Consensus and ordering in language dynamics

Xavier Castelló,[1] Andrea Baronchelli,[2] and Vittorio Loreto[3,4]

[1] *IFISC, Institut de Física Interdisciplinària i Sistemes Complexos (CSIC-UIB), Campus Universitat Illes Balears E07122 Palma de Mallorca (Spain)*

[2] *Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Campus Nord B4, 08034 Barcelona (Spain)*

[3] *Dipartimento di Fisica, "Sapienza" Università di Roma, P.le A. Moro 2, 00185 Roma (Italy)*

[4] *Fondazione ISI, Viale S. Severo 65, 10133 Torino (Italy)*

(Dated: January 24, 2009)

We consider two social consensus models, the AB-model and the Naming Game restricted to two conventions, which describe a population of interacting agents that can be in either of two equivalent states (A or B) or in a third mixed (AB) state. Proposed in the context of language competition and emergence, the AB state was associated with bilingualism and synonymy respectively. We show that the two models are equivalent in the mean field approximation, though the differences at the microscopic level have non-trivial consequences. To point them out, we investigate an extension of these dynamics in which confidence/trust is considered, focusing on the case of an underlying fully connected graph, and we show that the consensus-polarization phase transition taking place in the Naming Game is not observed in the AB model. We then consider the interface motion in regular lattices. Qualitatively, both models show the same behavior: a diffusive interface motion in a one-dimensional lattice, and a curvature driven dynamics with diffusing stripe-like metastable states in a two-dimensional one. However, in comparison to the Naming Game, the AB-model dynamics is shown to slow down the diffusion of such configurations.

PACS numbers: 64.60.Cn, 87.23.Ge

I. INTRODUCTION

The formalism, ideas and tools from statistical physics and complex systems have successfully been applied to different disciplines of science beyond the traditional research lines of physics, ranging from biology, to economics and the social sciences [1]. In particular, there has been a fruitful effort in providing models of language dynamics, including dynamics of language competition [2], language evolution [3, 4] and semiotic dynamics [5, 6].

In the field of language competition, the Abrams-Strogatz model [2] has triggered the development of several models which take into account the competition of many [7, 8] or few [9, 10] languages. A review of some of these models can be found in [11]. Building up upon a proposal by Minett and Wang [12, 13], the AB-model is a model of two non-excluding options, in which agents can be in two symmetric states (A or B) and in a third mixed (AB) state of coexisting options at the individual level [14]. It has been used to study the competition between two socially equivalent languages, where AB agents are associated to bilingualism. The model has been studied in two-dimensional and small world networks [14] and in networks with community structure [15, 16]. The final state of the system is always consensus in one of the options, A or B.

In semiotic dynamics, the Naming Game [5, 6, 17] describes a population of agents playing pairwise interactions in order to *negotiate* conventions, i.e., associations between forms and meanings, and elucidates the mechanisms leading to the emergence of a global consensus among them. For the sake of simplicity the model does not take into account the possibility of homonymy, so that all meanings are independent and one can work with only one of them, without loss of generality. An example of such a game is that of a population that has to reach the consensus on the name (i.e. the form) to assign to an object (i.e. the meaning) exploiting only local interactions. However it is clear that the model, originally inspired to robotic experiments [5], is appropriate to address general situations in which negotiation rules a decision process on a set of conventions (i.e. opinion dynamics, etc.). The Naming Game has been studied in fully connected graphs (i.e. in mean-field or homogeneous mixing populations) [5, 6, 17], regular lattices [18], small world networks [19] and complex networks [20, 21]. The final state of the system is always consensus, but stable polarized states can be reached introducing a simple confidence/trust parameter [22]. In this paper, we shall focus on the particular case in which only two options compete within the population [17, 22].

This paper is structured as follows: in Section II we present the microscopic description of the two models studied in this paper, the Naming Game restricted to two conventions and the AB-model. In Section III we look at the macroscopic description of the models, while in Sections IV and V we explore in detail the differences between the two models arising from the different microscopic interaction rules. Finally, we present in Section VI the conclusions as well as a discussion about the implications for language competition of the results obtained in this paper.

II. THE MODELS

We present here the two models considered in this paper: the generalized Naming Game restricted to two conventions [6, 22] and the AB-model [14], extended in such a way that confidence/trust is considered. In both models, we consider a set of N interacting agents embedded in a network. At each time step, and starting from a given initial condition, we select randomly an agent and we update its state according to the dynamical rules corresponding to each model.

In the Naming Game [6, 17], an agent is endowed with an internal inventory in which it can store an a priori unlimited number of conventions. Initially, all inventories are empty. At each time step, a pair of neighboring agents is chosen randomly, one playing as "speaker", the other as "hearer", and negotiate according to the following rules:

- the speaker selects randomly one of its conventions and conveys it to the hearer (if the inventory is empty, a new convention is invented by the speaker);
- if the hearer's inventory contains such a convention, the two agents update their inventories so as to keep only the convention involved in the interaction (*success*);
- otherwise, the hearer adds the convention to those already stored in its inventory (*failure*).

Here we are interested in the particular case in which a population deals with only two competing conventions (say A or B) [17]. We therefore assign to each agent one of the two conventions at the beginning of the process, preventing in this way further invention (that can happen only when the speaker's inventory is empty). Moreover, we adopt the generalized Naming Game scheme [22], in which a confidence/trust parameter β determines the update rule following a success: with probability β the usual dynamics takes place, while with the complementary probability $1-\beta$ nothing happens. The usual case is thus recovered for $\beta = 1$. For brevity we shall refer to this setting (generalized Naming Game restricted to two conventions) as the 2c-Naming Game. In this simplified case, it is easy to see that the transition probabilities are the following [22]:

$$p_{A\to AB} = n_B + \frac{1}{2}n_{AB}, \quad p_{B\to AB} = n_A + \frac{1}{2}n_{AB} \tag{1}$$

$$p_{AB\to A} = \frac{3\beta}{2}n_A + \beta n_{AB}, \quad p_{AB\to B} = \frac{3\beta}{2}n_B + \beta n_{AB} \tag{2}$$

where n_j (j=A, B, AB) are the fraction of agents storing in their inventory the conventions A, B or both A and B, respectively.

For the AB-model [14], an agent can be in three possible states: A, choosing option A (using language A), B, choosing option B (using language B) or AB, choosing both, A and B (using both languages, *bilingual* agent). An agent changes its state with a probability which depends on the fraction of agents in the other states, n_j (j=A, B, AB). The transition probabilities are the following[24]:

$$p_{A\to AB} = \frac{1}{2}n_B, \quad p_{B\to AB} = \frac{1}{2}n_A \tag{3}$$

$$p_{AB\to A} = \frac{1}{2}\beta(1-n_B), \quad p_{AB\to B} = \frac{1}{2}\beta(1-n_A) \tag{4}$$

An agent changes from the A or B state towards the AB state (equation (3)), with a probability proportional to the agents in the opposite option. The probability that an AB agent moves towards the A or B state (equation (4)) is proportional to the density of agents sharing that option, including those in the AB state ($1-n_i = n_j + n_{AB}$; $i,j = A, B$, $i \neq j$). In this paper, we extend the original model in analogy to the extension proposed for the Naming Game in [22]: an agent abandons an option or language according to the dynamics of the AB-model (changes from AB to A or B) with a probability β, while with a probability $1-\beta$ nothing happens. In the context of language competition, the parameter β can be interpreted as an inertia to stop using a language, and at the same time, as a reinforcement of the status of being bilingual, which was not taken into account in the original model (recovered by setting $\beta = 1$).

In both models, a unit of time is defined as N iterations, so that at every unit of time each agent has been updated on average once. To describe the dynamics of the system we use as an order parameter the *interface density* ρ, defined as the fraction of links connecting nodes in different states. When the system approaches consensus, domains grow in size, and the interface density decreases. Zero interface density indicates that an absorbing state, consensus, has been reached. We also use the average interface density, $\langle\rho\rangle$, where $\langle\cdot\rangle$ indicates average over realizations of the stochastic dynamics starting from different random initial conditions.

III. MACROSCOPIC DESCRIPTION

In Section II we have presented the microscopic description of the 2c-Naming Game and the AB-model, i.e., the set of local interactions among the agents. In order to have a macroscopic description of the dynamical evolution of the system as a whole we derive the mean-field equations for the fraction of agents in each state. For the 2c-Naming Game one has [22]:

$$\frac{dn_A}{dt} = -n_A n_B + \beta n_{AB}^2 + \frac{3\beta-1}{2}n_A n_{AB} \tag{5}$$

$$\frac{dn_B}{dt} = -n_A n_B + \beta n_{AB}^2 + \frac{3\beta-1}{2}n_B n_{AB} \tag{6}$$

and $n_{AB} = 1 - n_A - n_B$.

The stability analysis showed that there exist three fixed points [22]: (1) $n_A = 1, n_B = 0, n_{AB} = 0$; (2)

$n_A = 0, n_B = 1, n_{AB} = 0$ and (3) $n_A = b(\beta), n_B = b(\beta), n_{AB} = 1 - 2b(\beta)$ with $b(\beta) = \frac{1+5\beta-\sqrt{1+10\beta+17\beta^2}}{4\beta}$ (and b(0)=0). A non-equilibrium phase transition occurs for a critical value $\beta_c = 1/3$. For $\beta_c > 1/3$ consensus is stable. For $\beta_c < 1/3$ a change of stability gives place to a stationary coexistence of $n_A = n_B$ and a finite density of undecided agents n_{AB}, fluctuating around the average values $b(\beta)$ and $1 - 2b(\beta)$. In the Naming Game with invention, in fact, the one observed at $\beta_c = 1/3$ is the first of a series of transitions yielding the asymptotic survival of a diverging (in the thermodynamic limit) number of conventions as $\beta \to 0$ [22].

For the AB-model one has:

$$\frac{dn_A}{dt} = \frac{1}{2}(-n_A n_B + \beta n_{AB}^2 + \beta n_A n_{AB}) \tag{7}$$

$$\frac{dn_B}{dt} = \frac{1}{2}(-n_A n_B + \beta n_{AB}^2 + \beta n_B n_{AB}) \tag{8}$$

and $n_{AB} = 1 - n_A - n_B$.

The stability analysis shows that there exist three fixed points: (1) $n_A = 1, n_B = 0, n_{AB} = 0$; (2) $n_A = 0, n_B = 1, n_{AB} = 0$ and (3) $n_A = f(\beta), n_B = f(\beta), n_{AB} = 1 - 2f(\beta)$ with $f(\beta) = \frac{3\beta-\sqrt{\beta(\beta+4)}}{2(2\beta-1)}$.

Notice that in both models, at $\beta = 0$ the third fixed point becomes a stable absorbing state in which the system reaches consensus in the AB-state.

Surprisingly, the two original models ($\beta = 1$) are equivalent in the mean-field approximation. There is just a different time scale coming from the prefactor 1/2 in the AB-model (equations (7) and (8)). The mean-field approximation is exact in the thermodynamic limit, and valid for large systems in fully connected networks. However, the two models differ at their local interactions (see equations (1-4) for $\beta = 1$). To explore the effects of these differences at the microscopic level, in Section IV we investigate, in a complete graph, the role of the parameter β as described in equations (5-8), while in Section V we focus on the interface motion in regular lattices for the original case $\beta = 1$.

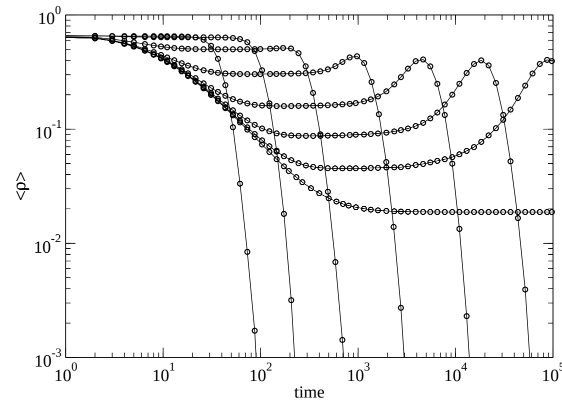

FIG. 1: AB-model: time evolution of the average interface density, $\langle\rho\rangle$, in a fully connected network of $N = 10000$ agents for different values of β. From left to right: $\beta = 1.0, 0.2, 0.05, 0.01, 0.002, 0.0005, 0.0001, 0.0$. Averaged over 1000 runs.

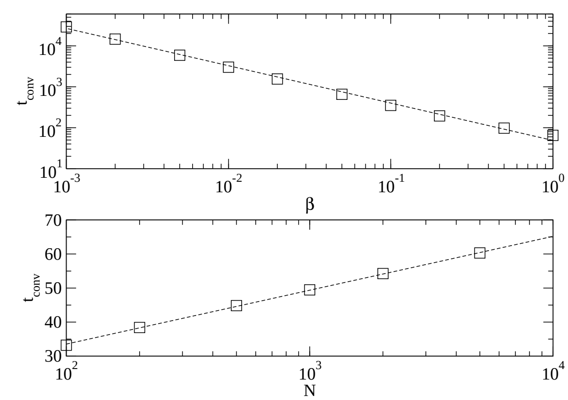

FIG. 2: AB-model. Top: scaling of the time of convergence with β for $N = 10000$: $t_{conv} \sim \beta^{-1}$. Averaged over 200-800 runs depending on the value of β. Bottom: scaling of the time of convergence with system size N for $\beta = 1$: $t_{conv} \sim ln(N)$. Averaged over 10000 runs.

IV. PHASE TRANSITION

Here we consider the extension of the AB-model presented above in a fully connected network, with the aim to explore a possible non-equilibrium phase transition in β similar to the one found in the 2c-Naming Game. In Figure 1 we show the time evolution of the average interface density, $\langle\rho\rangle$, for different values of the parameter β. For large values of β, $\langle\rho\rangle$ reaches a plateau followed by a finite size fluctuation that drives the system to an absorbing state. However, for $\beta \lesssim 0.01$ we observe that after $\langle\rho\rangle$ reaches the plateau, it increases again, reaching a maximum value after which a finite size fluctuation drives the system to consensus. In finite systems and for $\beta = 0$, the system reaches a constant value of $\langle\rho\rangle$, a frozen state corresponding to almost consensus in the AB-state, except for a small fraction of agents (less than 1% on average for $N = 10000$). This fraction decreases as N increases, and complete consensus in the AB-state is reached in the thermodynamic limit (see stability analysis in the previous section). For $\beta = 0$, $p_{AB\to A} = p_{AB\to B} = 0$, so the only possible evolution is that A and B agents move towards the AB state. At the last stage of the dynamics, when n_A and n_B approach to zero, as soon as one of the two single-option densities, n_i, vanishes, n_j remains constant ($i, j = A, B, i \neq j$) because $p_{j\to AB} \sim n_i$, giving rise to the small fraction of agents in the state j present in the frozen state. The time to convergence scales with beta as $t_{conv} \sim \beta^{-1}$ (Figure 2-top), as observed for the 2c-Naming Game for $\beta > \beta_c$ ($t_{conv} \sim (\beta - \beta_c)^{-1}$) [22].

Contrary to the phase transition described in Section III obtained in the 2c-Naming Game [22], there is no phase transition in the AB-model: at $\beta = 0$, the system reaches trivially a frozen state (dominance of the AB-state, with complete consensus in the thermodynamic limit); while for $\beta > 0$ the final absorbing state is, as usual, consensus in the A or B option. Even though the two original models are equivalent in the mean-field approximation (case $\beta = 1$), we observe two different behaviors when the parameter β is taken into account. This can be shown formally by looking at the time evolution of the *magnetization*, $m \equiv n_A - n_B$. For the 2c-Naming Game and the AB-model, we have respectively:

$$\frac{dm}{dt} = \frac{3\beta - 1}{2} n_{AB} m \tag{9}$$

$$\frac{dm}{dt} = \frac{1}{2} \beta n_{AB} m \tag{10}$$

In equation (9), we can observe the origin of the non-equilibrium phase transition described in Section III for the 2c-Naming Game. The time derivative of the magnetization, $\frac{dm}{dt}$, vanishes at the critical point $\beta_c = 1/3$. For $\beta_c > 1/3$, $sign(\frac{dm}{dt}) = sign(m)$, and therefore $|m| \rightarrow 1$: the system is driven to an absorbing state of consensus in the A or B option. For $\beta_c < 1/3$, $sign(\frac{dm}{dt}) = -sign(m)$ and $|m| \rightarrow 0$, giving rise to stationary coexistence of the three phases, with $n_A = n_B$ and a finite density of AB agents. For the AB-model, instead, we can see in equation (10) that for $\beta > 0$, $sign(\frac{dm}{dt}) = sign(m)$ so that consensus in the A or B option is always reached. The time derivative of the magnetization, $\frac{dm}{dt}$, vanishes at $\beta = 0$, where the dynamics gets stuck in an absorbing state corresponding to consensus in the AB state in the thermodynamic limit. Therefore, the phase transition observed in the 2c-Naming Game is not observed.

The reason of the difference shown above has to be found in the differences that these models have at the microscopic level. The fact that in the AB-model A and B agents do not feel the influence of AB agents is the key point which explains the different nature of the transition for this model. For the case $\beta = 1$, in the 2c-Naming Game the second term in equation (1) (influence of AB agents on the A or B agents) and the first term in equation (2) (influence of the A or B agents on the AB agents) combine in such a way that the mean-field equations are equivalent to the ones in the AB-model. However, when $\beta < 1$ the combination of these terms originate the phase transition from an absorbing final state towards a dynamical stationary state of coexistence, as found in [22].

In Figure 2-bottom, we observe for the AB-model the scaling of t_{conv} with the system size for $\beta = 1$: $t_{conv} \sim ln(N)$, indicating that t_{conv} increases slowly with the system size. As expected, this compares properly to the same scaling already obtained for the 2c-Naming Game in [17], since the two models are equivalent in the mean field for $\beta = 1$.

To understand the time evolution of the average interface density $\langle\rho\rangle$ shown in Figure 1 for $\beta \lesssim 0.01$, we show in Figure 3 the time evolution of ρ and the densities of agents in each state, n_A, n_B and n_{AB}, for a typical realization of the dynamics and a given small β. Because of the inertia of the AB agents to move away from their state (small β), at the beginning we observe an increase of n_{AB} together with the corresponding decrease of n_A, n_B and ρ. Then, ρ and the three densities reach a plateau. Most of the agents are in the AB state, while a competition between options A and B takes place, with $n_A \simeq n_B < n_{AB}$. This metastable state lasts longer as we increase the system size. At a certain point, however, a system size fluctuation drives the density of one of the two states (A in the figure) to zero, while the other (B in the figure) starts gaining ground. Since there are less and less agents in the state becoming extinct, and agents having one option do feel only the presence of agents in the opposite state, agents in the dominant state become more and more stable, until, when the other state disappears, they become totally stable. During this process, the interface density increases as n_{AB} decreases. The peak of ρ corresponds to the point where $n_{AB} = n_i$ (being i the state which takes over the whole system, B in the figure). When one of the states has vanished (A in the figure), the AB agents slowly move towards the remaining state (B in the figure) and the system reaches consensus.

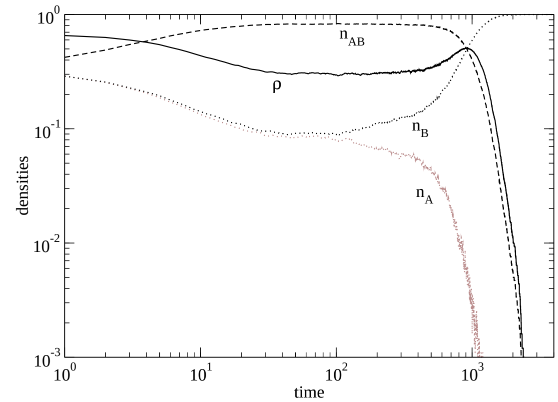

FIG. 3: AB-model. Density of agents in state A, n_A (dotted gray), in state B, n_B (dotted black), in state AB, n_{AB} (dashed); and interface density, ρ (solid line) for a typical realization of the AB-model. In the plateau, $n_A \sim n_B \simeq 0.1$, while the majority of agents are in the AB-state. Fully connected network, $\beta = 0.01$ and $N = 10000$ agents.

V. INTERFACE DYNAMICS: 1-D AND 2-D LATTICES

We study and compare here the interface dynamics in regular lattices with periodic boundary conditions for the two original models ($\beta = 1$). The 2c-Naming Game has been shown to exhibit a diffusive interface motion in a one-dimensional lattice, with a diffusion coefficient $D = 401/1816 \simeq 0.221$ [18]. We therefore focus on the AB-model, and, to analyze the interface dynamics in a one-dimensional lattice with N agents, we consider a single interface between two linear clusters of agents. In each of the clusters, all the agents are in the same state. We consider a cluster of agents in the state A on the left and another cluster of agents in the B state on the right. We call C_m an interface of m agents in state C (for clarity, here C labels an AB agent). Due to the dynamical rules, the only two possible interface widths are C_0, corresponding to a two directly connected clusters $\cdot\cdot AAABBB \cdot\cdot$, or C_1, corresponding to an interface of width one, $\cdot\cdot AAACBBB \cdot\cdot$. It is straightforward to compute the probability $p_{0,1} = 1/2N$ that a C_0 interface becomes a C_1 in a single time step. Otherwise, it stays in C_0. In the same way, $p_{1,0} = 1/2N$. We are now interested in determining the stationary probabilities of the Markov chain defined by the transition matrix

$$M = \begin{pmatrix} 1 - \frac{1}{2N} & \frac{1}{2N} \\ \frac{1}{2N} & 1 - \frac{1}{2N} \end{pmatrix} \tag{11}$$

in which the basis is $\{C_0, C_1\}$. The stationary probability vector, $\mathbf{P} = \{P_0, P_1\}$ is computed by imposing $\mathbf{P}(t+1) - \mathbf{P}(t) = 0$, i.e., $(M^T - I)\mathbf{P} = 0$. We obtain $P_0 = 1/2, P_1 = 1/2$. Since the interface has a bounded width, we assume that it can be modeled as a point-like object localized at position $x = (x_l + x_r)/2$, where x_l is the position of the rightmost site of cluster A, and x_r the leftmost site of cluster B. An interaction $C_m \to C_{m'}$ corresponds to a set of possible movements for the central position x. We denote by $W(x \to x \pm \delta)$ the transition probability that an interface centered in x moves to to the position $x \pm \delta$. The only possible transitions are: $W(x \to x \pm \frac{1}{2}) = \frac{1}{4N}P_0 + \frac{1}{4N}P_1$. Using the results obtained for the stationary probability vector we get $W(x \to x \pm \frac{1}{2}) = \frac{1}{4N}$. We are now able to write the master equation for the probability $P(x,t)$ to find the interface in position x at time t. In the limit of continuous time and space:

$$P(x,t+1) - P(x,t) \approx \delta t \frac{\partial P(x,t)}{\partial t}, \tag{12}$$

$$P(x+\delta x,t) \approx P(x,t) + \delta x \frac{\partial P(x,t)}{\partial x} + + \frac{1}{2}(\delta x)^2 \frac{\partial^2 P(x,t)}{\partial x^2} \tag{13}$$

In this limit, the master equation reads:

$$\frac{\partial P(x,t)}{\partial t} = \frac{D}{N} \frac{\partial^2 P(x,t)}{\partial x^2} \tag{14}$$

where the diffusion coefficient is $D = 1/16 = 0.0625$ (in the appropriate dimensional units $(\delta x)^2/\delta t$). These analytical results are confirmed by numerical simulations. In Figure 4 we show the time evolution of $P(x,t)$, which displays a clear diffusive behavior. The mean-square distance follows a diffusion law $\langle x^2 \rangle = 2D_{exp}t$, with $D_{exp} = 0.06205$ being the diffusion coefficient obtained numerically.

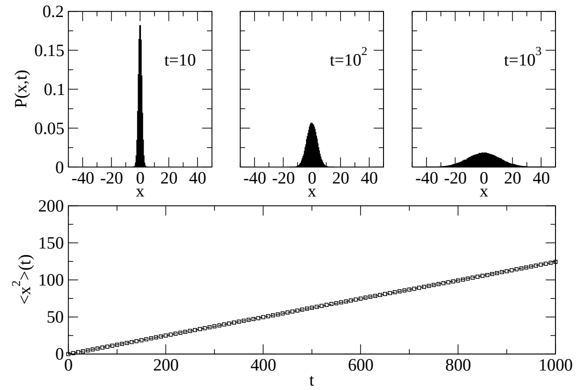

FIG. 4: AB-model: evolution of the position of an interface in a one-dimensional regular lattice. *Top*: time evolution of the distribution $P(x,t)$. *Bottom*: time evolution of the mean square displacement $\langle x^2(t) \rangle = 2D_{exp}t$. The value $D_{exp} = 0.06205$ obtained from the fitting is in perfect agreement with the theoretical prediction $D = 1/16 = 0.0625$.

FIG. 5: Initial conditions with one half of the lattice in the A state, and the other half in the B state. $N = 64^2$ *Top*: AB-model. Stripe-like metastable state. t=0, 100, 200 from left to right. *Bottom*: 2c-Naming Game: t=0, 50, 100 from left to right. Snapshots are selected taking into account the different time scale coming from the prefactor 1/2 in the meanfield equations (7) and (8).

Thus, the AB-model and the 2c-Naming Game display the same diffusive interface motion in one-dimensional lattices, but they differ in about one order of magnitude in the diffusion coefficient, indicating that in the AB-model interfaces diffuse much slower. It can also be seen that the growth of the typical size of the clusters is $\zeta \sim t^{\alpha}$, with $\alpha \simeq 0.5$, leading to the well known coarsening process found also in SFKI models [23] (not shown).

In two-dimensional lattices, on the other hand, it has been shown that starting from random initially dis-

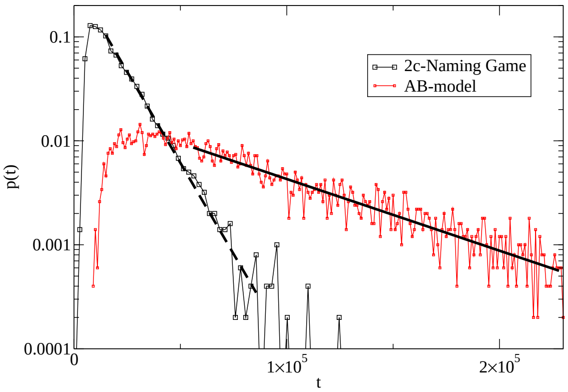

FIG. 6: Probability distribution for the time to reach consensus, starting with stripe-like configurations. Black: 2c-Naming Game, $\tau_{NG} \simeq 1.2 \times 10^4$; gray: AB-model, $\tau_{AB} \simeq 6.3 \times 10^4$. Averages are over 5000 runs.

tributed options among the agents, both models present a coarsening $\zeta \sim t^{\alpha}$, $\alpha \simeq 0.5$, with a curvature driven interface dynamics [14, 18] and AB-agents placing themselves at the interfaces between single-option domains. In Figure 5 we show snapshots comparing the two dynamics, starting from initial conditions where we have half of the lattice in state A, and the other half in state B. Given that the interface dynamics is curvature driven, flat boundaries are very stable. In both models these stripe-like configurations give rise to metastable states, already found in [14] for the AB-model: dynamical evolution of boundaries close to flat interfaces but with interfacial noise present. These configurations evolve by diffusion of the two walls (average interface density fluctuating around a fixed value) until they meet and the system is driven to an absorbing state. In the AB-model, also when starting from options randomly distributed through the lattice, 1/3 of the realizations end up in such stripe-like metastable states [14].We checked that the same turns out to be true also for the 2c-Naming Game. In the usual Naming Game with invention, on the other hand, stripes are better avoided since in that case the two convention state is usually reached when one cluster is already considerably larger than the other.

We show in Figure 6 the distribution of survival times for the two models, i.e., the time needed for a stripe-like configuration to reach an absorbing state. The distribution displays an exponential tail, $p(t) \sim e^{-t/\tau}$ with a characteristic time τ. The characteristic time for the AB-model is however larger than the one for the 2c-Naming Game ($\tau_{AB} > \tau_{NG}$), confirming that the AB-model interface dynamics slows down the diffusion of configurations such as stripes in two dimensional lattices, or walls in one dimensional lattices. Notice that in both cases, the differences found are beyond the trivial different time scale corresponding to the prefactor $1/2$ in the mean field equations for the AB-model (equations (7) and (8)).

VI. DISCUSSION AND CONCLUSION

We have analyzed and compared the 2c-Naming Game and the AB-model, originally defined in the context of language emergence and competition, respectively. We have shown that although these two models are equivalent in mean-field, their microscopic differences give rise to different behaviors. In particular, we have focused on (1) the extension of the models by introducing the parameter β, describing the inertia of the agents to abandon an acquired option, and (2) the interface dynamics in one and two-dimensional lattices.

As for the extension of the models incorporating the parameter β, even though the original models are equivalent in the mean field approximation for $\beta = 1$, an important difference concerns the existence of a phase transition. While the 2c-Naming Game features a non-equilibrium phase transition between consensus and stationary coexistence of the three phases present in the system, in the AB-model such a transition does not exist, the model featuring a trivial frozen state for $\beta = 0$ (dominance of the AB-state, with complete consensus in the thermodynamic limit), and the usual consensus in the A or B state, for $0 < \beta \leq 1$.

As for the interface dynamics, we have shown in one-dimensional lattices that the AB-model has a diffusive interface motion analogous to the one already found in the 2c-Naming Game, but with a diffusion coefficient nearly one order of magnitude smaller. In two-dimensional lattices, we have studied the time evolution of stripe-like configurations, which are metastable in both models but have a larger life time in the AB-model. Both results indicate that in comparison to the 2c-Naming Game, the AB-model interface dynamics slows down the diffusion of these configurations.

It is interesting to discuss the implications of our results on the AB-model in the context of dynamics of language competition. In the original AB-model ($\beta = 1$), the density of bilingual individuals remains small during the language competition process (around 20%), and in the end bilingual individuals disappear together with the language facing extinction. When introducing the parameter β, interpreted here as a sort of inertia to stop using a language, and, at the same time, as a reinforcement of the status of being bilingual, we observed the following. When β is small enough, bilingual agents rapidly become the majority, while the two monolingual communities compete between each other and have similar sizes. The smaller the parameter β, the larger the bilingual community is at this point. After this stage of coexistence, a symmetry breaking takes place and one of the two monolingual communities starts to grow, while the other looses ground. When this monolingual community faces extinction, the language spoken in that community survives in the bilingual community until this community also disappears.

In other words, contrary to the original model, the extinction of a language takes place in two steps. At first, the agents who speak just that language disappear, but the language does not, as it is still spoken in the society by the bilingual agents. Then bilingual agents disappear, too, which leads to the extinction of the language. Within the limited framework of the AB-model, in which there does not exist any political measure enhancing the prestige of an endangered language, these results come to support the idea that, in societies with two languages, the disappearance of a monolingual community using a language as its only way of communication could represent the first step in the extinction of that language. The other language could indeed become eventually the only spoken one, as the bilingual agents would eventually end up using only the language spoken by the remaining monolingual community.

The dynamics of the original Naming Game as well as that of the AB-model are strongly affected by the underlying interaction network, as it has been shown in [14, 15, 16, 18, 19, 20, 21, 22] (times to consensus, apparition of trapped metastable states, etc). In order to understand the implications of different complex social networks (small world effect, community structure, etc) for the extension of the model presented here, it is worth investigating in future this extension in topologies of increasing complexity.

Acknowledgments

X. Castelló acknowledges financial support from a phD fellowship of the Govern de les Illes Balears (Spain). A. Baronchelli acknowledges support from the DURSI, Generalitat de Catalunya (Spain) and from Spanish MEC (FEDER) through project No: FIS2007-66485-C02-01. This research has been party supported by the TAGora project funded by the Future and Emerging Technologies program (IST-FET) of the European Commission under the European Union RD contract IST-034721.

---

[1] C. Castellano, S. Fortunato, and V. Loreto. Statistical physics of social dynamics. *Reviews of Modern Physics*, 2007. (in press) [e-print arXiv:0710.3256].

[2] D. M. Abrams and S. H. Strogatz. Modelling the dynamics of language death. *Nature*, 424:900, 2003.

[3] M.A. Nowak and D.C. Krakauer. The evolution of language. *PNAS*, 96(14):8028–8033, 1999.

[4] M. A. Nowak, N. L. Komarova, and P. Niyogi. Evolution of universal grammar. *Science*, 291(5501):114–118, 2001.

[5] L. Steels. A self-organizing spatial vocabulary. *Artificial Life*, 2(3):319–332, 1995.

[6] A. Baronchelli, M. Felici, V. Loreto, E. Caglioti, and L. Steels. Sharp transition towards shared vocabularies in multi-agent systems. *Journal of Statistical Mechanics*, P06014, 2006.

[7] D. Stauffer and C. Schulze. Microscopic and macroscopic simulation of competition between languages. *Physics of Life Reviews*, 2:89–116, 2005.

[8] T. Tesileanu and H. Meyer-Ortmanns. Competition of languages and their hamming distance. *International Journal of Modern Physics C*, 17:259–278, 2006.

[9] K. Kosmidis, J. M. Halley, and P. Argyrakis. Language evolution and population dynamics in a system of two interacting species. *Physica A*, 353:595–612, 2005.

[10] D. Stauffer, X. Castelló, V. M. Eguíluz, and M. San Miguel. Microscopic Abrams-Strogatz model of language competition. *Physica A*, 374:835–842, 2007.

[11] C. Schulze and D. Stauffer. Recent developments in computer simulations of language competition. *Computing in Science and Engineering*, 8:60–67, 2006.

[12] William S-Y. Wang and James W. Minett. The invasion of language: emergence, change and death. *Trends in Ecology and Evolution*, 20:263–269, 2005.

[13] J. W. Minett and W. S.-Y. Wang. Modelling endangered languages: the effects of bilingualism and social structure. *Lingua*, 118:19–45, 2008.

[14] X. Castelló, V. M. Eguíluz, and M. San Miguel. Ordering dynamics with two non-excluding options: Bilingualism in language competition. *New Journal of Physics*, 8:308–322, 2006.

[15] X. Castelló, R. Toivonen, V. M. Eguíluz, J. Saramäki, K. Kaski, and M. San Miguel. Anomalous lifetime distributions and topological traps in ordering dynamics. *Europhysics Letters*, 79:66006 (1–6), 2007.

[16] R. Toivonen, X. Castelló, V. M. Eguíluz, J. Saramäki, K. Kaski, and M. San Miguel. Broad lifetime distributions for ordering dynamics in complex networks. *Physical Review E*, 2008. (in press) [e-print arXiv:0808.3318].

[17] A. Baronchelli, V. Loreto, and L. Steels. In-depth analysis of the naming game dynamics: the homogeneous mixing case. *Interntaional Journal of Modern Physics C*, 19:785, 2008.

[18] A. Baronchelli, L. Dall'Asta, A. Barrat, and V. Loreto. Topology induced coarsening in language games. *Physical Review E*, 73:015102, 2005.

[19] L. Dall'Asta, A. Baronchelli, A. Barrat, and V. Loreto. Agreement dynamics on small-world networks. *Europhysics Letters*, 73(6):969–975, 2006.

[20] L. Dall'Asta, A. Baronchelli, A. Barrat, and V. Loreto. Nonequilibrium dynamics of language games on complex networks. *Physical Review E*, 74(3):036105, 2006.

[21] L. Dall'Asta and A. Baronchelli. Microscopic activity patterns in the naming game. *Journal of Physics A: Mathematical and General*, 39:14851–14867, 2006.

[22] A. Baronchelli, L. Dall'Asta, A. Barrat, and V. Loreto. Nonequilibrium phase transition in negotiation dynamics. *Physical Review E*, 76(5):051102, 2007.

[23] J. D. Gunton, M. San Miguel, and P. Sahni. *Phase Transitions and Critical Phenomena*, volume 8, chapter The dynamics of first order phase transitions, pages 269–446. Academic Press, London, 1983.

[24] In the original model [12] languages with different prestige were considered. The factor 1/2 in the AB-model corresponds to the case of socially equivalent languages.