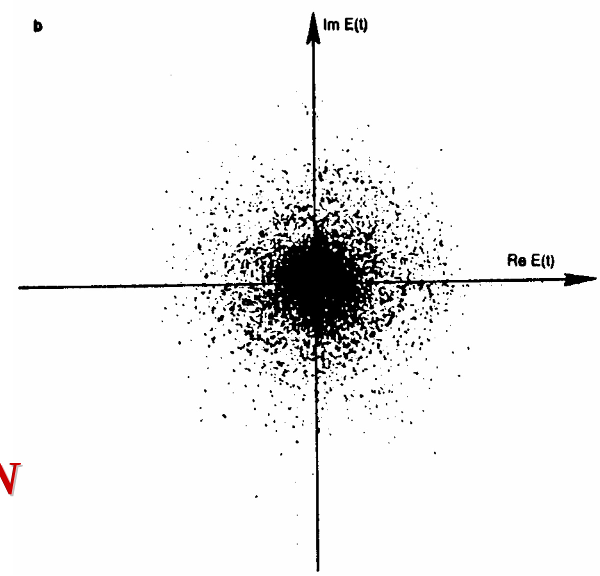


LASER FLUCTUATIONS

$$\partial_t E = aE - |E|^2 E + \sqrt{\varepsilon}\,\xi(t) = -\frac{\partial F}{\partial E^*} + \sqrt{\varepsilon}\,\xi(t), \qquad E = \sqrt{I}\,e^{i\varphi}$$

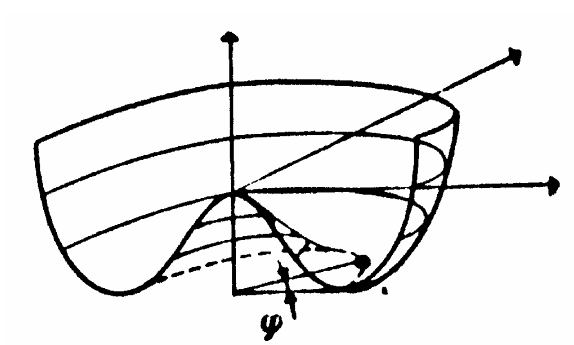
BELOW THRESHOLD

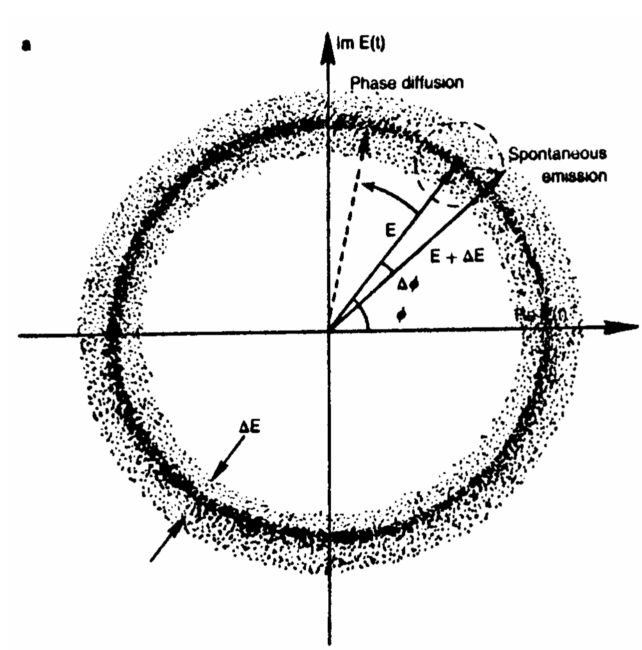




SPONTANEOUS EMISSION

ABOVE THRESHOLD





PHASE DIFFUSION

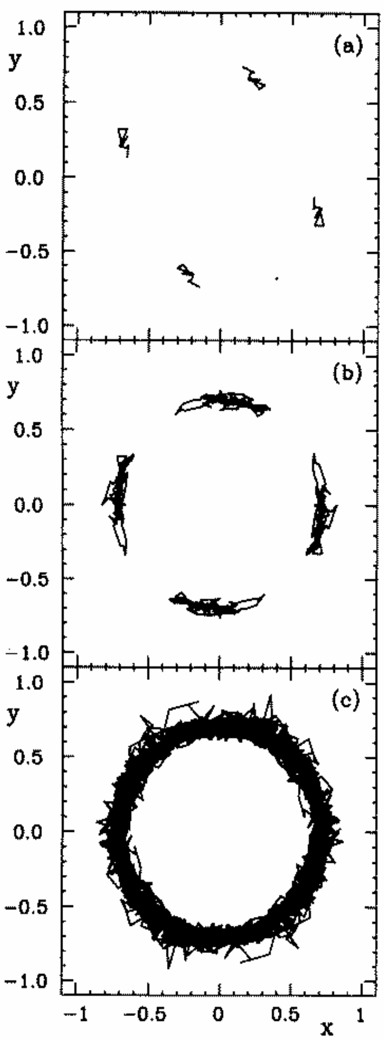
IMEDEA

FLUCTUATIONS IN TRANSVERSE LASER PATTERNS

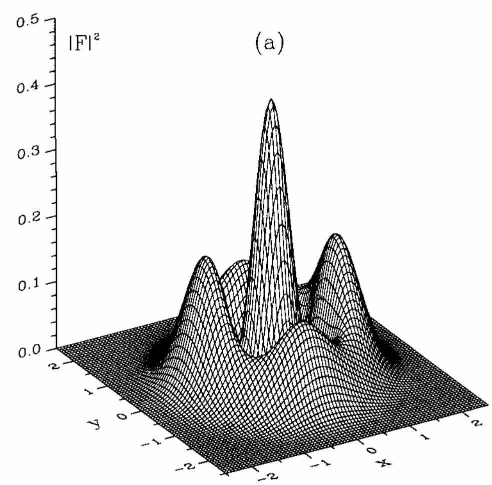
$$E(\rho,\varphi) = \sum_{i=1}^{3} f_i(t) A_i(\rho,\varphi)$$

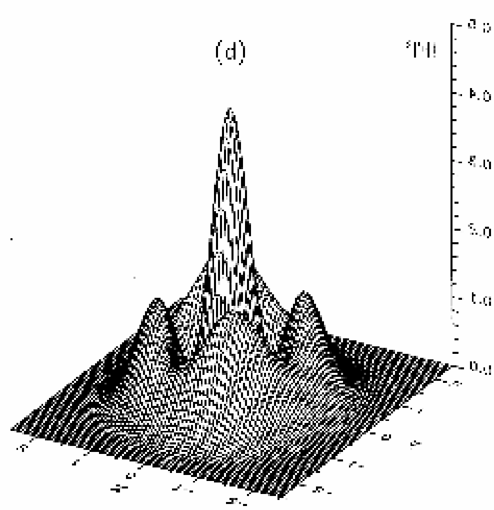
$$\mathrm{d}_t\, f_i = -f_i + 2C\left(M_i f_i - \sum_{i=1}^{3} A_{ijkl}\, f_j\, f_k\, f_l^{*} \right) + \xi_i(t)$$

Marginal phase associated with pattern orientation diffuses.
Broken rotational symmetry restored by noise



time

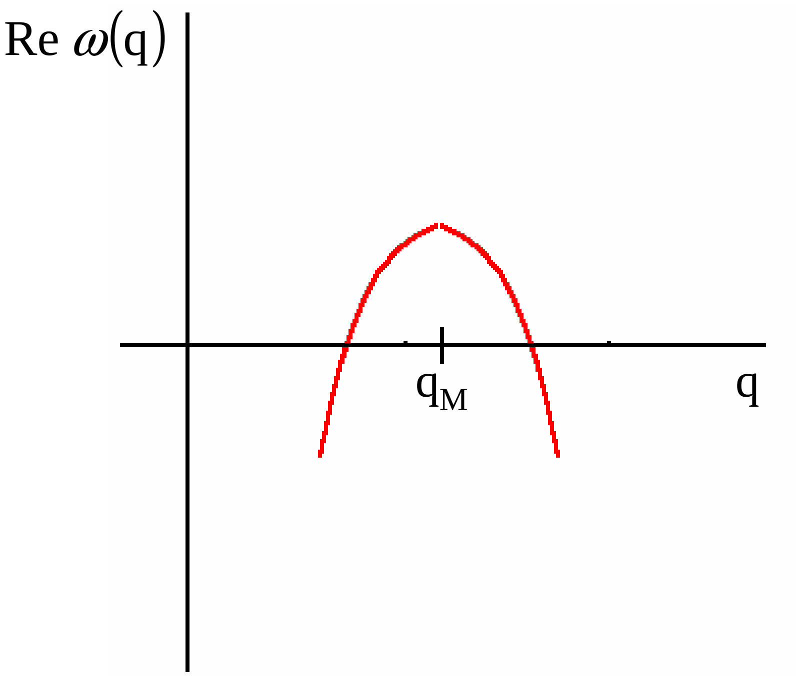




http://www.imedea.uib.es

IMEDEA

PHASE DIFFUSION IN d=1 PATTERN FORMATION



$$\Gamma = A e^{i q_M x} + c.c.$$

$$A = R e^{i\psi} e^{iqx}$$

PHASE MODE: $x \rightarrow x_0 + x \quad \Rightarrow \quad \psi \rightarrow \psi_0 + \psi$

PHASE EQUATION: $\partial_t \psi = D(q) \partial_x^2 \psi + \xi(x,t)$

ECKHAUS STABLE: $D(q) > 0$

PHASE FLUCTUATIONS: $\langle \psi_k^2 \rangle \approx k^{-2}$

$$d = 2 \quad \Rightarrow \quad \int d\mathbf{k} \langle \psi_k^2 \rangle \approx \ln L$$

Symmetry restoring by noise:

NO LONG RANGE ORDER FOR d < d_c=2

IMEDEA

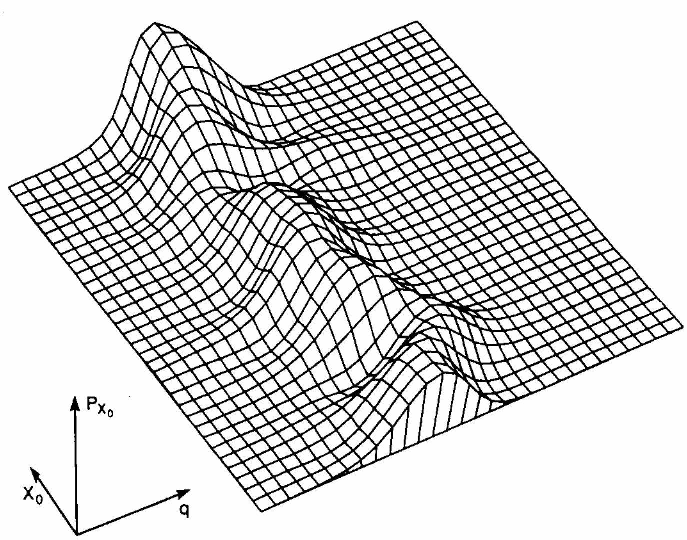
STOCHASTIC d=1 SWIFT-HOHENBERG EQUATION

$$\partial_t \Gamma(x,t) = \left[\mu - \left(q_M + \partial_x^2\right)^2\right]\Gamma - \Gamma^3 + \sqrt{\varepsilon}\,\xi(x,t)$$

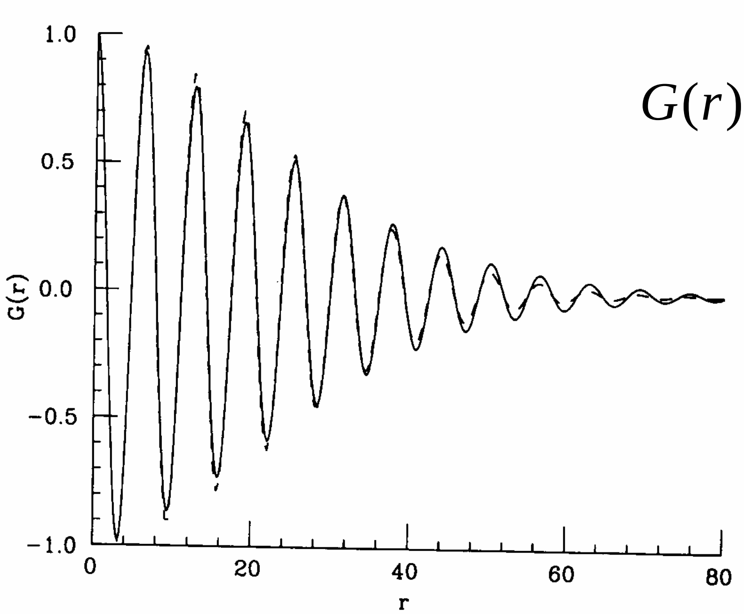
LOCAL POWER SPECTRUM:

$$P_{x_0} \approx \left(\left|\Gamma_q\right|^2\right)_{x_0}$$

- ***LOCAL WAVE NUMBER***
- ***BROAD SPECTRUM***



CORRELATION FUNCTION:



$$G(r) = G(0)\, e^{-\left(\frac{r}{r_0}\right)^2} \cos(q_M r)$$

Correlation length $r_0 \sim L/50$

NO LONG RANGE ORDER

IMEDEA

FLUCTUATIONS AND STRIPE PATTERNS ABOVE THRESHOLD

$$O(\mathbf{r},t) = A_+ e^{iq_M x} + A_- e^{-iq_M x}; \qquad A_+ = R\, e^{i(\phi+\psi)},\ A_- = R\, e^{i(\phi-\psi)}$$

$$O(\mathbf{r},t) = R\, e^{i\phi} \cos(q_M x + \psi)$$

SWIFT-HOHENBERG EQ.

$$\partial_t \Gamma(\mathbf{r},t) = \left[\mu - \left(q_M + \nabla^2\right)^2\right]\Gamma - \Gamma^3 + \sqrt{\varepsilon}\, \xi(\mathbf{r},t)$$

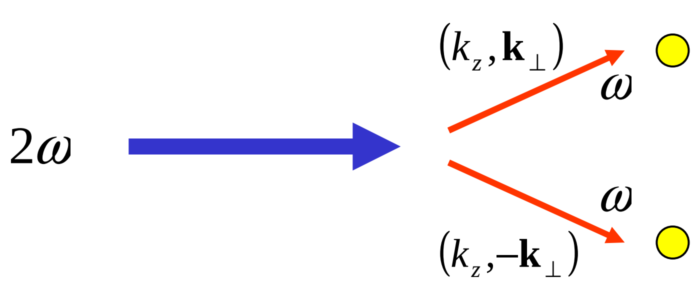
$$O(\mathbf{r},t)\ \text{real}: \quad A_+ = A_-^*, \quad \phi = 0, \quad \psi \ \text{arbitrary}$$

Noise effects: $\begin{cases} I_+ = I_- \ \text{for any noise event} \quad \left(I_\pm = |A_\pm|^2\right) \\ \psi \ \text{diffuses} \end{cases}$

Degenerate OPO:

$$\partial_t A_0 = \gamma_0\left[-(1+i\Delta_0)A_0 + E_0 + i a_0 \nabla^2 A_0 + 2iK_0 A_1^2\right] + \sqrt{\varepsilon_0}\, \xi_0(\mathbf{r},t)$$
$$\partial_t A_1 = \gamma_1\left[-(1+i\Delta_1)A_1 + i a_1 \nabla^2 A_1 + iK_0 A_1^* A_0\right] + \sqrt{\varepsilon_1}\, \xi_1(\mathbf{r},t)$$

$$\mathrm{O}(\mathbf{r},t) \Rightarrow A_1 \ \text{complex}, \quad q_M(\Delta_0), \quad \phi(\Delta_1)\ \text{fixed}, \quad \psi \ \text{arbitrary}$$



$$|\mathbf{k}_\perp| = q_M$$

Noise effects: $\left\{ \begin{array}{l} I_+ - I_- \ \text{small} \quad \left(I_\pm = |A_\pm|^2\right) \\ \psi \ \text{large fluctuations} \end{array} \right\}$ ***COMPLEMENTARITY***

IMEDEA

NOISY PRECURSORS and QUANTUM IMAGES

SWIFT-HOHENBERG EQ.

$$\partial_t \Gamma(\mathbf{r},t) = \left[\mu - \left(q_M + \nabla^2\right)^2\right]\Gamma - \Gamma^3 + \sqrt{\varepsilon}\,\xi(\mathbf{r},t)$$

Linear analysis below threshold:

$$\partial_t \Gamma_{\mathbf{k}} = \omega(|\mathbf{k}|^2)\Gamma_{\mathbf{k}} + \sqrt{\varepsilon}\xi_{\mathbf{k}}, \quad \Gamma_{\mathbf{k}} = \Gamma^*_{-\mathbf{k}}, \quad \omega(|\mathbf{k}|^2) = (\mu - (q_M - |\mathbf{k}|^2)^2) < 0$$

Power Spectrum (“Far field”):

$$I_{\mathbf{k}} = I_{-\mathbf{k}} = |\Gamma_{\mathbf{k}}|^2, \quad \langle I_{\mathbf{k}} \rangle = -\frac{\varepsilon}{\omega(|\mathbf{k}|^2)}$$

$$\langle \Delta I_{\mathbf{k}}(t)\Delta I_{\mathbf{k}'}(t)\rangle = \left(\delta_{\mathbf{k},\mathbf{k}'} + \delta_{\mathbf{k},-\mathbf{k}'}\right)\frac{\varepsilon^2}{\omega^2(|\mathbf{k}|^2)} \quad I_{\mathbf{k}}$$

- Ring of maximum power: $|\mathbf{k}|^2 = q_M$
- $\mathbf{k}$ correlated with $\pm\mathbf{k}$ (Γ real)

Degenerate OPO:

$$\partial_t A_0 = \gamma_0\left[-(1+i\Delta_0)A_0 + E_0 + ia_0\nabla^2 A_0 + 2iK_0 A_1^2\right] + \sqrt{\varepsilon_0}\,\xi_0(\mathbf{r},t)$$

$$\partial_t A_1 = \gamma_1\left[-(1+i\Delta_1)A_1 + ia_1\nabla^2 A_1 + iK_0 A_1^* A_0\right] + \sqrt{\varepsilon_1}\,\xi_1(\mathbf{r},t)$$

Linear analysis below threshold:

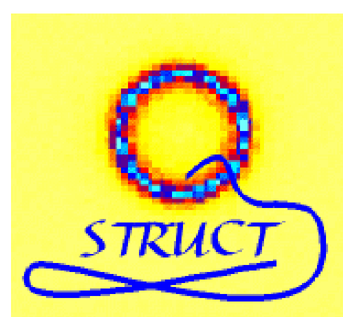
$$\partial_t A_{1,\mathbf{k}} = \nu(|\mathbf{k}|^2)A_{1,\mathbf{k}} + iK_0 A^*_{1,-\mathbf{k}} A_0^s + \sqrt{\varepsilon_1}\,\xi_{\mathbf{k}}, \quad \nu(|\mathbf{k}|^2) = \gamma_1\left[-(1+i\Delta_1) - ia_1|\mathbf{k}|^2\right]$$

Power Spectrum (“Far field”):

$I_{\mathbf{k}} = |A_{1,\mathbf{k}}|^2, \quad I_{\mathbf{k}} \neq I_{-\mathbf{k}}$ but $\mathbf{k}$ and $-\mathbf{k}$ linearly coupled

[illegible]

[illegible]

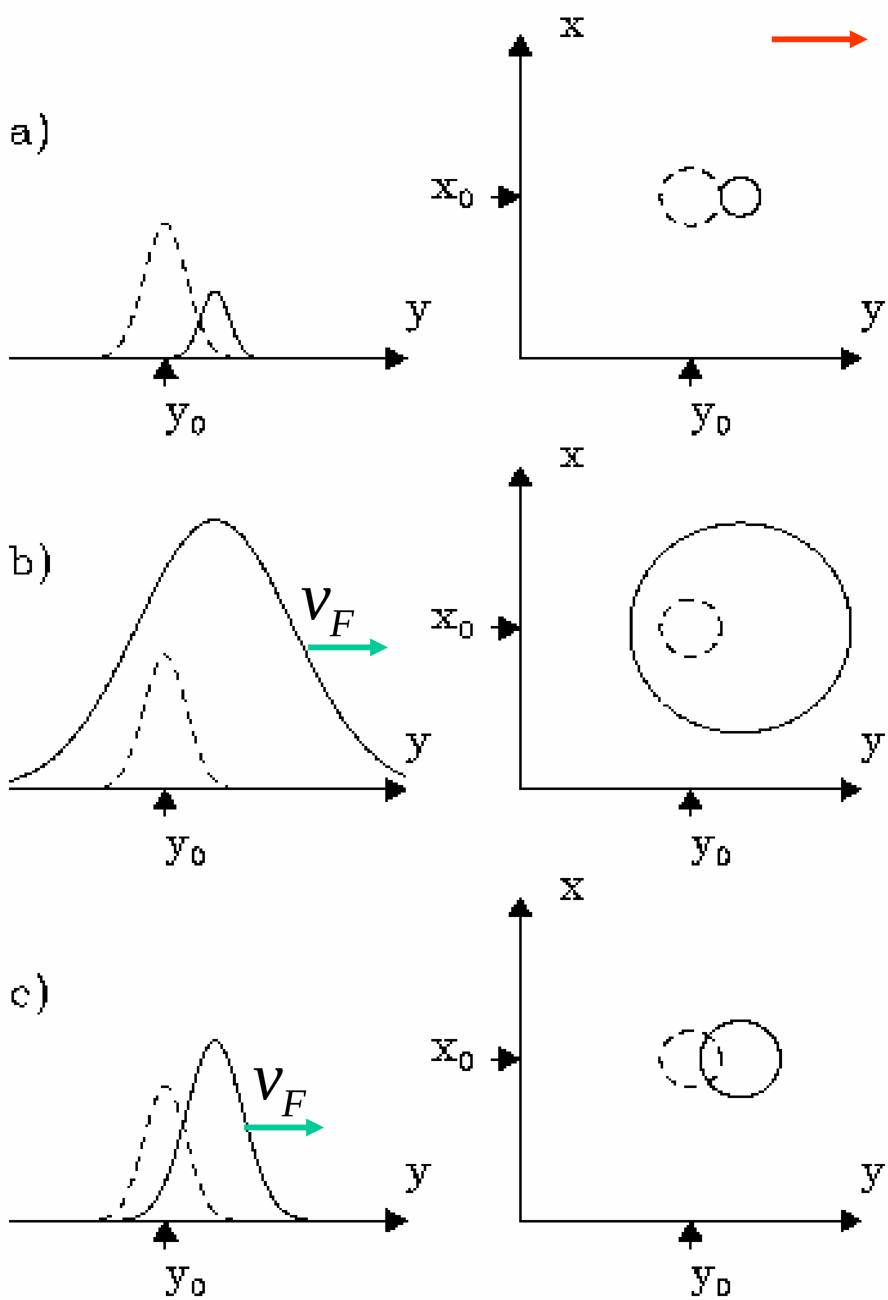


Beware: Pattern selection is a nonlinear mechanism !!

IMEDEA

CONVECTIVE INSTABILITY

$\longrightarrow v\partial_y$



Absolutely Stable

Absolutely Unstable $v_F > v$

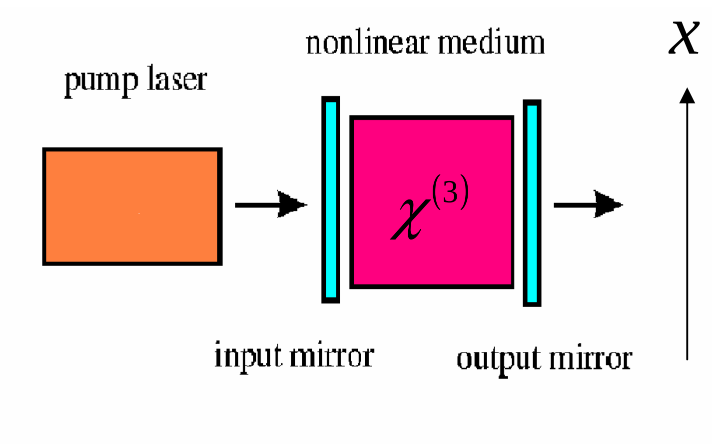
Convectively Unstable $v_F < v$

Absolute Instability Threshold: $v = v_F$

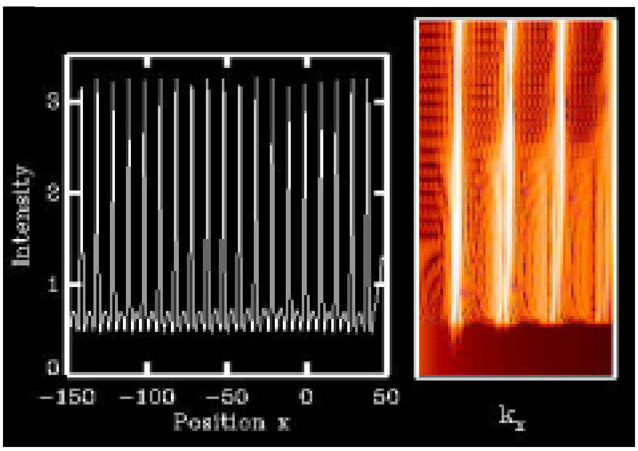
Convectively Unstable Regime: Noise Sustained Structure

IMEDEA

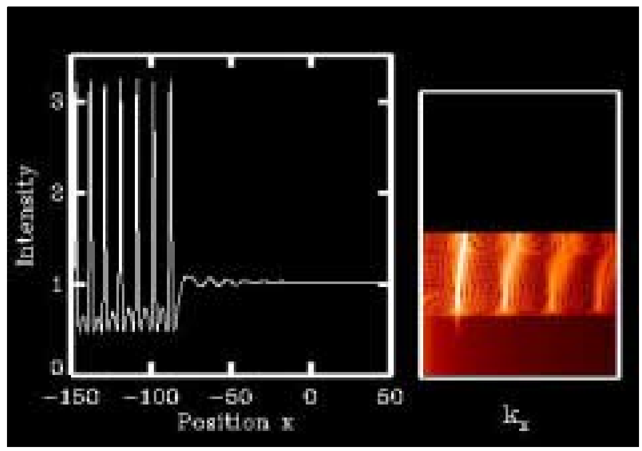
Noise Sustained Structure in a Kerr Resonator



∂_x : Tilted Pump

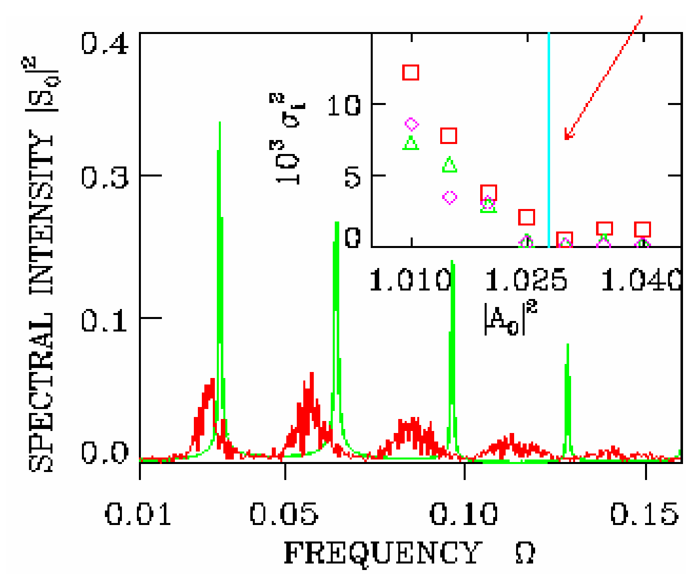


Absolutely Unstable



Convectively Unstable

Spectral Narrowing identifies transition from a NSS to a Deterministic Pattern



http://www.imedea.uib.es

CONCEPTS IN NOISE SUSTAINED STRUCTURES

PHENOMENON OF NOISE AMPLIFICATION, while quantum images are weakly damped critical fluctuations

NOISE NEEDED AT ALL TIMES: A laser requires amplification of spontaneous emission, but when the laser is lasing noise is no longer required to maintain the oscillation

PRECURSOR OF AN ABSOLUTELY UNSTABLE REGIME: An optical amplifier is in a convective regime, but there is no regime of absolute instability

Examples

Fabry-Perot Cavities filled with a cubic (Kerr) nonlinear medium
Transverse drift due to tilted pump

- One-dimensional (roll pattern)
  Santagiustina et al., Phys Rev. Lett. 79, 3633 (97)
- Two-dimensional (hexagonal pattern)
  Santagiustina et al., Quant. Sem. Opt., to appear (99)

Ring optical parametric oscillators (quadratic nonlinearity)
Transverse walk-off due to birefringence

- Type I (one polarization)
  Santagiustina et al., Phys Rev. E 58, 3843 (97)
- Type II (two polarizations)
  New

Type II Optical Parametric Oscillator

$$\partial_t A_0 = \gamma_0\left[-(1+i\Delta_0)A_0 + E_0 + ia_0\nabla^2 A_0 + 2iK_0 A_1 A_2\right] + \sqrt{\varepsilon_0}\,\xi_0(\mathbf{r},t)$$

$$\partial_t A_1 = \gamma_1\left[-(1+i\Delta_1)A_1 + ia_1\nabla^2 A_1 + iK_0 A_2^* A_0\right] + \sqrt{\varepsilon_1}\,\xi_1(\mathbf{r},t)$$

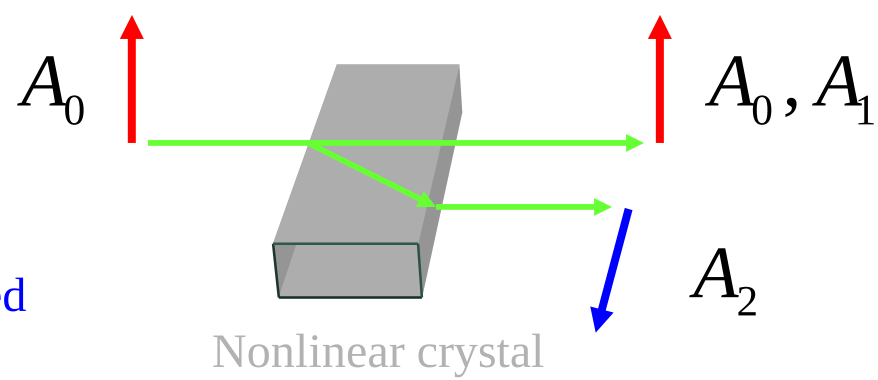
$$\partial_t A_2 = \gamma_2\left[-(1+i\Delta_2)A_2 + ia_2\nabla^2 A_2 + iK_0 A_1^* A_0 + \boxed{\rho_2}\,\partial_y A_2\right] + \sqrt{\varepsilon_2}\,\xi_2(\mathbf{r},t)$$

Walk-off

A_0 Pump field, Ordinary Polarized

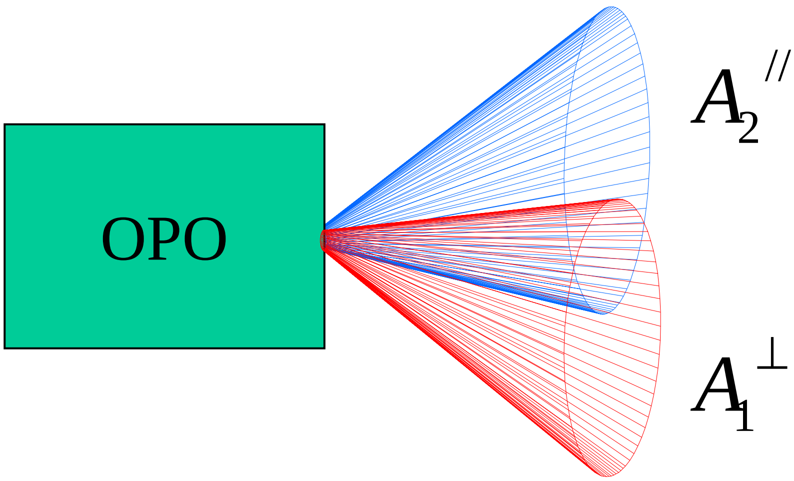
A_1 Signal field, Ordinary Polarized

A_2 Idler field, Extraordinary Polarized



IMEDEA

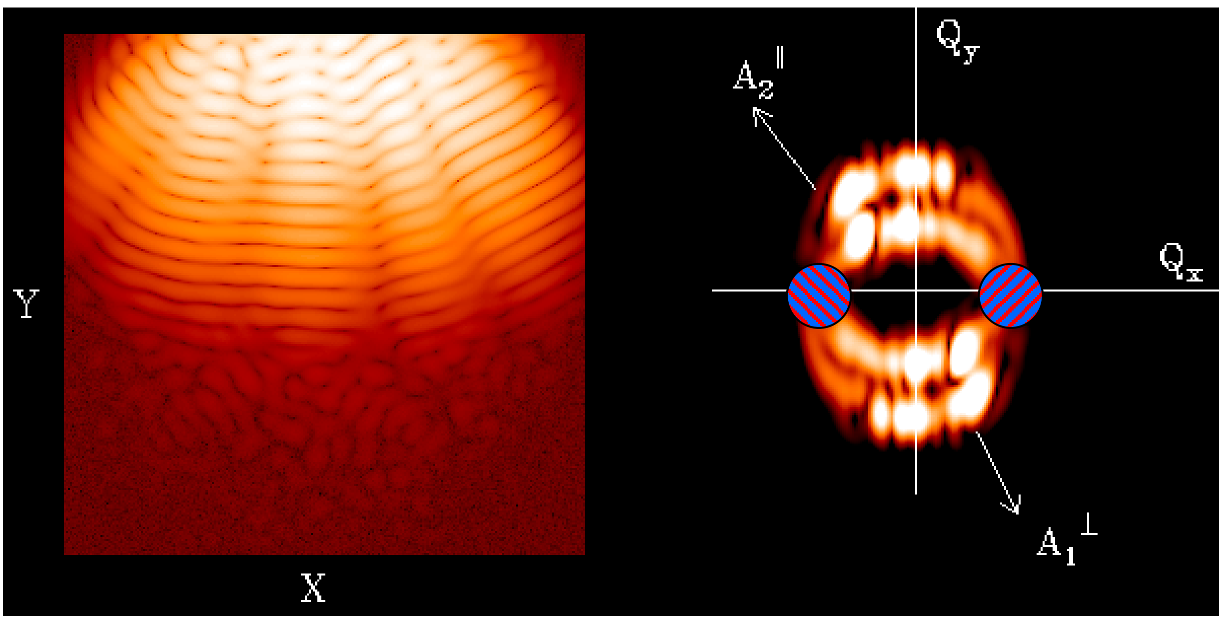
Type II OPO: CONVECTIVE REGIME



Noise Amplification in Optical Cavity

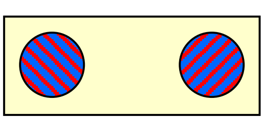
A_1

NEAR FIELD



FAR FIELD

State



1) Nonlinear Q. Correlations $\mathbf{k}_{\perp}, -\mathbf{k}_{\perp}$

2) Polarization Entanglement in a macroscopic Noise Sustained Structure