Spontaneous ordering against an external field in non-equilibrium systems

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Abstract. We study the collective behavior of non-equilibrium systems subjected to an external field with a dynamics characterized by the existence of non-interacting states. Aiming at exploring the generality of the results, we consider two types of model according to the nature of their state variables: (i) a vector model, where interactions are proportional to the overlap between the states, and (ii) a scalar model, where interactions depend on the distance between states. The phase space is numerically characterized for each model in a fully connected network and in random and scale-free networks. For both models, the system displays three phases: two ordered phases, one parallel to the field and another orthogonal to the field, and one disordered phase. By placing the particles on a small-world network, we show that an ordered phase in a state different from the one imposed by the field is possible because of the long-range interactions that exist in fully connected, random and scale-free networks. This phase does not exist in a regular lattice and emerges when long-range interactions are included in a small-world network.

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Contents

1. Introduction

From the pioneering work by Lenz [1] to modern approaches to complex systems, a rather general question considered in the framework of statistical physics of interacting particles (particles, spins and agents) is the competition between local particle–particle interactions (collective self-organization) and particle interaction with a global externally applied field or with a global mean field [2]–[11]. The commonly accepted answer to this question is that a strong external field dominates local particle–particle interactions and orders the system by aligning particles with the broken symmetry imposed by the field. However, the question arises: is this essential equilibrium concept generally valid for generic non-potential interactions?

The increasing interest of physicists in the statistical modeling of complex systems of sociological origin, such as populations of agents whose interaction patterns are described through graphs or networks [11], has provided the scenarios for investigating new forms of particle–particle and particle–field interactions and for studying many collective phenomena in non-equilibrium systems [12]–[19]. In the context of these studies, it has been shown that a sufficiently intense external field can induce disorder [7, 20, 21], in contrast to the behavior in, for example, Ising-type systems, where the particles tend to acquire the state of the field [22].

In this paper, we challenge the expected effect of an external field by reporting the collective ordering of non-equilibrium systems in a state different from the one imposed by the forcing field. The external field might break the symmetry in a given direction, but the system orders, breaking the symmetry in a different direction. We show that this phenomenon happens in two well-studied non-equilibrium models [11, 13], [23]–[25]. What is common to these two models is that the particle–particle interaction rule is such that no interaction exists for some relative values characterizing the states of the particles that compose the system. This type of interaction is common in social systems where there is often some bound or restriction for the occurrence of interaction between particles, such as a similarity condition for the state variable [23]–[28].

A subsidiary question addressed in this paper is the dependence of this phenomenon on the topology of the network of interactions. We show that the phenomenon is not found for particles interacting with their nearest neighbors in a regular lattice, but occurs in a globally coupled system, in a small-world network, and in random and scale-free networks: it emerges as long-range links in the network that are introduced when going from the regular lattice to a random network via small-world networks [29].

Section 2 discusses a vector model based on the Axelrod model [23] in fully connected, random and scale-free networks; in section 3, we consider a continuous scalar model [24] in the same type of networks. Section 4 considers both models in small-world networks to analyze the role of long-range interactions in the phenomena discussed. Section 5 summarizes our conclusions. In the appendix, we discuss the connection of our study with the site percolation problem in the limit in which particles only interact with an external field.
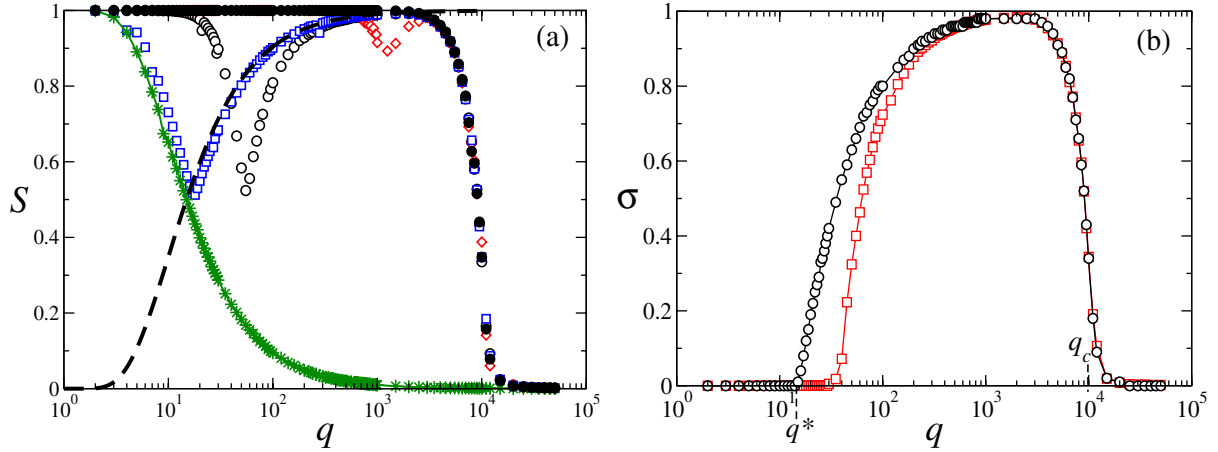
2. The vector model

The first system that we consider is based on the dynamics of cultural dissemination of the Axelrod model [23]. It consists of a set of N particles located at the nodes of an interaction network. The state of particle i is given by an F-component vector C_i^f ($f = 1, 2, \ldots, F$) where each component can take any of q different values $C_i^f \in \{0, 1, \ldots, q-1\}$. The external field, defined as an F-component vector $M^f \in \{0, 1, \ldots, q-1\}$, can interact with any particle in the system.

Starting from a random initial condition, at any given time, a randomly selected particle can interact either with the external field or with one of its neighbors in the interaction network. The dynamics of the system is defined by iterating the following steps:

1. Select at random a particle i.
2. Select the *source of interaction*: with probability B the particle i interacts with the field, while with probability $(1 - B)$ it interacts with one of its nearest neighbors j.
3. The overlap between the selected particle and the source of interaction is the number of shared components between their respective vector states, $d = \sum_{f=1}^F \delta_{C_i^f, X^f}$, where $X^f = M^f$ if the source of interaction is the field, or $X^f = C_j^f$ if i interacts with j. If $0 \leqslant d < F$, with probability d/F, choose h randomly such that $C_i^h \neq X^h$ and set $C_i^h = X^h$; if $d = 0$ or F the state of the particle does not change.

The strength of the field is represented by the parameter $B \in [0, 1]$ that measures the probability for the particle–field interactions. In the absence of an external field, $B = 0$, the system reaches a stationary configuration in any finite network, where for any pair of neighbors i and j, $d(i, j) = 0$ or $d(i, j) = F$. A *domain* is a set of connected particles with the same state. A homogeneous or ordered phase corresponds to $d(i, j) = F$, $\forall i, j$. There are q^F equivalent configurations for this ordered phase. In an inhomogeneous or disordered phase several domains coexist. To characterize the ordering properties of this system, we consider as order parameters the normalized average size of the largest domain S, and of the largest domain displaying the state of the field S_M in the system. The dynamics in a regular and random network displays a critical point q_c that separates two phases: an ordered phase ($S \simeq 1$) for $q < q_c$ and a disordered phase ($S \ll 1$) for $q > q_c$ [13], [30]–[32].

First, we analyze the model in a fully connected network. In the absence of an external field, i.e. $B = 0$, the system spontaneously reaches an ordered phase for values $q < q_c$ (figure 1(a)). For $B \to 0$ and $q < q_c$, the external field M^f imposes its state in the system, as in a two-dimensional network [20]. For $B = 1$, the particles only interact with the external field; in this case, only those particles that initially share at least one component of their vector state with the components of M^f will converge to the field state M^f. The probability that a particle has feature f different from the external field is $(q - 1)/q$; thus, the probability that all F features

Figure 1. (a) S versus q in the vector model on a fully connected network for $B = 0$ (solid circles), $B = 0.005$ (diamonds), $B = 0.05$ (empty circles), $B = 0.5$ (squares) and $B = 1$ (stars). The continuous line is the analytical curve $1 - (1 - 1/q)^F$, whereas the dashed line corresponds to the curve $(1 - 1/q)^F$. (b) $\sigma = S - S_M$ versus q for $B = 0.8$ (circles) and $B = 0.1$ (squares). The values of $q_c \simeq 10^4$ and $q^* \simeq 14$ (for $B = 0.8$). Parameter values are $N = 2500$, $F = 10$. Each data point is an average over 100 independent realizations.

are different from the field is $(1 - 1/q)^F$. The fraction of particles that converge to M^f is the fraction of particles that initially share at least one feature with the external field

$$S_M(B = 1) = 1 - (1 - 1/q)^F . \tag{1}$$

Figure 1(a) shows both the numerically calculated values of S and the analytical curve given by equation (1) for different values of q. Both the quantities are in good agreement, indicating that the largest domain in the system has a vector state equal to that of the external field when $B = 1$.

For intermediate values of B, the spontaneous order emerging in the system for parameter values $q < q_c$ due to the particle–particle interactions competes with the order being imposed by the field. This competition is manifested in the behavior of the order parameter S which displays a sharp local minimum at a value $q^*(B) < q_c$ that depends on B, whereas the value of q_c is found to be independent of the intensity B, as shown in figure 1(a). To understand the nature of this minimum, we plot, in figure 1(b), the quantity $\sigma = S - S_M$ as a function of q. For $q < q^*(B)$ the largest domain corresponds to the state of the external field, $S = S_M$, and thus $\sigma = 0$. For $q > q^*(B)$, the largest domain no longer corresponds to the state of the external field M^f but to other states non-interacting with the external field, i.e. $S > S_M$, and $\sigma > 0$. The value of $q^*(B)$ can be estimated for the limiting case $B \to 1$, for which $S_M \approx 1 - (1 - 1/q)^F$ and the largest domain different from the field is $S \approx 1 - S_M$. Therefore the condition $S = S_M$ yields

$$q^*(B \to 1) = \left[1 - (1/2)^{1/F} \right]^{-1} . \tag{2}$$

For $F = 10$ it gives $q^*(B \to 1) = 15$ in good agreement with the numerical results. The order parameter σ reaches a maximum at some value of q between q^* and q_c. For larger values of q the order decreases in the system and both $S \to 0$ and $S_M \to 0$. As a consequence, σ starts to decrease.
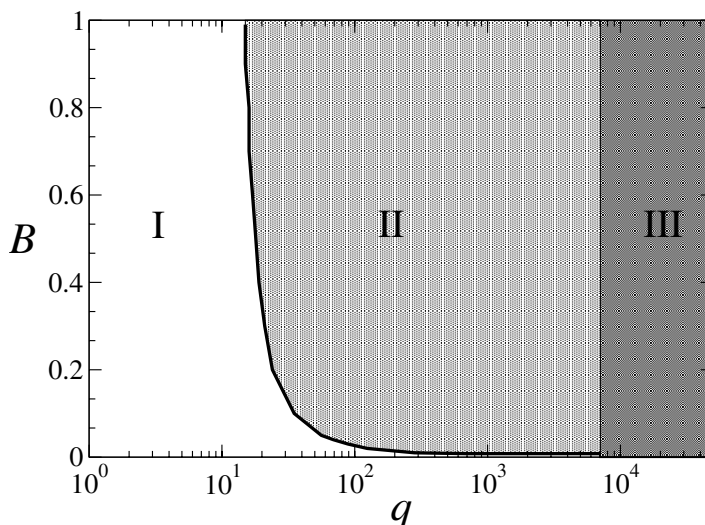
Figure 2. Phase space on the plane (q, B) for the vector model on a fully connected network subject to an external field, with fixed $F = 10$. Regions where phases I, II and III occur are indicated.

The collective behavior of the vector model on a fully connected network subject to an external field can be characterized by three phases on the space of parameters (q, B), as shown in figure 2: (I) an ordered phase induced by the field for $q < q^*$, for which $\sigma = 0$ and $S = S_M \sim 1$; (II) an ordered phase in a state *orthogonal* to the field (i. e. the overlap between the ordered state and the external field is zero) for $q^* < q < q_c$, for which σ increases and $S > S_M$, with $S \sim 1$; and (III) a disordered phase for $q > q_c$, for which σ decreases and $S \rightarrow 0$, $S_M \rightarrow 0$.

For parameter values $q < q_c$ for which the system orders due to the interactions among the particles, a sufficiently weak external field is always able to impose its state to the entire system (phase I). However, for intermediate values of $q < q_c$ if the probability B of interaction with the field exceeds a critical value, the system spontaneously orders in a state orthogonal to the field (phase II).

The emergence of an ordered phase with a state orthogonal to that of an applied field also occurs in complex networks. Figure 3 shows the order parameter S as a function of q for different values of the intensity of the field B for the vector model defined on random and scale-free networks with average degree $\langle k \rangle$ [33]. Again, we observe a local minimum in S at a value $q^*(B) < q_c$. For $q < q^*(B)$ the largest domain corresponds to the state of the external field, $S = S_M$. For $q > q^*(B)$, the largest domain no longer corresponds to the state of the external field but to other states orthogonal to that of the field, i.e. $S_M < S$. However, in contrast to the fully connected network, the size of this alternative largest domain is not big enough to cover the entire system, i.e. $S < 1$.

We note that the limiting case $B = 1$ can be mapped exactly to site percolation with a proper definition of the occupancy probability (see the appendix).

3. Scalar model with continuous states

Models of continuous states based on bounded interactions provide other examples of non-equilibrium systems where induced and spontaneous order compete in the presence of an
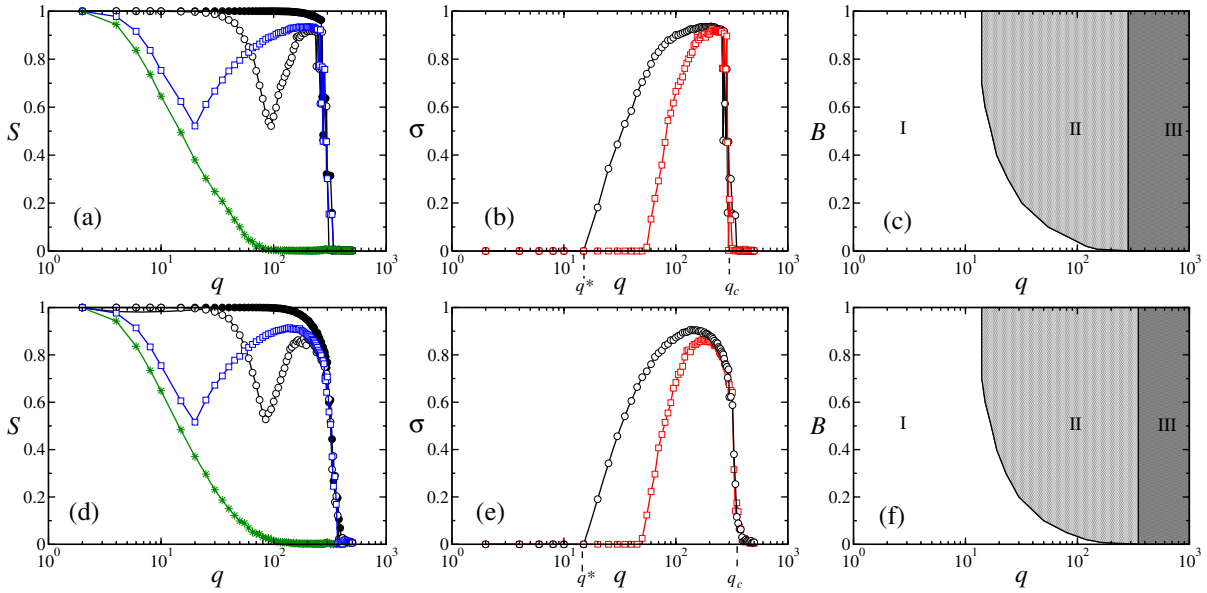
Figure 3. Top panels: the vector model on random networks with $\langle k \rangle = 8$, $N = 5000$, $F = 10$. For these values, $q_c = 285$. (a) S versus q for different values of the parameter B: $B = 0$ (solid circles), $B = 0.05$ (empty circles), $B = 0.5$ (squares) and $B = 1$ (stars). (b) σ versus q for $B = 0.8$ (circles) and $B = 0.1$ (squares). (c) Parameter space on the plane (q, B). Bottom panels: the vector model on scale-free networks with $\langle k \rangle = 8$, $N = 5000$, $F = 10$. For these values, $q_c = 350$. (d) S versus q for different values of the parameter B: $B = 0$ (solid circles), $B = 0.05$ (empty circles), $B = 0.5$ (squares) and $B = 1$ (stars). (e) σ versus q for $B = 0.8$ (circles) and $B = 0.1$ (squares). (f) Parameter space on the plane (q, B). For all simulations, each data point is an average over 100 independent realizations of the underlying network and the dynamics.

external field. Consider, for example, the bounded confidence model [24]. It consists of a population of N particles in an interaction network where the state of particle i is given at time t by a real number $C_i^t \in [0, 1]$. We introduce an external field $M \in [0, 1]$ that can interact with any of the particles in the system. The strength of the field is again described by a parameter $B \in [0, 1]$ that measures the probability for the particle–field interactions, as in the vector model.

We start from a uniform, random initial distribution of the states of the particles. At each time step, a particle i is randomly chosen:

1. with probability B, particle i interacts with the field M: if $|C_i^t - M| < d$, then

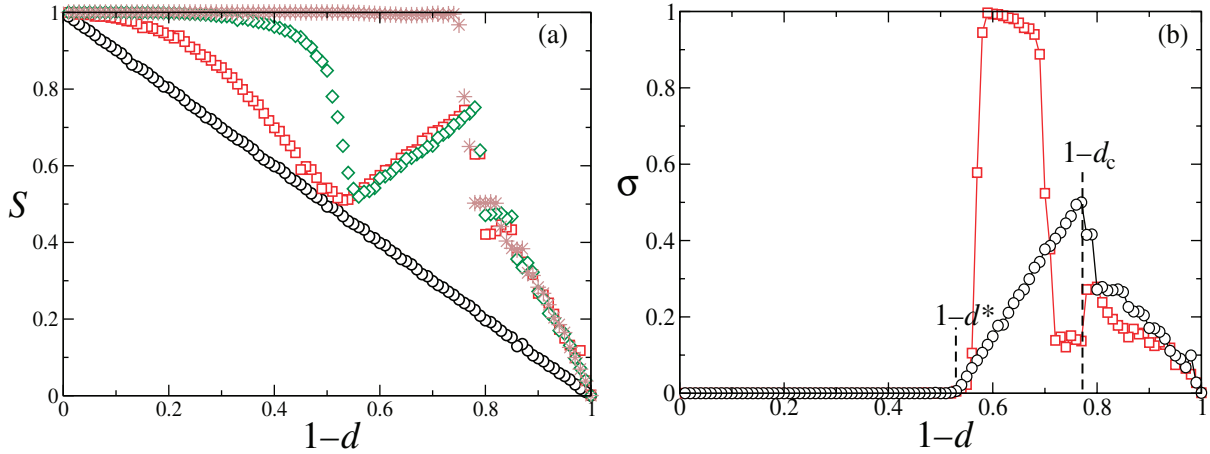$$C_i^{t+1} = \frac{1}{2}(M + C_i^t); \tag{3}$$

   if $|C_i^t - M| \geqslant d$ the state of particle i does not change.

2. Otherwise, a nearest neighbor j in the network is selected at random: if $|C_i^t - C_j^t| < d$, then

$$C_i^{t+1} = C_j^{t+1} = \frac{1}{2}(C_j^t + C_i^t); \tag{4}$$

   if $|C_i^t - C_j^t| \geqslant d$ the state of particle i does not change.

Figure 4. (a) S versus $1-d$ in the continuous model on a fully connected network for $B=0$ (stars), $B=0.5$ (diamonds), $B=0.8$ (squares) and $B=1$ (circles). (b) σ versus d for $B=0.8$ (circles) and $B=0.1$ (squares). The values of $1-d_c \simeq 0.77$ and $1-d^* \simeq 0.5$ (for $B=0.8$). System size $N=2500$. Each data point is an average over 100 independent realizations.

The parameter d defines a threshold distance for interaction. In our simulations, we set $M=1$.

We first consider a fully connected network and calculate the normalized average size of the largest domain S in the system as a function of $1-d$,[4] for different values of B, as shown in figure 4(a). For $B=0$, the system spontaneously reaches a homogeneous state $C_i=0.5, \forall i$, characterized by $S=1$, for values $1-d < 1-d_c \approx 0.77$, with $d_c \approx 0.23$, whereas for $1-d > 1-d_c$ several domains are formed yielding $S < 1$ [24].

For $B=1$, particles only interact with the field; in this case the value of M is imposed on the largest domain whose normalized size increases with the threshold, i.e. $S=S_M=d$. For intermediate values of B, the spontaneous order emerging in the system for values of $1-d < 1-d_c$ due to the interactions between the particles competes with the order being induced by the field. The quantity S exhibits a sharp local minimum at a value $1-d=1-d^* < 1-d_c$, as shown in figure 4(a). In figure 4(b), we plot the order parameter $\sigma = S - S_M$ as a function of $1-d$, for different values of B. For $1-d < 1-d^*$ the largest domain reaches a state equal to M, that is, $S=S_M$, and thus $\sigma=0$. At $1-d = 1-d^*$, the state of the field no longer corresponds to the largest domain, i.e. $S > S_M$, and σ starts to increase as $1-d$ increases. For a small value of B, the quantity σ reaches a maximum close to one, indicating that the spontaneously formed largest domain almost occupies the entire system, i.e. the field is too weak to compete with the attracting homogeneous state $C_i=0.5, \forall i$. However, when B is increased, the maximum of σ is about 0.5, i.e. the attraction of the field $M=1$ increases and the size of the domain with a state equal to M is not negligible in relation to the size of the largest domain. In contrast, in the vector model the maximum $\sigma \to 1$ in the region $q^* < q < q_c$, independently of the value of B.

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[4] We use $1-d$ as the control parameter to compare the scalar model with the vector model introduced in section 2: In the vector model, increasing q decreases the states with whom a given particle can interact; in the scalar model, the same behavior is observed for $1-d$.
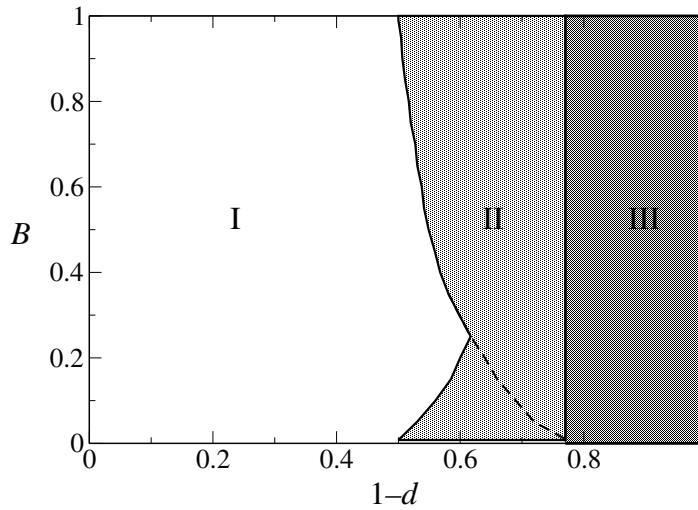
Figure 5. Phase space on the plane $(1-d, B)$ for the scalar model on a fully connected network subject to an external field. Regions where phases I, II and III occur are indicated. The dashed line in phase II separates two regions: one where the maximum of $\sigma \to 1$ (below this line) and another where $\sigma \leqslant 0.5$ (above this line).

The value of d^* in the scalar model depends on B and it can be estimated for $B \to 1$. In this case, $S_M \approx d$ and $S \approx 1-d$; thus the condition $S = S_M$ yields $d^* \approx 0.5$ when $B \to 1$. The quantity σ reaches a maximum at the value $1-d \approx 1-d_c$, above which disorder increases in the system, and both S and S_M decrease. As a consequence, σ decreases for $1-d > 1-d_c$.

As in the vector model, the collective behavior exhibited by the scalar model on a fully connected network subject to an external field can be characterized by three phases: (I) an ordered phase parallel to the field for $1-d < 1-d^*$, for which $\sigma = 0$ and $S = S_M \simeq 1$; (II) a ordered phase for $1-q^* < 1-d < 1-d_c$, for which σ increases and $S > S_M$; and (III) a disordered phase for $1-d > 1-d_c$, for which σ decreases and both S and S_M decrease. Figure 5 shows the phase diagram on the plane $(1-d, B)$ for the scalar model subject to an external field. The continuous curve separating phases I and II gives the dependence $d^*(B)$.

We have also found these three phases in the scalar model defined on random and scale-free networks [34]. Figure 6 shows the order parameter S as a function of $1-d$ for several values of B. The quantity S exhibits a local minimum at a value $1-d = 1-d^* < 1-d_c$. The ordered phase with a state orthogonal to that of the field occurs for $1-d^* < 1-d < 1-d_c$, for which $S > S_M$. As in the vector model, the limiting case $B = 1$ can be mapped to site percolation with a proper definition of the occupancy probability in the underlying network (see the appendix).

4. Short-range interactions

To analyze the role of the connectivity in the emergence of an ordered phase different from the one forced by the external field, we consider a small-world network [29], where the rewiring probability can be varied in order to introduce long-range interactions between the particles. We start from a two-dimensional lattice with nearest-neighbor interactions (degree $k = 4$). Each link is rewired at random with probability p. The value $p = 0$ corresponds to a two-dimensional
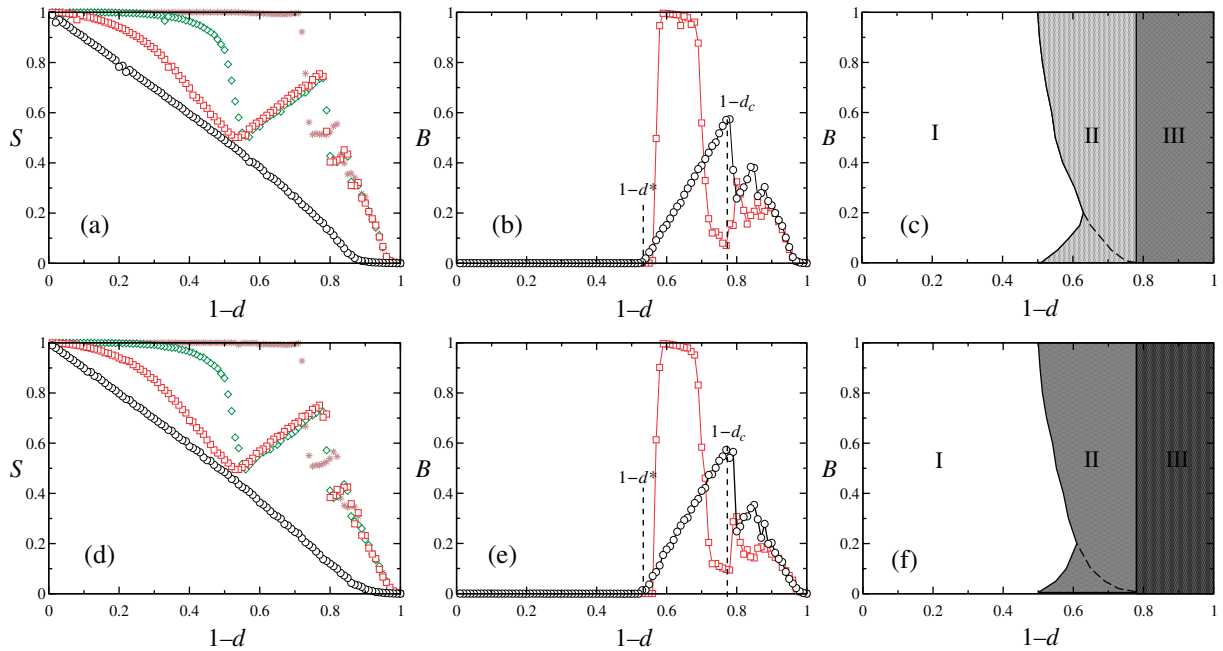
Figure 6. Top panels: The scalar model on random networks with $\langle k \rangle = 8$, $N = 2500$. (a) S versus $1-d$ for different values of the parameter B: $B = 0$ (stars), $B = 0.5$ (diamonds), $B = 0.8$ (squares) and $B = 1$ (circles). (b) σ versus $1-d$ for $B = 0.8$ (circles) and $B = 0.1$ (squares). (c) Parameter space on the plane $(1-d, B)$. Bottom panels: The scalar model on scale-free networks with $\langle k \rangle = 8$, $N = 2500$. (d) S versus $1-d$ for different values of the parameter B: $B = 0$ (stars), $B = 0.5$ (diamonds), $B = 0.8$ (squares) and $B = 1$ (circles). (e) σ versus $1-d$ for $B = 0.8$ (circles) and $B = 0.1$ (squares). (f) Parameter space on the plane $(1-d, B)$. For all simulations, each data point is an average over 100 independent realizations of the underlying network and the dynamics.

regular network with nearest-neighbor interaction, whereas $p = 1$ corresponds to a random network with average degree $\langle k \rangle = 4$.

Figure 7 shows the order parameter S as a function of q in the vector model defined on this network for different values of the rewiring probability p and for a fixed value of the intensity of the field B [30]. The critical value q_c where the order–disorder transition takes place increases with p, which is compatible with the large value of q_c observed in a fully connected network. When the long-range interactions between particles are not present, i.e. $p = 0$, the external field is able to impose its state on the entire system for $q < q_c$. Spontaneous ordering different from the state of the external field appears as the probability of having long-range interactions increases. The size of this alternative largest domain increases with p, but it does not grow enough to cover the entire system (see the inset of figure 7).

Increasing the rewiring probability in the scalar model also produces a behavior similar to the vector model, as shown in figure 8. Thus, in systems whose dynamics is based on a bounded interaction, the presence of long-range connections allows the emergence of spontaneous ordering not associated with the state of an applied external field.
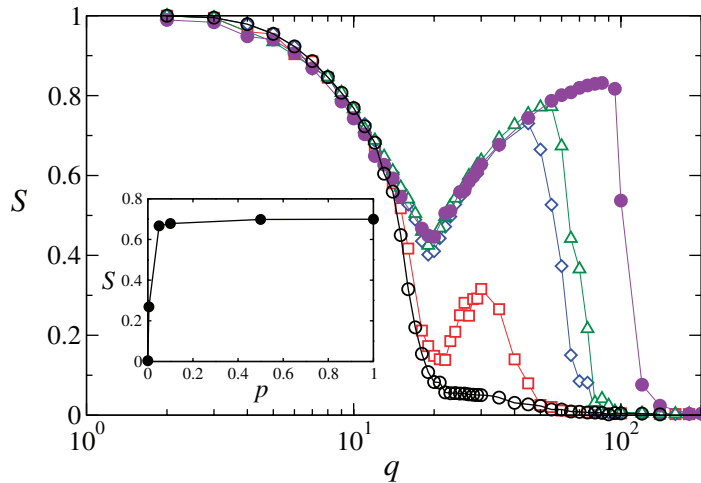
Figure 7. S versus q in the vector model on a small-world network with $\langle k \rangle = 4$, $N = 2500$, $B = 0.5$, $F = 3$, for different values of the probability p: $p = 0$ (empty circles), $p = 0.005$ (squares), $p = 0.05$ (diamonds), $p = 0.1$ (triangles) and $p = 1$ (solid circles). Inset: S versus p for fixed values $q = 40 > q^*$ and $B = 0.5$. Each data point is an average over 100 independent realizations of the underlying network and the dynamics.
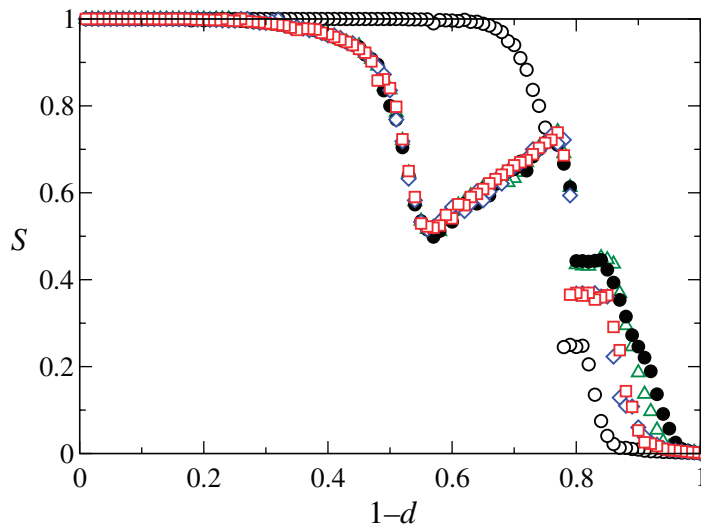
Figure 8. S versus $1 - d$ in the scalar model on a small-world network with $\langle k \rangle = 4$, $N = 10^4$ and $B = 0.5$ for different values of the probability p: $p = 0$ (empty circles), $p = 0.005$ (squares), $p = 0.05$ (diamonds), $p = 0.1$ (triangles) and $p = 1$ (solid circles). Each data point is an average over 100 independent realizations of the underlying network and the dynamics.

5. Conclusions

In summary, we have addressed the question of the competition between collective self-organization and external forcing in non-equilibrium dynamics, as well as the role of network topology in this competition. We have considered two non-equilibrium models with a common

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feature: the existence of non-interacting states in their dynamics. Studying these two models on fully connected, random and scale-free networks, we have found three phases depending on parameter values: two ordered phases, one having the state imposed by the external field, and the other one consisting of a large domain on a state orthogonal to the one selected by the field, and a disordered phase. We have traced back the existence of a self-organized ordered phase in a state different from the external forcing to the presence of long-range connections in the underlying networks considered. This claim is substantiated by considering the two models in a small-world network: we find that such a phase does not exist in a regular network and emerges as long-range interactions are included in a small-world network.

Our results suggest that the emergence of an ordered phase with a state different from that of an external field should arise in other non-equilibrium systems provided they allow for non-interacting states. Potential candidates to show this phenomena are biological systems able to display clustering, aggregation and migration, whose dynamics usually possess a bound condition for interaction. This is the case with models including the presence of motile elements (such as swarms, fish shoals, bird flocks and bacteria colony growth) and non-local interactions in population dynamics [35]–[40].
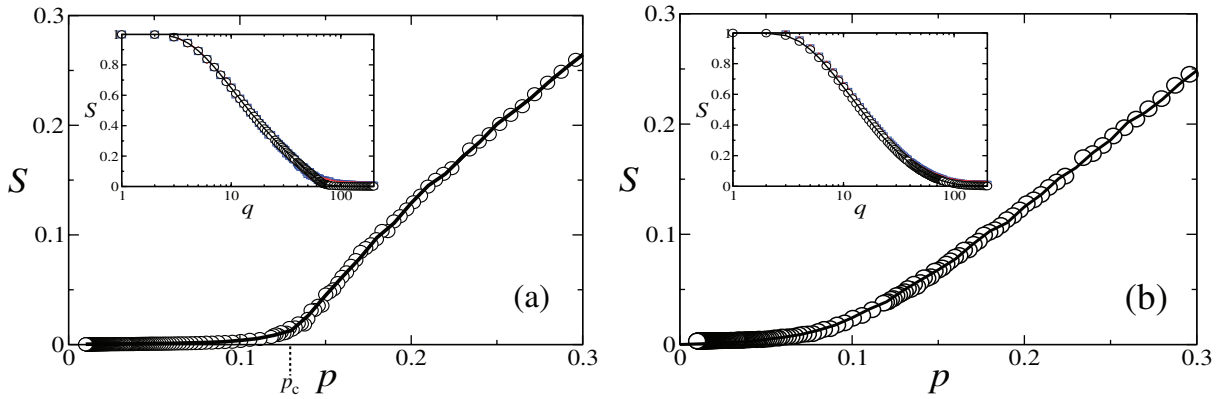
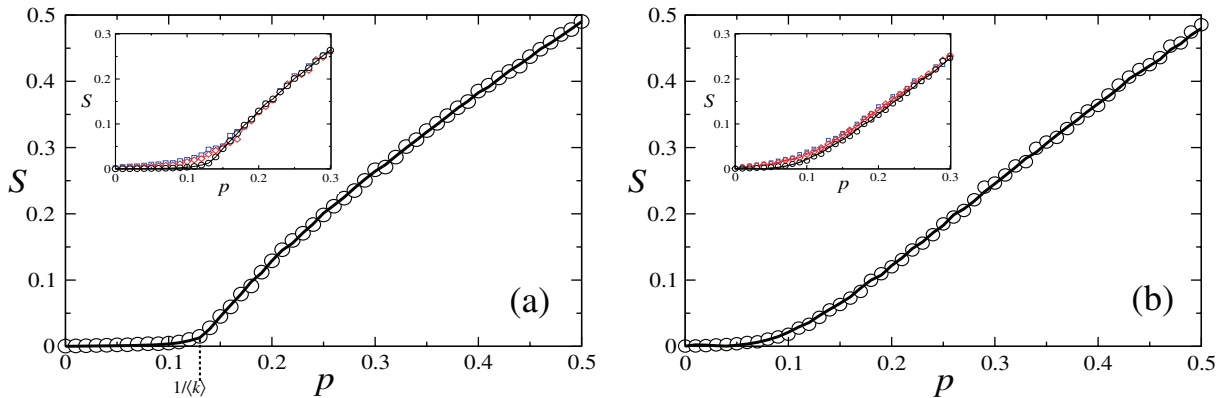## Appendix. External field and site percolation

In this appendix, we address the relation between the limiting case $B = 1$ in the models defined in this paper and site percolation. Site percolation refers to the dependence of the connected components of a network on the fraction of the occupied nodes [41, 42]: given a network, each node can be either occupied with probability p or empty with probability $1 - p$. Following the traditional notation in percolation, a *cluster* is defined as a set of neighboring occupied nodes. In our notation, this is what we defined as a domain. Similarly, the order parameter is the (normalized) size of the largest domain (cluster). Percolation in a complex network typically displays a phase transition for a critical value of the occupancy parameter p [43, 44].

For the models introduced in this paper, the limiting case $B = 1$ corresponds to the situation where the particles only interact with the external field. The particles that change their state are those with overlap different from zero in the vector model (that is, they share at least one feature with the external field) or with state in the range $[1 - d, 1]$ in the scalar model. In these cases, the state of the particles becomes the same as the one of the external field. For zero overlap or states outside the range $[1 - d, 1]$, particles keep their original state. As a consequence, the ordered state in this limiting case $B = 1$ is the one determined by the field. The largest domain for random initial conditions has the state of the external field.

In analogy with site percolation, an occupied node corresponds to a particle that has the state of the external field. In the vector model, only those particles that initially share at least one component of their vector states with the external field will converge to the field state. The

Figure A.1. S versus p for the site percolation model (line) and the vector model with external field $B = 1$ (circles) on (a) a random and (b) a scale-free network with $\langle k \rangle = 8$. For the vector model $p = 1 - (1 - 1/q)^F$, $F = 10$ and $B = 1$. System size $N = 104$. Insets: S versus q in the site percolation model (line) and vector model (symbols) for different system sizes, $N = 10^4$ (circles), 10^3 (diamonds) and 500 (squares). The relation between q for the vector model and the occupancy probability p in site percolation is $q = [1 - (1 - p)^{(1/F)}]^{-1}$. Each data point is an average over 100 independent realizations.

Figure A.2. S versus p for the site percolation model (line) and the scalar model with external field $B = 1$ (circles) on (a) a random and (b) a scale-free network with $\langle k \rangle = 8$. System size $N = 10^4$. Insets: S versus p for site percolation (line) and the scalar model (symbols) for system size $N = 10^4$ (circles), 10^3 (diamonds) and 500 (squares). The relation between d for the scalar model and the occupancy probability p in site percolation is $d = p$. Each data point is an average over 100 independent realizations.

fraction of particles that converge to M^f is given by equation (1)

$$p = 1 - (1 - 1/q)^F. \tag{A.1}$$

Thus, the quantity p can be interpreted as the probability of interaction between the external field and any particle in the system. Similarly, the parameter $p = d$ in the scalar model can be seen as the probability for particle–field interaction when $B = 1$. Therefore, the quantity p in

either model measures the probability that any particle in the system reaches the state of the field, whereas the complementary probability $(1 - p)$ indicates the fraction of particles that do not converge to the state of field. The largest cluster in the site percolation problem can be viewed as the size of the largest domain S in both the vector and the scalar models when $B = 1$.

In order to illustrate the mapping of the models discussed in this paper in an external field with $B = 1$ and site percolation, we have performed simulations of these models in several complex networks. Figure A.1 (figure A.2) shows the normalized size of the largest domain S as a function of the probability p of occupied sites for the site percolation model, and as a function of the quantity $p = 1 - (1 - 1/q)^F$ $[p = d]$ (equation (A.1)) for the vector (scalar) model subject to an external field with $B = 1$, on a random and a scale-free network. The critical values for the onset of a spanning cluster (the ordered state) is $p_c = 1/\langle k \rangle = 0.125$ and $p_c = 0$, respectively, for the random and the scale-free network [43, 44].

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