Computational Studies of the Complex Ginzburg-Landau Equation

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The Complex Ginzburg-Landau Equation (CGLE) is a model equation describing the dynamics of extended systems near a Hopf-bifurcation. Examples of physical context include amplitude waves in binary fluid convection, transversally extended lasers and electrohydrodynamic convection in liquid crystals. Suitable scaling of the complex amplitude A, space and time allows the CGLE to be written as a function of two real parameters b_1 and b_3

$$\partial_t A = A + (1 + ib_1)\nabla^2 A - (b_3 - i) \mid A \mid^2 A, \;\; b_3 > 0, \, x \in [0, L] \,. \tag{1}$$

Recent work[1] in the one-dimensional case has identified numerically regions of parameter $[b_1$–$b_3]$-space displaying different kinds of regular and chaotic behavior. This numerical "phase diagram" and the "transition lines" contained on it deserve a better understanding as well as a theoretical justification. Graham and co-workers[2] have constructed an approximate nonequilibrium potential (NEP) for the 1-d CGLE that could be useful for describing the different phases. In this paper, we numerically (and analytically) check the validity of this approximate nonequilibrium potential as a Lyapunov functional. Our results can be summarized as follows.

- In the no-chaotic region of the parameter space this NEP monotically decrease in time towards the plane wave attractors, as expected for a Lyapunov functional.
- NEP also works for the strange attractor in the phase turbulence region near the Benjamin-Feir instability line.
- NEP is not a correct Lyapunov functional for regions where the singularities of the coherent structures make no longer valid the hypotheses for its approximate construction.
- This NEP has been found to have local minima on spatially chaotic functions. Thus it is of interest to study *per se* the statistical mechanics of such a new model with chaotic ground state.

References

1 H. Chaté, Nonlinearity **7**, 185 (1994) and in *Spatiotemporal Pattern in Nonequilibrium Complex Systems* (Addison-Wesley, New York, 1994), "Santa Fe Institute in the Sciences of Complexity".

2 O. Descalzi and R. Graham, Phys. Lett. A **170**, 84 (1992) and Z. Phys. B **93**, 509 (1994).