Microscopic Abrams-Strogatz model of language competition

Dietrich Stauffer*, Xavier Castelló, Víctor M. Eguíluz, and Maxi San Miguel

IMEDEA (CSIC-UIB), Campus Universitat Illes Balears
E-07122 Palma de Mallorca, Spain

* Visiting from Institute for Theoretical Physics, Cologne University, D-50923 Köln, Euroland

e-mail: {xavi,maxi,victor}@imedea.uib.es, stauffer@thp.uni-koeln.de

Abstract: The differential equation of Abrams and Strogatz for the competition between two languages is compared with agent-based Monte Carlo simulations for fully connected networks as well as for lattices in one, two and three dimensions, with up to 10^9 agents. In the case of socially equivalent languages, agent-based models and a mean field approximation give grossly different results.
Keywords: Monte Carlo, language competition

1.INTRODUCTION

Language competition and extinction is being considered from the point of view of complex systems. Language competition studies the dynamics of language use due to social interactions. It is known that most of the 6000 languages spoken today are in danger, with around 50% of them facing extinction in the current century. Perhaps more important is the distribution of speakers, with 4% of languages accounting for 96% of people and 25% having fewer than 1000 speakers.[1]

Many computer simulations of the competition between different languages have appeared, mostly in physics journals, since the publication in 2003 of a model by Abrams-Strogatz [2] for the competition between two languages. Some of them use a mean field approximation [3, 4, 5, 6], while others implement more realistic agent-based-models for many [7, 8, 9, 10] or few languages [11, 12]. A more complete review is given in [13], and a shorter one in [14]. Other studies address learning processes of a language [15, 16], a question that we do not take into account here.

Our main goal in this work is to check to what extent the results of the mean field approximation of the Abrams-Strogatz model are confirmed by agent-based simulations with many individuals. For the social interaction, we will consider a completely connected network as well as a regular lattice with nearest neighbour interaction in 1, 2, and 3 dimensions.

2.THE ABRAMS-STROGATZ MODEL

The model of Abrams-Strogatz studies the competition between two languages, X and Y, in a given society. An individual changes her/his language from Y to X, taking into account: 1) the total number of people speaking this language; 2) its perceived status, a parameter that reflects the attractiveness of a language: access to culture, personal and professional development,... The dynamics is as follows:

$$\mathrm{d}x/\mathrm{d}t = (1-x)p_{YX} - xp_{XY} \tag{1}$$

where the probability p_{YX} to switch from language Y to language X, and the probability p_{XY} for the inverse switch, are given by,

$$p_{YX} = sx^a, \quad p_{XY} = (1-s)(1-x)^a \tag{2}$$

Here x is the proportion of people speaking language X, and s its perceived status; $1-x$ is the proportion of people speaking Y and $1-s$ its corresponding status. From now on, we will use the word *prestige* for Abrams-Strogatz *status*, following common linguistics terminology.

The resulting Abrams-Strogatz differential equation for the competition of a language X with prestige s against another language Y with prestige $1-s$ is

$$\mathrm{d}x/\mathrm{d}t = (1-x)x\left(x^{a-1}s - (1-x)^{a-1}(1-s)\right) \tag{3}$$

Prestige is a parameter in the range $0 < s \leq 1$. The case $s < 1/2$, models the situation of a language with lower prestige X, competing against a more prestigious language Y. Fitting data for several endangered languages, $a \simeq 1.3$ was obtained in [2].

This equation has three fixed points for $a \neq 1$: For $a > 1$: x=0 and x=1 are stable, and a third one $0 < x^* < 1$ is unstable. For $a < 1$, stable fixed points become unstable, and vice versa.

We have considered a situation where both languages have initially the same number of speakers, $x(t = 0) = 1/2$. Fig. 1 shows exponential decay for $a = 1.31$ as well as for the simpler linear case $a = 1$. From now on we use $a = 1$. This choice simplifies (3) into equation (4), similar to the logistic equation which was applied to languages before, as reviewed by [17].

$$\mathrm{d}x/\mathrm{d}t = (2s - 1)(1 - x)x \tag{4}$$

We have now two fixed points For long times and $s < 1/2$, the fraction of speakers x has an exponential decay $e^{(2s-1)t}$. For $s = 1/2$ any value of x is a marginally stable stationary solution.

3.AGENT BASED MODELS

Differential equation (4) is a mean-field approximation, ignoring the fate of individuals and the resulting fluctuations. To take into account a discrete society, we build an agent-based model with N individuals which in a completely connected network feel the influence of all individuals, while on the d-dimensional lattice they feel only the influence of their $2d$ nearest neighbors. For lattices therefore, in the probabilities p_{YX} to switch from language Y to language X, and p_{XY} for the inverse switch, x is no longer the global fraction of speakers of language X, but a *local density*: fraction of X speakers within the $2d$ nearest neighbours. Initially each person speaks one of the two languages with equal probability: $x(t = 0) = 0.5$.

We have considered two different asynchronous updatings: 1) *regular updating*: the state of every node is updated going through them in an indexed order. 2) *random updating*: at each iteration, we choose one agent i at random, and change its language according to the probabilities mentioned above. This is more realistic, but takes more time. In both cases, a time step is defined as N iterations, with every node updated once on average. We find that our results agree qualitatively for both updatings.

Our results in the case of non equivalent languages $s < 1/2$ are shown in Fig.2 for the fully connected case and in Fig.3 for the square lattice. Results are qualitatively similar for the completely connected network and the square lattice, as well as for the original differential equation, giving a fast exponential decay of the less prestigious language, until it faces extinction.

4.SOCIALLY EQUIVALENT LANGUAGES

We consider now the symmetric case $s = 1/2$ of competition between two socially equivalent languages. The mean field approximation fails grossly in this case: the differential equation has x staying at $1/2$ for all times, while random fluctuation for finite population systems destabilize this situation and let one of the two languages win over the other, with x going to zero or unity with equal probability. In this symmetric situation, our lattice model becomes similar to the voter model [18].

The symmetric case in a regular lattice can be described in a unified way by looking at the number of lattice neighbours speaking a language different from the centre site. It corresponds to an energy, ϵ, in the Ising magnet and measures microscopic interfaces. Initially this number equals d on average. This magnitude is related to the averaged interface density $<\rho>$, used previously in [18] to analyze the voter model:

$$<\epsilon>= 2d <\rho> \tag{5}$$

We present here the results for d-dimensional lattices, with $d = 1, 2, 3$. Fig.4 shows the results for a square lattice. The energy defined above decays to zero, first possibly as a power law of exponent 0.1 (compatible with the decaying obtained for the voter model, $1/\ln t$), and then exponentially after a time which increases with increasing lattice size. The first decay describes a coarsening phenomenon, while the exponential decay is triggered by finite size fluctuations. One- and three-dimensional lattices have also been considered, for a more complete analysis. In Fig.5 we can observe how in one dimension the initial decay follows a power law, $t^{-1/2}$, while in three dimensions an initial plateau is reached, and thus no coarsening process occurs. This is followed after a time increasing with size by an exponential decay in $d = 1, 3$ as in two dimensions. Results for both updatings are not quantitatively identical, but give the same qualitative behaviour, including the exponents for the power laws.

Fig.6 shows that the average of $|x(t) - 1/2|$ increases in two dimensions roughly as the square-root of time until it saturates at $1/2$, indicating random walk behavior. (Note that first averaging over x and then taking the absolute value $| < x > -1/2|$ would not give appropriate results since $< x >$ would always be $1/2$ apart from fluctuations.)

Fig.7. shows the dependence on system size of the relaxation time for the extinction of a language associated to the exponential decay mentioned above. Regular updating is shown in Fig.7a and random updating in Fig.7b. Both figures are quite similar, with scaling laws for the characteristic time which are compatible with the ones obtained for a voter model [18]: $\tau \simeq N^2$

in $d = 1$, $\tau \simeq N \ln N$ in $d = 2$, and $\tau \simeq N$ in $d = 3$, where $N = L^d$.

Comparisons not shown here between this model for $s = 1/2$ and voter model, show how prestige, being a factor reducing the maximum probability to switch to 1/2, introduces a time delay to the whole dynamics compared to the voter model, but keeps all its qualitative behaviour.

CONCLUSIONS

We conclude that agent-based simulations agree qualitatively for non equivalent languages in the topologies studied. However, the results differ appreciably from the results of the mean-field approach for the symmetric case $s = 1/2$ of two socially equivalent languages: while Eqs.(1,2) predict x to stay at $x = 1/2$, our simulations in Fig.4 and later show that after a decay time everybody speaks the same language, bringing the other language to extinction. In a fully connected network and in $d = 3$ the decay is triggered by a finite size fluctuation, while in $d = 1, 2$ the intrinsic dynamics of the system causes an initial ordering phenomena in which spatial domains of speakers of the same language grow in size.

Other aspects that can be introduced in agent-based models of language competition are the presence of bilingual individuals as well as a complex social structure [5, 19, 20].

We acknowledge financial support from the MEC (Spain) through project CONOCE2 (FIS2004-00953).

References

[1] D. Crystal, Language death (Cambridge: CUP, 2000).

[2] D.M. Abrams and S.H. Strogatz, Nature 424 (2003) 900.

[3] M. Patriarca and T. Leppänen, Physica A 338 (2004) 296.

[4] W.S.Y. Wang and J.W. Minett, Trans. Philological Soc.103 (2005) 121.

[5] J.Mira and A. Paredes, Europhys. Lett. 69 (2005) 1031.

[6] J.P. Pinasco and L. Romanelli, Physica A 361 (2006) 355.

[7] C. Schulze and D. Stauffer, Int. J. Mod. Phys. C 16 (2005) 781; Physics of Life Reviews 2 (2005) 89;

[8] T. Teşileanu and H. Meyer-Ortmanns, Int. J. Mod. Phys. C 17, No. 3, 2006, in press.

[9] D. Stauffer, C. Schulze, F.W.S. Lima, S. Wichmann and S. Solomon, e-print physics/0601160 at arXiv.org.

[10] V.M. de Oliveira, M.A.F. Gomes and I.R. Tsang, Physica A 361 (2006) 361; V.M. de Oliveira, P.R.A. Campos, M.A.F. Gomes and I.R. Tsang, e-print physics/0510249 at arXiv.org for Physica A.

[11] K. Kosmidis, J.M. Halley and P. Argyrakis, Physica A, 353 (2005) 595; K.Kosmidis, A. Kalampokis and P.Argyrakis, physics/0510019 in arXiv.org to be published in Physica A.

[12] V. Schwämmle, Int. J. Mod. Phys. C 16 (2005) 1519; *ibidem* 17, 2006, in press.

[13] D. Stauffer, S. Moss de Oliveira, P.M.C. de Oliveira, J.S. Sa Martins, *Biology, Sociology, Geology by Computational Physicists*, Elsevier, Amsterdam 2006.

[14] C. Schulze and D. Stauffer, Comput. Sci. Engin. 8 (2006) in press.

[15] M.A. Nowak, N.L. Komarova and P. Niyogi, Nature 417 (2002) 611.

[16] A. Baronchelli, M. Felici, E. Caglioti, V. Loreto, L. Steels, e-prints physics/0509075, 0511201 and 0512045 at arXiv.org.

[17] W.S.Y. Wang, J. Ke, J.W. Minett, in: Computational linguistics and beyond, eds. C.R. Huang and W. Lenders (Academica Sinica : Institute of Linguistics, Taipei, 2004); www.ee.cuhk.edu.hk/∼wsywang

[18] R. Holley and T.M. Liggett, Ann. Probab. 3 (1975) 643; K. Suchecki, V.M. Eguíluz and M. San Miguel, Phys. Rev. E 72 (2005) 0361362 and Europhys. Lett. 69 (2005) 228; M. San Miguel, V.M. Eguíluz, R. Toral and K. Klemm, Comp. Sci. Engin. 7 (Nov/Dec 2005) 67.

[19] J.W. Minett, W. S-Y. Wang. (unpublished).

[20] X. Castelló, V.M. Eguíluz, M. San Miguel; communication 10.51, AK-SOE meeting 2006. http://www.dpg-physik.de/static/fachlich/aksoe/

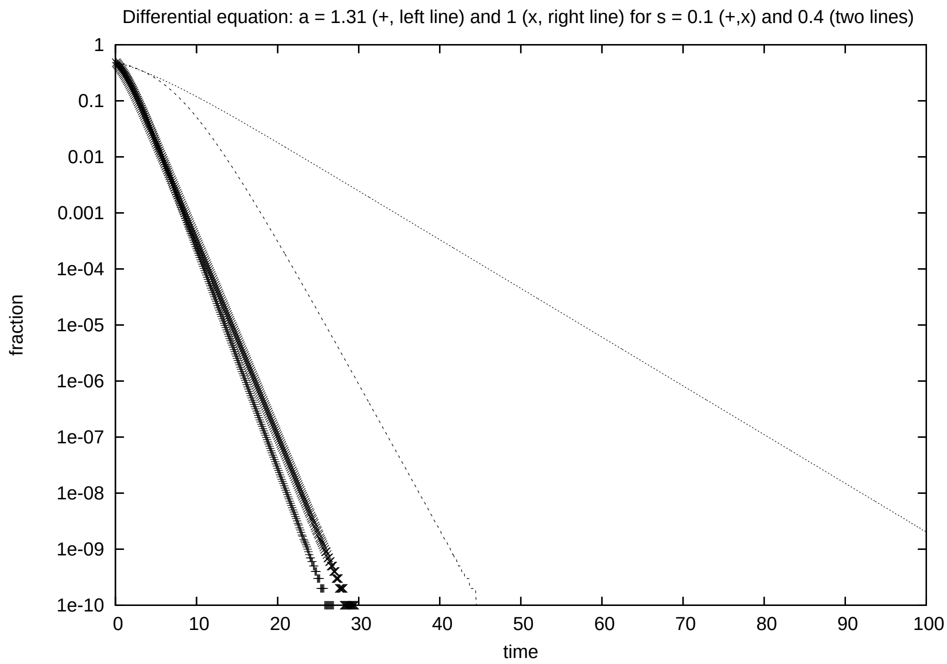


Figure 1: Fraction of X speakers from Abrams-Strogatz differential equation with $a = 1.31$ and $a = 1$, at status $s = 0.1$ (heavy symbols at left) and $s = 0.4$ (two lines at right). For $a = 1.31$ the decay is faster than for $a = 1$.

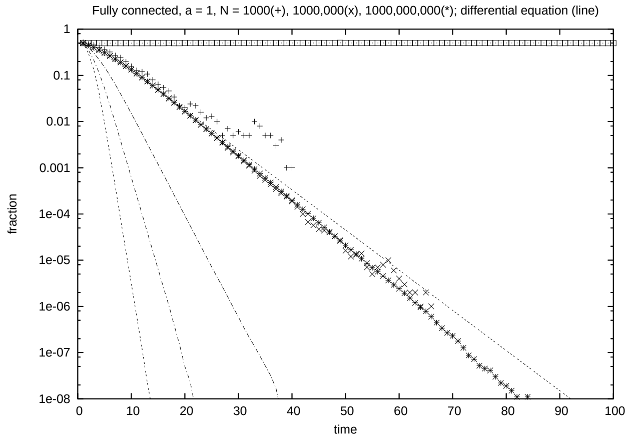


Figure 2: Fully connected model with $10^3,\ 10^6,\ 10^9$ agents at $s = 0.4$ compared with differential equation (rightmost line) at $s = 0.4$. The three left lines correspond to $s = 0.1,\ 0.2,\ 0.3$ from left to right for $N = 10^9$. The thick horizontal line corresponds to $s = 0.5$ and $N = 10^6$ and changes away from 1/2 only for much longer times. Figs. 2 and 3 use one sample only and thus indicate self-averaging: The fluctuations decrease for increasing population. In this figure, as well as in Fig.3, results are obtained with regular updating.

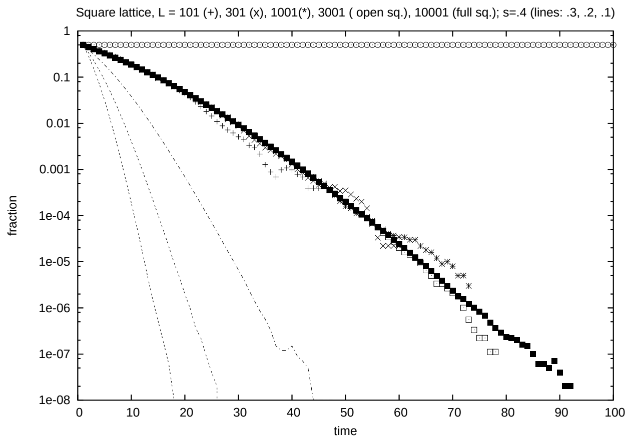


Figure 3: $L \times L$ square lattice with $L = 101$ to 10,001 at $s = 0.4$. The three left lines correspond to $s = 0.1,\ 0.2,\ 0.3$ from left to right for $L = 10,001$. The thick horizontal line corresponds to $s = 0.5,\ L = 10,001$ and might deviate only for $t > 10^7$.

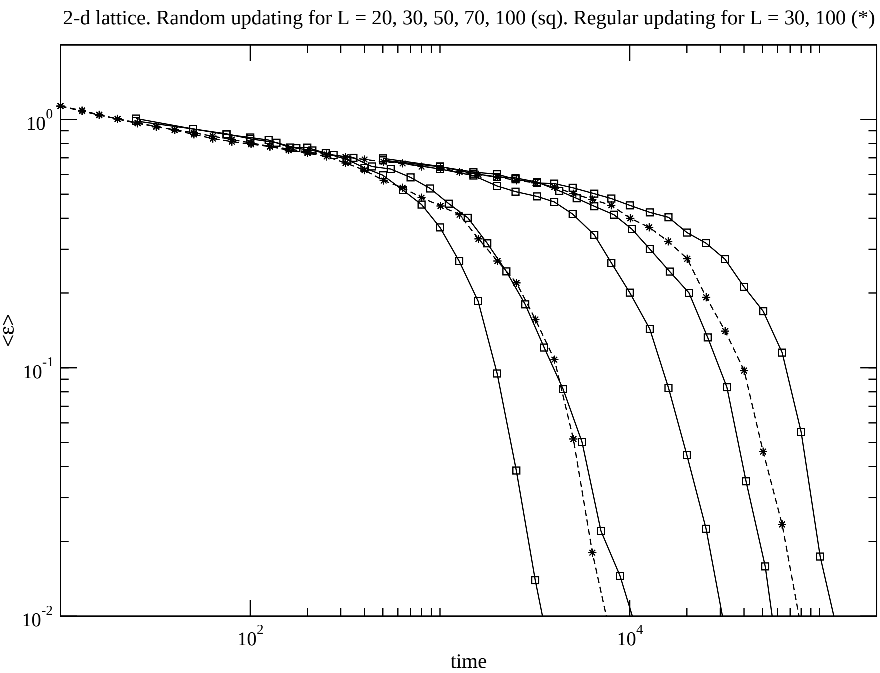
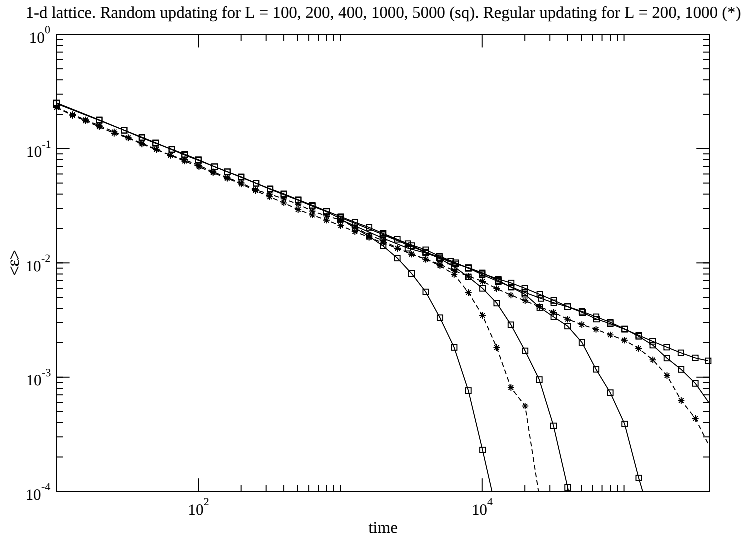


Figure 4: Decay of unstable symmetric solution $x = 1/2$ for $s = 1/2$ for square lattices of various sizes, with system size increasing from left to right. A semilogarithmic plot, not shown, indicates a simple exponential decay. Simulations shown are done with random updating (straight lines). Some system sizes are also represented for regular updating for comparison (dashed lines). Average over 100 samples.



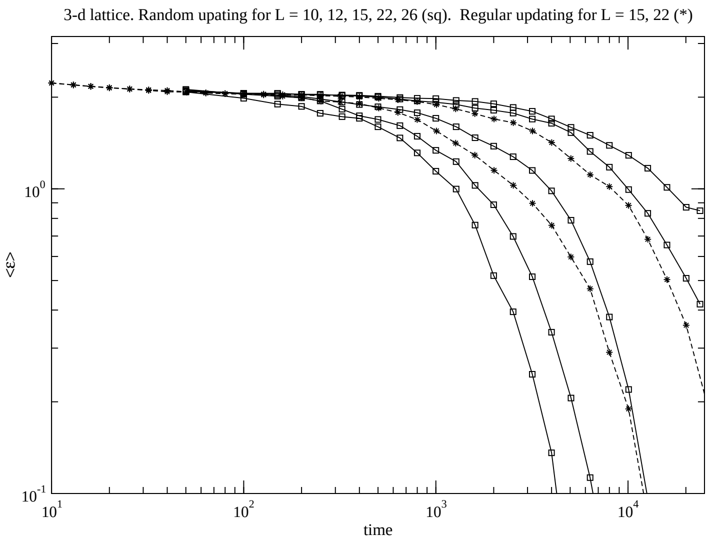


Figure 5: Same as Fig.4 but in one (top) and three (bottom) dimensions. For one-dimensional lattice, due to larger times for the separation from the power law decay, and self-averaging for large systems, $L = 100, 200$ are averaged over 1000 samples; $L = 400, 1000$ over 200 samples; and $L = 5000$ over 50 (random updating). For regular update we average over 100 samples. For three-dimensional lattice we average over 100 samples.

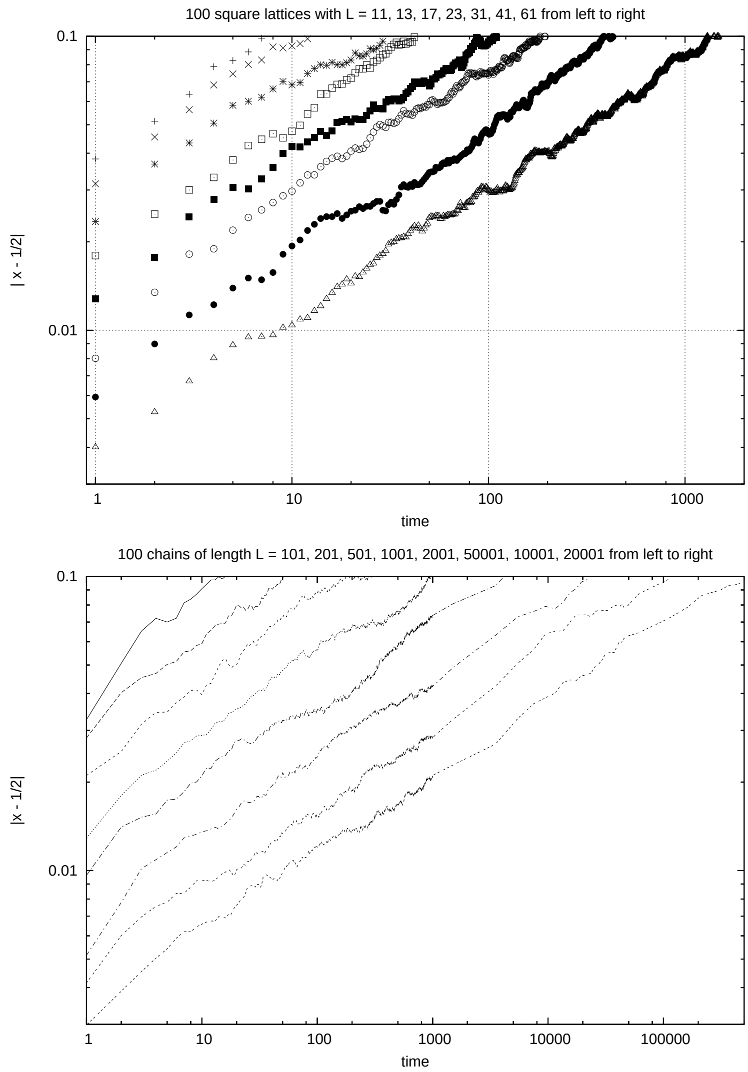


Figure 6: Absolute difference between $x(t)$ and $x(t=0) = 1/2$ for $d = 2$ (part a) and 1 (part b, averages for $t > 1000$). Average over 100 samples.

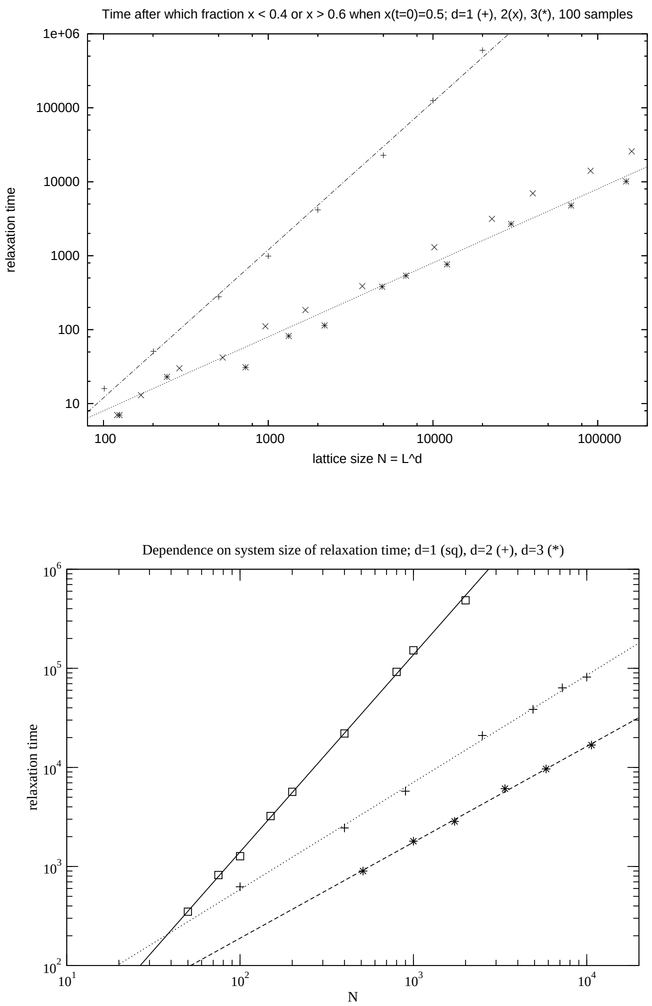


Figure 7: Time for the energy ϵ (number of lattice neighbours speaking different language) to reach some constant fraction of its initial value, versus population $N = L^d$, in one (+), two (x) and three (*) dimensions. Averaged over 100 samples. Part a uses regular updating: we checked when the fraction x, initially 1/2 leaves the interval $[0.4, 0.6]$ on its way to zero or one. The straight lines have slope 1 for $d = 2, 3$, and 2 for $d = 1$. Part b uses random updating: we checked when the energy reaches a small fraction of its initial value, taken as $2/L$, 0.04, 0.6 for $d = 1, 2, 3$. The exponents for the fitted power laws are 1.99 ($d = 1$, straight line), 1.08 ($d = 2$ dotted line) and 0.97 ($d = 3$ dashed line).