Institut de Física Interdisciplinària i Sistemes Complexos

Collective phenomena in social dynamics: consensus problems, ordering dynamics and language competition

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Chapter 1

Introduction

1.1 Collective phenomena: physics, complexity and social sciences

Understanding the complex collective behaviour of many particle systems in terms of a microscopic description based on the interaction rules among the particles is the well established purpose of Statistical Physics. This micro-macro paradigm [1] is also shared by Social Science studies based on agent interactions. In many cases parallel research in both disciplines goes far beyond superficial analogies. For example, Schelling's model [1] of residential segregation is mathematically equivalent to the zero-temperature spin-exchange Kinetic Ising model with vacancies. Cross-fertilisation between these research fields opens interesting new topics of research [2, 3].

On the one hand, the formalism and tools from statistical physics and complex systems theory are becoming a powerful framework to model and understand social systems: emergence of collective phenomena, non-equilibrium dynamics, coarsening, phase transitions, bifurcations [4]. Social phenomena appear to be complex systems of increasing interest for the physicists community. On the other hand, interactions in complex networks is a relatively recent paradigm in statistical physics [5]. Complex topologies can be seen as the skeleton of a complex system and are found *everywhere* in science: from neuroscience and molecular biology, to financial markets and social networks. The works at the end of the 90s by Watts and Strogatz [6], and Barabási and Albert [7] opened a novel approach from the point of view of statistical physics to the modelling and understanding of complex networks. The possibility to have simple models accounting for the main features observed in these topologies, such as the small world and the scale-free effects together with the increasing power of computing, triggered a coherent effort to model and study through network theory many different systems beyond the traditional physics research, ranging from biology to economy and the social sciences. In particular, network theory applied to complex social networks makes possible an analysis of the effect of the network structure on non-equilibrium dynamics proposed in order to model social behaviour.

In this way, the micro-macro approach to the study of social systems is reinforced, extended and renewed by statistical physics and complex systems theory. The microscopic interaction rules include two main ingredients: (i) structure: a set of interacting agents, which can account for individuals, groups of individuals, organisations, institutions, etc., which are embedded in a framework of interaction modelled by a network; and (ii) dynamics: the interaction mechanism between particles/agents which defines the non-equilibrium dynamics mentioned above. These dynamics have been proposed in parallel to the explosion of the complex networks research field, and deal with problems such as opinion dynamics [8], cooperation [9], culture dissemination [10], epidemics [11], language dynamics [12], dynamics of financial markets [13], game theory [14] etc. This micro-macro approach is generally known as Agent Based Modelling (ABM) in computational and social sciences [15].

Today, researchers working with Agent Based Models come from very different backgrounds; physics, mathematics, engineering, computer sciences, economics, sociology or cognitive science, and start to share a common formalism which is both, theoretical and experimental [16]. Theoretical because the micro-macro paradigm inherited from complex systems is underlying in the conceptual modelling; and experimental in the sense that computer simulations make it possible to generate the data needed by scientists in order to study the models proposed, which can be statistically analysed. Regarding real data analysis, the recent explosion of the *internet society* has been crucial, as it has increased enormously the amount of real data available to study social systems: mailing networks, collaborative tagging systems, online communities, etc. Although scientists from different disciplines think and develop models in a different way according to their own background, agent based modelling is currently giving a common framework and enhancing interdisciplinary research in many areas of science.

1.2 The consensus problem. Mechanisms and models

The consensus problem is a general one of broad interest, recently addressed by statistical physics: the question is to establish when the dynamics of a set of interacting agents that can choose among several options leads to a consensus in one of these options, or alternatively, when a state with several coexisting social options prevails [17]. For an equilibrium system the analogy would be with an order-disorder transition. For nonequilibrium dynamics we rely on ideas of studies of domain growth and coarsening in the kinetics of phase transitions [18], where dynamics is dominated by interface motion.

Several models have been proposed to account for different mechanisms of social interaction in the dynamics of social consensus. The idea is to capture the essence of different social behaviours by simple interaction rules: following the idea of universality classes [4], in collective emergent phenomena details might not matter. There are several examples of this mechanisms that have given fruitful results in the last years: (i) imitation (voter Model [19]), (ii) social pressure (Ising-like models [18]), (iii) homophily (Axelrod model for cultural dissemination [10]), (iv) majority convinces (Sznajd model [20]), (v) interactions bounded by a threshold (Granovetter model [21]), (vi) interaction through small groups (Galam model [22]) (vii) game theory framework [14].

1.3 Language competition

Language competition belongs to the general class of processes that can be modelled by the interaction of heterogeneous agents as an example of collective phenomena in problems of social consensus, and it motivates the present work. Language competition occurs today worldwide. Different languages coexist within many societies and the fate of a high number of them in the future is worrying: most of the 6000 languages spoken today are in danger, with around 50% of them facing extinction in the current century. Even more striking is the distribution of speakers, since 4% of the languages are spoken by 96% of the world population, while 25% have fewer than 1000 speakers. New pidgins and creoles are also emerging, but their number is relatively small compared with the language loss rate [23]. In this scenario, and beyond Weinreich 's *Languages in Contact* [24], numerous sociolinguistic studies have been published in order to: (1) reveal the level of endangerment of specific languages [25]; (2) find a common pattern that might relate language choice to ethnicity, community identity or the like [26]; and (3) claim the role played by social networks in the dynamics of language competition, which has given rise to the monographic issue [27].

In recent years, language competition, which studies the dynamics of language use and learning due to social interactions, has also been tackled with from a different approach. Abrams and Strogatz model [12] for the dynamics of endangered languages (from now on, AS-model) has triggered a coherent effort to understand the mechanisms of language dynamics outside the traditional linguistic research. Their study considers a two-state society, that is, one in which there are speakers of either a language A or a language B, and accounts for data of extinction of endangered languages such as Quechua (in competition with Spanish), Scottish Gaelic and Welsh (both in competition with English). This seminal work, as well as others along the same line [28, 29], belongs to the general class of studies of population dynamics based on nonlinear ordinary differential equations for the populations of speakers of different languages. In addition, other studies implement discrete agent based models with speakers of many [30, 31] or few languages [32, 33], as reviewed in [34]. These studies consider spatially distributed social structures in which agents are connected with one another.

A specific feature of language competition is that agents can share two of the social options that are chosen by the agents in the consensus dynamics. These are the bilingual agents, that is, agents that use both languages A and B, who have been claimed to play a relevant role in the evolution of multilingual societies [29, 35]. In this work, we will follow the proposal by Minett and Wang for a dynamics of language competition including bilingualism [36] which is an extension of the Abrams-Strogatz model. We are interested in comparing both models, and analyse the effects of adding bilingualism. Following Milroy [37], we expect that also social structure might be an important factor in language competition, and, therefore, we are interested in the study of these models in networks, providing in this way a quantitative analysis that is wanting in the field of sociolinguistics, as noted by de Bot and Stoessel [38].

Other different problems of language dynamics in which statistical physics can play a relevant role, are those regarding language evolution (dynamics of language structure) and language cognition (learning processes). These include evolution of universal grammar [39], utterance selection models [40], and social impact theory applied to language change [41]. Among these, *semiotic dynamics*, considered in the context of language games such as the *naming game* [42, 43], is another relevant example of the consensus problem. In the naming game, a shared lexicon among agents emerges from peer interaction. It has been studied in complex networks [44], and the special case of two words [45] has similarities with the AB-model [46], the model we present in Chapter 2 and which is the core of the present work.

1.4 Ordering dynamics: towards models of two non-excluding options

The microscopic version [33] of the Abrams and Strogatz model for the competition of two equivalent languages and marginal volatility (presented in detail in Chapter 2) is equivalent to the voter model [19, 47, 48]. The voter model is a prototype lattice spin-model of nonequilibrium dynamics for which $d = 2$ is a critical dimension [49]: For regular lattices with $d > 2$ coarsening does not occur and, in the thermodynamic limit, the system does not reach one of the homogeneous absorbing states (consensus states). The same phenomenon occurs in complex networks of interaction of effective large dimensionality where a finite system gets trapped in long-lived heterogeneous metastable states [50, 51, 52]. From the point of view of interaction mechanisms, the voter model is one of random imitation of a state of a neighbour. A different mechanism (for $d > 1$) of majority rule is the one implemented in a zero-temperature spin-flip kinetic Ising (SFKI) model [1]. Detailed comparative studies of the consequences of these two mechanisms in different interaction networks have been recently reported [55]. From the point of view of coarsening and interface dynamics, a main difference is that, in the voter model coarsening is driven by interfacial noise, while for a SFKI model coarsening is curvature driven with surface tension reduction.

The voter and SFKI models are two-option models (spin $+1$ and spin -1) with two equivalent global attractors for the system. Kinetics of multi-option models like Potts or clock models were addressed long ago [56, 57]. More recently, a related model proposed by Axelrod [10] has been studied in some detail [58, 59, 60]. This is a multi-option model but, in general, its nonequilibrium dynamics does not minimise a potential leading to a thermodynamic equilibrium state like in traditional statistical physics [61]. On the other hand, the kinetics of the simplest three-options models [62, 63, 64] has not been studied in great detail.

We are here interested in the class of 3-state models for which two states are equivalent (spin ± 1, state A or B) and a third one is not (spin 0, state AB). Motivated by studies of language competition as mentioned in the previous Section, we consider the extension of the the Abrams and Strogatz model for the dynamics of endangered languages [12] proposed by Minett and Wang, in which bilingualism is taken into account [36]. The possible state of the agents are speaking either of these languages (A or B) or a third non-equivalent bilingual state (AB). In the context of the consensus problem this introduces a special ingredient in the sense that the options are not excluding: there is a possible state of the agents (bilinguals or AB agents) in which there is coexistence of two possible options at the individual level. In a more general framework, the problem addressed here is that of competition or emergence of social norms [65] in the case where two norms can coexist at the individual level. In other words: the general problem of ordering dynamics with two non-excluding options.

In this work and within this general framework, we study in detail a 3-state extension of the voter model, the *AB-model*, which reduces to the voter model when AB agents are not taken into account [46]. We aim to explore possible mechanisms for the stabilisation of a coexistence between the two options, possible metastable sates, and the role of AB agents (bilingual in-

[1] A different majority rule based in group interaction is considered in [53, 54]

dividuals) and the interaction network (social structure) in these processes. To this end, we analyse the growth mechanisms of A or B spatial domains (monolingual domains), the dynamics at the interfaces (linguistic borders), and the scaling laws for the times to reach consensus (dominance/extinction of a language). This is studied in regular lattices and in complex networks of interaction. A regular lattice structure captures a topology where interactions are based on geographical proximity. However, it has been shown that social networks are far from being regular, and they are not totally random either [6]. Quite on the contrary, what has been found as a main character in most of the real social networks analysed (e.g., mobile phone calls [66]) is the small world phenomenon, which describes the effect of long range social interactions among the agents. Therefore, the analysis of the AB-model in small world networks is crucial to understand the role of these long range interactions in the dynamics.

Generally speaking, we find that allowing for the AB-state (bilinguals) modifies the nature and dynamics of interfaces: agents in the AB-state define thin interfaces and coarsening processes change from voter-like dynamics to curvature driven dynamics. This change of coarsening mechanism is also shown to originate for a class of perturbations of the voter model.

The work presented in this master thesis has given rise to a publication in The New Journal of Physics [46]. Some of the results, also correspond to a previous work published in Physica A, [33].

1.5 Outline

The outline of this work is as follows: Chapter 2 describes the microscopic models studied: the Abrams-Strogatz model and the Minett-Wang model, which reduce to the voter Model and the AB-model in the case of socially equivalent languages and marginal volatility. A mean field analysis of the ordinary differential equations is also presented. Chapter 3 contains the results obtained in the analysis of numerical simulations of the latter models in fully connected networks, two-dimensional lattices and small world networks. Finally, in Chapter 4 we expose our conclusions and we give an outline of further research to be addressed.

Chapter 2

The models

2.1 The Abrams-Strogatz model

We present here the microscopic version of the AS-model [33], a two-states model proposed for the competition between two languages. In this model, an agent i sits in a node within a network of N individuals and has k_i neighbours. It can be in the following states: A, agent using language A (monolingual A); or B, agent using language B (monolingual B)[1].

The state of an agent evolves according to the following rules: starting from a given initial condition, at each iteration we choose one agent i at random and we compute the local densities for each of the communities in the neighbourhood of node i, $\sigma_{i,l}$ (l=A, B). The agent changes its state according to the following transition probabilities:

$$p_{i,A\to B} = (1-s)(\sigma_{i,B})^a \quad , \qquad p_{i,B\to A} = s(\sigma_{i,A})^a \tag{2.1}$$

Equations (2.1) give the probabilities for an agent i to change from community A to B, or viceversa. They depend on the local densities ($\sigma_{i,A}$, $\sigma_{i,B}$) and on two free parameters: the *prestige* of the language A, $0 \leqslant s \leqslant 1$ (the one of language B is $1-s$); and the *volatility*, $a \geqslant 0$. On the one hand, the prestige gives a measure of the different status between the two languages, that is, which is the language that gives an agent more possibilities in the social and personal spheres. The case of socially equivalent languages is $s = 1/2$. On the other hand, the volatility is a parameter which gives shape to the functional form of the transition probabilities (see Figure 2.1). The case a=1 is the marginal situation, where the transition probabilities depend linearly on the local densities. A high volatility regime regime exists for $a < 1$, with a probability of changing language state above the marginal case, and therefore agents change its state rather frequently. A low volatility regime exists

[1] Notice that we consider *use* of a language rather than *competence*.

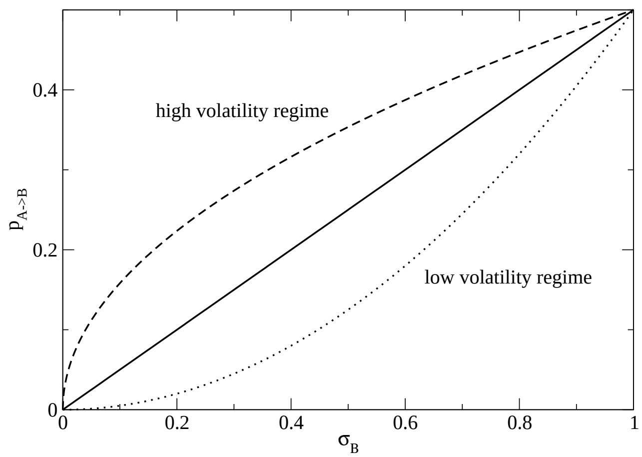

Figure 2.1: Volatility parameter, a: marginal case ($a = 1$, solid line), high volatility regime ($a < 1$, dashed line) and low volatility regime ($a > 1$, dotted line).

for $a > 1$ with a probability of changing language state below the marginal case, where agents have a larger inertia to change its state.

In the thermodynamic limit, the model can be described by a differential equation for the total population density of agents Σ_A ($\Sigma_B = 1 - \Sigma_A$):

$$\mathrm{d}\Sigma_A/\mathrm{d}t = \Sigma_A(1-\Sigma_A)[s(\Sigma_A)^{a-1} - (1-s)(1-\Sigma_A)^{a-1}] \tag{2.2}$$

This population dynamics approach was the initial proposal by Abrams-Strogatz [2] in [12].

When $a \neq 1$, the stability analysis shows that there are three fixed points: (i) $\Sigma_A^* = 0$ and (ii) $\Sigma_A^* = 1$, which correspond to consensus in the state A or B; and (iii) $\Sigma_A^* = [(\frac{s}{1-s})^{\frac{1}{a-1}} + 1]^{-1}$. For $a > 1$, the two first fix points are stable, and the third one is not, leading to stable consensus. For $a < 1$ instead, the stability changes via a transcritical bifurcation, and consensus becomes unstable giving rise to stable coexistence of the two states.

For $a = 1$, and $s \neq 1/2$, the ODE becomes the logistic-Verhulst equation [33]:

$$\mathrm{d}\Sigma_A/\mathrm{d}t = (2s-1)\Sigma_A(1-\Sigma_A) \tag{2.3}$$

[2] Abrams and Strogatz found an exponent a=1.31 when fitting to real data from the competition between Quechua-Spanish, Scottish Gaelic-English and Welsh-English. They considered only this case.

In this case, there exist just two fixed points: (i) $\Sigma_A^* = 0$ and (ii) $\Sigma_A^* = 1$. For $s < 1/2$ (i) is stable and (ii) unstable; while for $s > 1/2$ it happens the opposite. For the case $s = 1/2$, we obtain a null system with a degenerate line of fixed points, and therefore, any initial condition is a fixed point of the dynamics.

2.2 The Minett-Wang model

We present here the extension of the AS-model proposed by Minett and Wang (from now on, MW-model), which takes into account the presence of a third possible state: the bilingual agents[3].

In this model, the agents can also be in a third possible state, *AB*, bilingual agent using both languages, A and B; and there are three local densities to compute for each node *i*: $\sigma_{i,l}$ ($l = A, B, AB$). The agent changes its state according to the following transition probabilities:

$$p_{i,A\to AB} = (1-s)(\sigma_{i,B})^a \quad , \qquad p_{i,B\to AB} = s(\sigma_{i,A})^a \tag{2.4}$$

$$p_{i,AB\to B} = (1-s)(1-\sigma_{i,A})^a \quad , \qquad p_{i,AB\to A} = s(1-\sigma_{i,B})^a \tag{2.5}$$

Equations (2.4) give the probabilities for changing from a monolingual community, A or B, to the bilingual community AB, while equations (2.5) give the probabilities for an agent to move from the AB community towards the A or B communities. Notice that the latter depend on the local density of agents using the language to be adopted, including bilinguals ($1 - \sigma_{i,l} = \sigma_{i,j} + \sigma_{i,AB}$, $l, j = A, B$; $l \neq j$). It is important to stress that a change from state A to state B or viceversa, always implies an intermediate step through the AB-state.

In the mean field limit, the model can as well be described by differential equations for the total population densities of agents Σ_A, Σ_B ($\Sigma_{AB} = 1 - \Sigma_A - \Sigma_B$).

$$\mathrm{d}\Sigma_A/\mathrm{d}t = s(1-\Sigma_A-\Sigma_B)(1-\Sigma_B)^a - (1-s)\Sigma_A(\Sigma_B)^a \tag{2.6}$$

$$\mathrm{d}\Sigma_B/\mathrm{d}t = (1-s)(1-\Sigma_A-\Sigma_B)(1-\Sigma_A)^a - s(\Sigma_A)^a\Sigma_B \tag{2.7}$$

The stability analysis shows that there are three fixed points: $(\Sigma_A, \Sigma_B, \Sigma_{AB}) = (1,0,0), (0,1,0)$, which correspond to consensus in the state A or B; and $(\Sigma_A^*, \Sigma_B^*, \Sigma_{AB}^*)$, with $\Sigma_l^* \neq 0$ ($l = A, B, AB$) [4]. For $a \geqslant 0.63$, the two first fixed points are stable, and the third one is not, leading to stable consensus. For

[3]Notice that this extension was proposed in a working paper in 2005 (see also [36]). The final version of the paper [67] differs slightly on the transition probabilities. However, we analyse here their initial proposal.

[4]There is no closed expression for Σ_l^* ($l = A, B, AB$). Numerical analysis is needed.

$a < 0.63$ instead, the stability is reversed, consensus becomes unstable and a stable state of coexistence of the three states becomes possible.

An implementation of these two models in a two-dimensional lattice has been done by designing a Java Applet in which one can tune the parameters described above, set different initial conditions, and see the simulations in real time[5]. An interactive exploration of the parameter space *(a,s)* can be done there.

2.3 The voter model and the AB-model

The models presented above can account for the more general framework of a consensus problem, where there exist a competition between two social norms. In this way, the AB-state represents the case when two norms can coexist at the individual level (an agent using two languages in the case of language competition). In this work, we will concentrate in analysing these models in detail for the case of two socially equivalent norms or options (languages), $s = 0.5$, and volatility $a = 1$ [46].

On the one hand, the transition probabilities of the microscopic AS-model for the agent i reduce to:

$$p_{i,A\to B} = \frac{1}{2}\sigma_{i,B} \quad , \qquad p_{i,B\to A} = \frac{1}{2}\sigma_{i,A} \tag{2.8}$$

Except for a time scale coming from the prefactor $1/2$, the AS-model becomes the voter Model, which have been extensively studied [19, 47, 48, 49, 50, 51, 52, 68]. Equation (2.8) gives probabilities for an agent to change between the two states, which are proportional to the local density of agents in the opposite option. Notice that the voter model rules are equivalent to the adoption by the agents of the opinion of a randomly chosen neighbour.

In the mean field approximation, the voter model reduces to the equation:

$$\mathrm{d}\Sigma_A/\mathrm{d}t = 0, \tag{2.9}$$

predicting that any given initial density of agents in state A would persist forever. Simulations of the corresponding stochastic discrete voter model in a lattice [19] indicate a very different behaviour with one of the two options eventually becoming dominant.

On the other hand, the MW-model becomes:

[5] Applet can be found at: $http://ifisc.uib.es/eng/lines/complex/APPLET\_LANGDYN.html$

$$p_{i,A\to AB} = \frac{1}{2}\sigma_{i,B} \quad , \qquad p_{i,B\to AB} = \frac{1}{2}\sigma_{i,A} \tag{2.10}$$

$$p_{i,AB\to B} = \frac{1}{2}(1-\sigma_{i,A}) \quad , \qquad p_{i,AB\to A} = \frac{1}{2}(1-\sigma_{i,B}) \tag{2.11}$$

Equation (2.10) gives the probabilities for an agent i to move away from a single-option community, A or B, to the AB community. They are proportional to the density of agents in the opposed single-option state in the neighbourhood of i. On the other hand, equation (2.11) gives the probabilities for an agent to move from the AB community towards the A or B communities. They are proportional to the local density of agents with the option to be adopted, including those in the AB-state ($1-\sigma_{i,l} = \sigma_{i,j}+\sigma_{i,AB}$, *l, j*=A,B; $l \neq j$). It is important to remind that a change from state A to state B or viceversa, always implies an intermediate step through the AB-state. The dynamical rules (2.10) and (2.11) are fully symmetric under the exchange of A and B, so that states A and B are equivalent with no preference for any of the two options. Reaching consensus in either of these two states is a symmetry breaking process. These dynamical rules, which define a modification of the two state voter model to account for a third mixed AB-state, reflect the special character of this state as one of coexisting options. We will refer to the model defined by (2.10) and (2.11) as the *AB-model*.

From now on, we will focus our attention in the novel results we have obtained for the AB-model, comparing them to the already extensively studied voter model. In this way, we will address the effect of the extension of the latter model through the addition of the AB-state.

In a fully connected network and in the limit of infinite population size, the AB-model can be described by coupled differential equations for the total population densities Σ_A, Σ_B ($\Sigma_{AB} = 1-\Sigma_A-\Sigma_B$):

$$\mathrm{d}\Sigma_A/\mathrm{d}t = 1/2[1-\Sigma_A+(\Sigma_B)^2-2\Sigma_B] \tag{2.12}$$

$$\mathrm{d}\Sigma_B/\mathrm{d}t = 1/2[1-\Sigma_B+(\Sigma_A)^2-2\Sigma_A] \tag{2.13}$$

The analysis of these mean field equations shows the existence of three fixed points: two of them stable and equivalent, corresponding to consensus in the state A or B: $(\Sigma_A, \Sigma_B, \Sigma_{AB}) = (1,0,0), (0,1,0)$; and another one unstable, with non-vanishing values for the global densities of agents in the three states: $(\Sigma_A^*, \Sigma_B^*, \Sigma_{AB}^*)$, with $\Sigma_l^* \neq 0$ ($l = A, B, AB$). Figure 2.2 shows the phase portrait of the system, i.e., the trajectories of the system in the space (Σ_A,Σ_B). We can observe the location of the fixed points of the system and the two basins of attraction corresponding to the stable fixed points, which are separated by the line Σ_A=Σ_B, the stable manifold of the saddle.

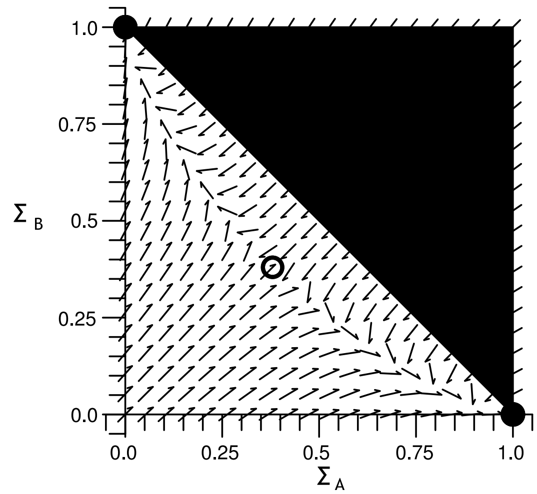

Figure 2.2: Phaseportrait: trajectories in the Σ_A-Σ_B space for the bilingual model. Each arrow shows the direction of change of the system at that state, allowing trajectories and fixed points to be inferred. Stable fixed points are marked by filled circles; unstable ones are marked by unfilled circles. The points such that $\Sigma_A + \Sigma_B > 1$ are not shown (black area), as they are not physical; remember that $\Sigma_A + \Sigma_B + \Sigma_{AB}$=1.

In this work, we go beyond the simple mean field description of these models (2.12)-(2.13). We describe the microscopic dynamics in which discrete and finite size effects, as well as the topology of the network of interaction are taken into account.

Chapter 3

Results

We present in this chapter the results obtained in numerical simulations for the voter model and the AB-model in different topologies: fully connected networks, lattices and small world networks. In our simulations we use random asynchronous node update: at each iteration or time step a single node is randomly chosen and updated according to the transition probabilities (2.8) and (2.10-2.11). We normalise time so that in every unit of time each node has been updated on average once. Therefore, a unit of time includes N iterations. In most of our simulations we start from random initial conditions: in the AB-model, random distribution in the network of 1/3 of the population in state A, 1/3 in state B and 1/3 in state AB (in the voter model, 1/2 of the population in state A and 1/2 in state B).

For a quantitative description of the ordering dynamics towards consensus in the A or B state we use as an order parameter the ensemble average interface density $\langle \rho \rangle$. This is defined as the density of links joining nodes in the network which are in different states [49, 68]. The ensemble average, indicated as $\langle .. \rangle$, denotes average over realisations of the stochastic dynamics starting from different random initial conditions. For our random initial conditions in the AB-model $\langle \rho(t = 0) \rangle = 2/3$: a given node has probability $2/3$ of being connected to a node in a different state ($\langle \rho(t = 0) \rangle = 1/2$ in the voter model). This is valid for any network in which there are no correlations in the random initial distribution of states among the nodes of the network. During the time evolution, the decrease of ρ from its initial value describes the ordering dynamics by a coarsening process with growth of spatial domains in which agents are in the same state. The minimum value $\rho = 0$ corresponds to an absorbing state where all the agents have reached consensus in the same state.

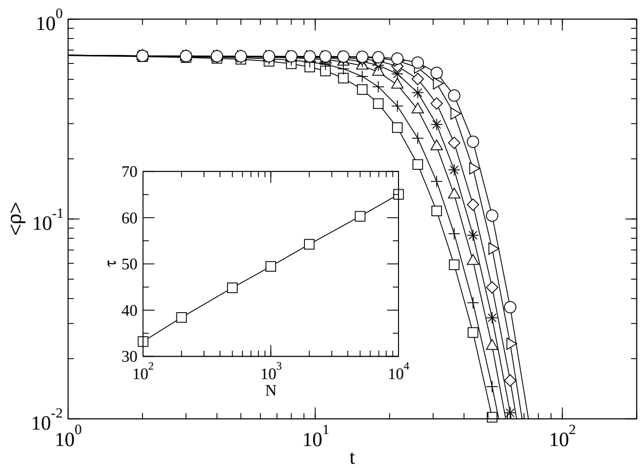

Figure 3.1: Time evolution of the average interface density $\langle \rho \rangle$ for the AB-model in a fully connected network for different system sizes. Random initial conditions. From left to right: $N = 100$ (□), 200 (+), 500 (△), 1000 (∗), 2000 (◇), 5000 (▷), 10000 (○). Averages are calculated over 10000 realisations. Inset: dependence of the average time to reach an absorbing state τ with the system size: $\tau \sim \ ln(N)$.

3.1 Fully connected networks

As a first step, we consider the dynamics of the AB-model in a fully connected network of N individuals, i.e., a network where all agents interact with one another. In this way, we go beyond population dynamics described by ODEs, and we account for the finite size effects of the resulting stochastic dynamics.

Figure 3.1 shows the time evolution of the average interface density in fully connected networks for different system sizes starting from random initial conditions. Consensus in the A or B option is always reached, with equal probability. The system reaches a plateau and approaches exponentially the absorbing state after a finite size fluctuation $\langle \rho(t) \rangle \sim e^{-kt}$ (the exponent k of the exponential decay is a constant independent of system size). In the inset of this figure, the average time to reach an absorbing state, τ, is shown to scale logarithmically with the system size as $\tau \sim \ ln(N)$.

In the limit of infinite system size, the dynamics is exactly described by the ODEs (2.12)-(2.13). When starting from random initial conditions, the system lies on top of the stable manifold of the saddle point corresponding to coexistence of the three phases (Figure 2.2). Therefore, the system moves until reaching the saddle fixed point and stays there; consensus is never reached. When we consider finite size fully connected networks instead, the system moves towards the saddle and fluctuates around this fixed point (stage corre-

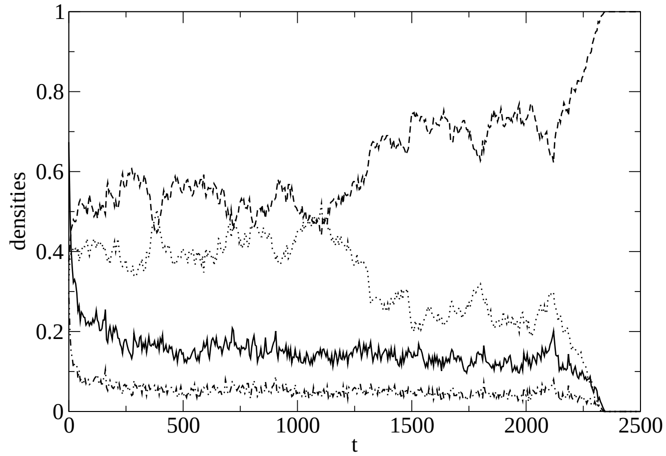

Figure 3.2: Time evolution of the total densities of agents in the three states, Σ_i (*i*= *A, B, AB*), and the interface density, ρ for the AB-model in a 2-dimensional regular lattice. One realisation in a population of $N = 400$ agents is shown. From top to bottom: Σ_A (dashed line), Σ_B (dotted line), ρ (solid line), Σ_{AB} (dot-dashed line).

sponding to the plateau in Figure 3.1). At a certain point, a finite size fluctuation is large enough to drive the system far enough towards one of the two basins of attraction of the two stable fixed points, so that the system reaches consensus in the A or B option exponentially (stage corresponding to the decay of $\langle\rho(t)\rangle$ in Figure 3.1).

In comparison, the results for the voter model in fully connected networks when starting from random initial conditions are the following: for a single realisation, $\rho(t)$ fluctuates grossly until a finite size fluctuation drives it to the absorbing state. The time evolution of the average interface density, though, is $\langle\rho(t)\rangle \sim e^{-2t/N}$; giving an exponential decay depending on system size. This is related to the fact that the probability that a simulation reaches consensus at time t decays as $\sim e^{-t/N}$ [69]. In addition, the time to reach an absorbing state, τ, scales with the system size as $\tau \sim N$ [48], giving rise to a much slower path to consensus compared to the AB-model ($\tau \sim \; ln(N)$).

Moreover, the ensemble average magnetisation is conserved in the voter model in networks with an homogeneous degree distribution [47] [1]. Therefore, the fraction of runs which lead to consensus in the A option are proportional to the fraction of initial agents in the state A. This is not the case in the AB-model, where the average magnetisation, defined as $\langle m(t)\rangle \equiv \langle\Sigma_A(t) - \Sigma_B(t)\rangle$, is not conserved:

[1] In heterogeneous networks, only an ensemble average magnetisation weighted by the degree of the node is conserved [68].

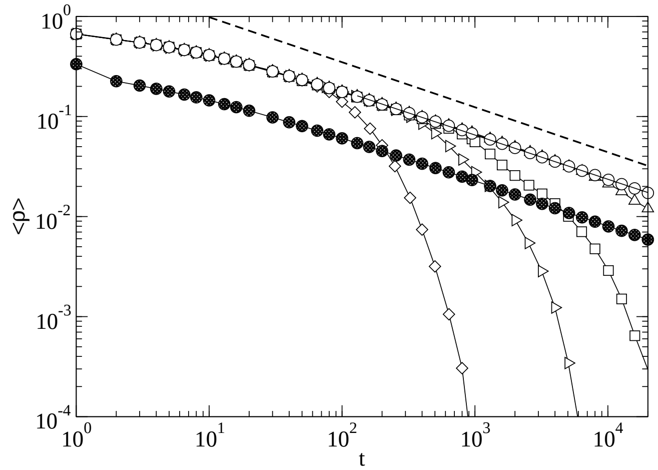

Figure 3.3: Time evolution of the average interface density $\langle\rho\rangle$ for the AB-model in a 2-dimensional regular lattice for different system sizes. Empty symbols: from left to right: $N = 10^2$ (◇), 20^2 (▷), 30^2 (□), 100^2 (△), 300^2 (○). The average global density of AB agents, $\langle\Sigma_{AB}\rangle$, for $N = 300^2$ agents is also shown (•). Averages are calculated over 100-1000 realisations depending on the system size. Dashed line for reference: $\langle\rho\rangle \sim t^{-0.45}$.

$$\frac{d\langle m\rangle}{dt} = \frac{1}{2}\langle\Sigma_{AB}\rangle\langle m\rangle \tag{3.1}$$

$\langle\Sigma_{AB}\rangle \geqslant 0, \forall t$ so that $sign(\frac{d\langle m\rangle}{t}) = sign(\langle m\rangle)$: if there is a bias in the initial conditions towards one of the two options, this option will be the one who will take over the system.

3.2 Two-dimensional lattices

In order to take into account a spatial distribution of agents, we consider next the dynamics of the AB-model on a 2-dimensional regular lattice with four neighbours per node [46]. In Figure 3.2 we show, for a typical realisation, the time evolution of the total densities of agents in state A, Σ_A, in state B, Σ_B, and in state AB, Σ_{AB}; and the density of interfaces, ρ. State A takes over the system, while the opposite option B disappears. Consensus in either of the two equivalent states A or B is always reached (with equal probability to reach consensus in state A or B). We observe an early very fast decay of the interface density and of the total density of agents in the state AB, Σ_{AB}, followed by a slower decay corresponding to the coarsening dynamical stage. This stage lasts until a finite size fluctuation causes the dominance of one of

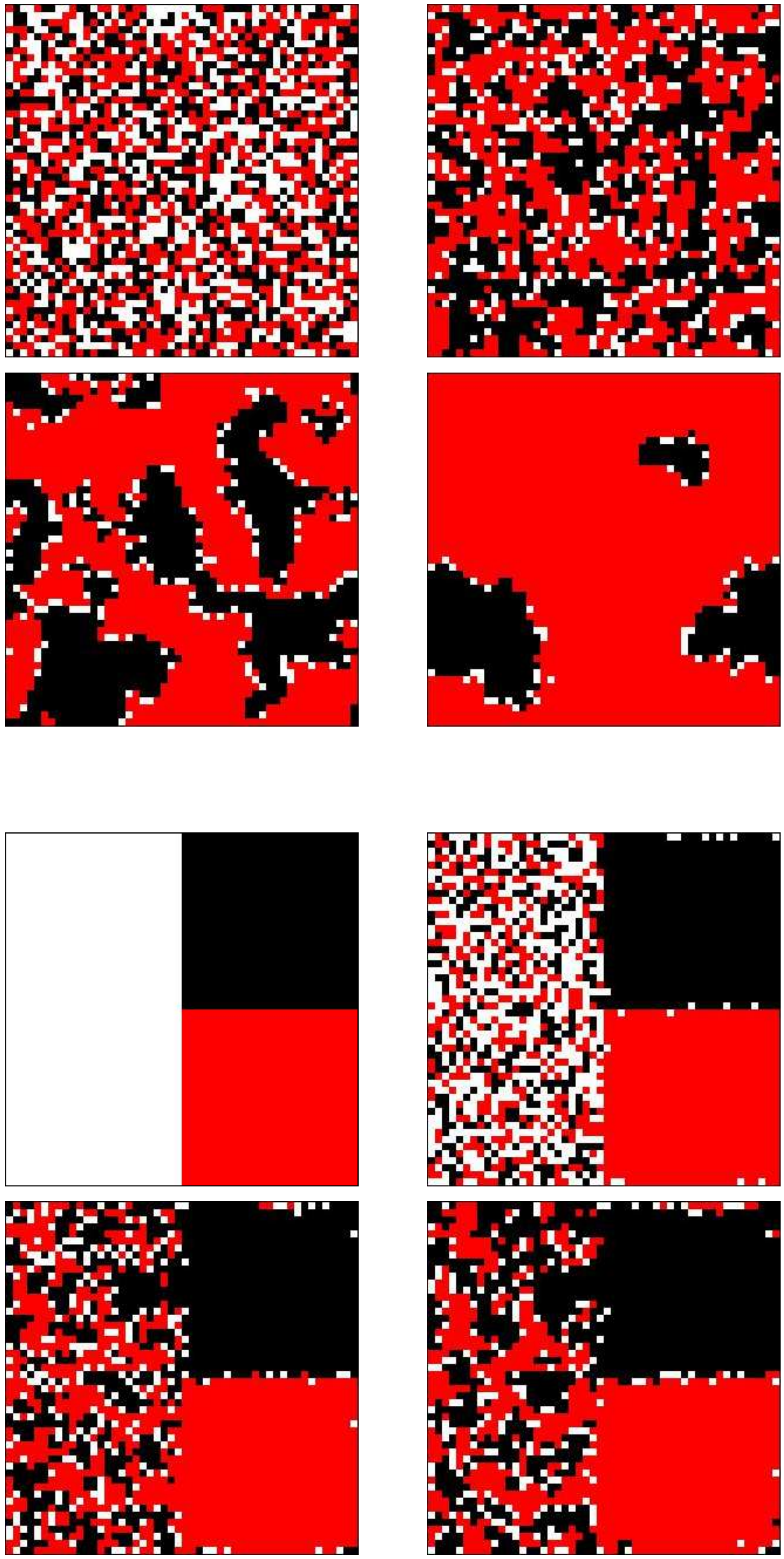

Figure 3.4: *Top*: Random initial conditions: snapshots of a typical simulation of the dynamics in a regular lattice of 2500 individuals. t=0, 8, 80, 800 from left to right. *Bottom*: Disintegration of an initial bilingual community in a regular network: 2500 individuals. t=0, 1, 5, 10 from left to right. Grey: monolinguals A, black: monolinguals B, white: bilinguals, AB

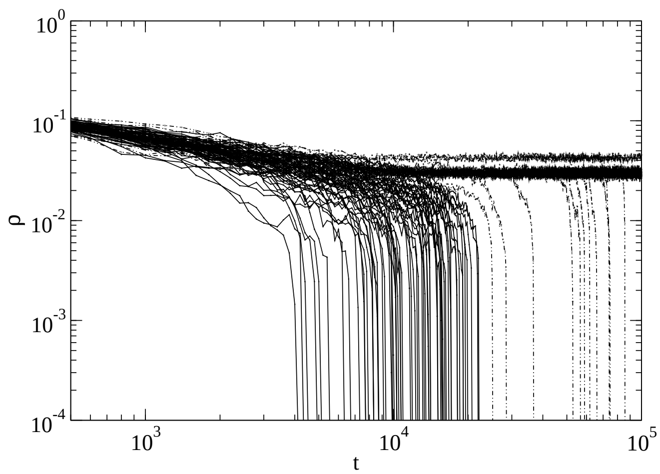

Figure 3.5: The time dependence of the interface density ρ in a regular lattice for the AB-model is shown for 100 realisations. We observe two types of realisations: most of them decay by a finite size fluctuation to an absorbing state after the stage of coarsening (solid lines); however, around 1/3 of them get trapped in dynamical metastable states, identified by an essentially constant value of ρ, until they eventually decay (dotted lines).

the states A or B, and the density of AB agents disappears together with the density of agents in the state opposite to the one that becomes dominant.

In Figure 3.3 we show the time evolution of the average interface density and of the average total density of AB agents, averaged over different realisations. For the relaxation towards one of the absorbing states (dominance of either A or B) both the average interface density and the average density of AB agents decay following a power law with the same exponent, $\langle\rho\rangle \sim \langle\Sigma_{AB}\rangle \sim t^{-\gamma}$, $\gamma \simeq 0.45$. This indicates that the evolution of the average density of the AB agents is correlated with the interface dynamics. Several systems sizes are shown in order to see the effect of finite size fluctuations. During the coarsening stage described by the power law behaviour, spatial domains of the A and B community are formed and grow in size. From the dependence of $\langle\rho\rangle$ with time, it follows that the typical size of a domain, $\langle\xi\rangle$, grows as $\langle\xi\rangle \sim t^{\gamma}$, $\gamma \simeq 0.45$. Eventually a finite size fluctuation occurs (as the one shown in Figure 3.2) so that the whole system is taken to an absorbing state in which there is consensus in either the A or B option.

During the coarsening process spatial domains of AB agents are never formed. Rather, during an early fast dynamics AB agents place themselves in the boundaries between A and B domains. This explains the finding that the density of AB agents follows the same power law than the average density of interfaces. We can observe in Figure 3.4-top snaphots a typical realisation of

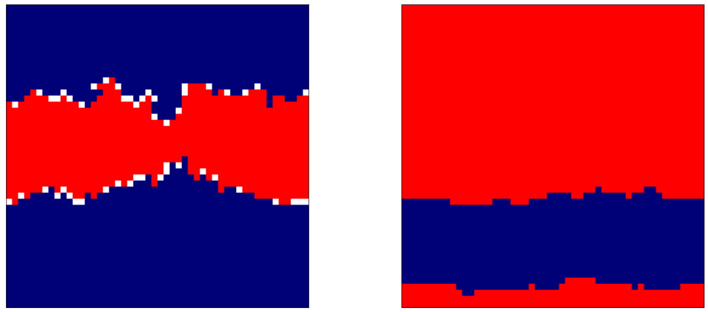

Figure 3.6: Snapshots of simulations which get trapped in stripe-like dynamical metastable states. $N = 50^2$ agents. Legend: Black: state A, grey: state B, white: state AB. Left panel: AB-model in a two dimensional regular lattice; 4 neighbours per site. Right panel: ϵ-model ($\epsilon = 1.0$). Two dimensional regular lattice; 8 neighbours per site.

the dynamics: the fast decay of the amount of AB agents, the formation of A and B domains, and the presence of AB agents only at the interfaces between them. We have also checked the intrinsic instability of an AB community: an initial AB domain disintegrates very fast into smaller A and B domains, with AB agents just placed at the interfaces, as shown in the set of snaphots in Figure 3.4-bottom.

Our result for the growth law of the characteristic length of A or B domains is compatible with the well known exponent 0.5 associated with domain growth driven by mean curvature and surface tension reduction observed in SFKI models [18]. However, systematic deviations from the exponent 0.5 are observed. These deviations are at least partially due to the fact that on closer inspection there are two type of qualitatively different realisations, which we show in Figure 3.5: while many of them have a coarsening stage followed by a finite size fluctuation which drives the system to an absorbing state, a finite fraction of the realisations ($1/3$ of them, for large enough systems) get trapped in long-lived metastable states. These metastable states are reminiscent of the ones found [70] in the analysis of a two states majority rule dynamics based on group interaction [53]. They correspond to stripe-like configurations for an A or B domain. The boundaries of these stripe-shaped domains are close to flat interfaces but with interfacial noise present (Figure 3.6-Left). Although long-lived, these configurations continue to evolve and in this sense they are different from the stripe-like frozen states with completely flat boundaries found in a zero temperature SFKI model [71]. When a realisation falls in such dynamical metastable states, coarsening stops (the average interface density fluctuates around a fixed value), until eventually a finite size fluctuation makes the two walls collide and takes the system to one of the

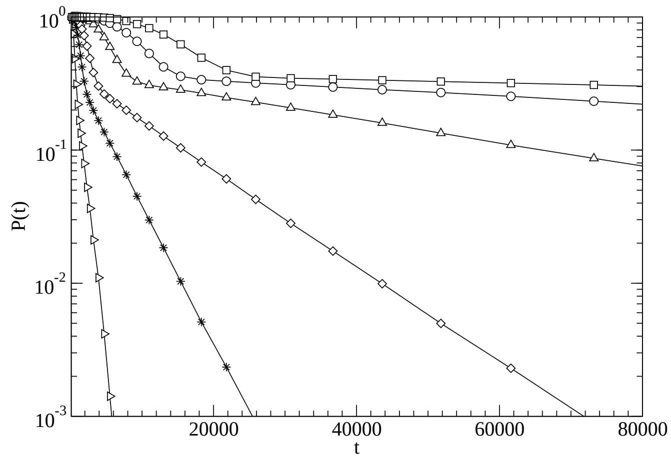

Figure 3.7: Time evolution of the fraction of alive runs, $P(t)$, for the AB-model in a 2-dimensional regular lattice for different system sizes. From left to right: $N = 20^2$ ($\triangleright$), 30^2 ($*$), 40^2 ($\diamond$), 60^2 ($\triangle$), 80^2 ($\circ$), 100^2 ($\square$). Averages are calculated over 5000-20000 realisations depending on the system size. The exponential tail corresponds to the stripe-like metastable states.

absorbing states (see Figure 3.5).

If the realisations that fall into long-lived dynamical metastable states are removed when computing the average interface density, the power law exponent for the decay of $\langle\rho\rangle$ increases, approaching the value $\gamma = 0.5$ characteristic of curvature driven coarsening. Other deviations from the exponent $\gamma = 0.5$ can be due to non trivial logarithmic corrections. In 1 and 3-dimensional lattices, we also find an exponent close to 0.5, which substantiates the claim that curvature reduction is the dominant mechanism at work for the coarsening process in the AB-model.

The existence of two type of realisations gives rise to two different characteristic times. Starting from random initial conditions we consider the distribution of survival times, $p(t)$, i.e., the distribution of the times for a simulation to reach an absorbing state. From numerical simulations, it has been proven that this distribution displays an exponential tail corresponding to the realisations that involve a metastable state. The characteristic time to reach consensus can be calculated from,

$\tau = \int_0^\infty tp(t)\,dt = \int_0^T tp_1(t)\,dt + \int_T^\infty tp_2(t)\,dt$

where $p_1(t)$ corresponds to the first type of realisations, $p_2(t)$ to the second type, and T is the time where there remain only stripe-like metastable states ($1/3$ of the realisations, as mentioned above) and where $p(t)$ becomes exponential.

For the first type of realisations, the ones that do not get trapped in long

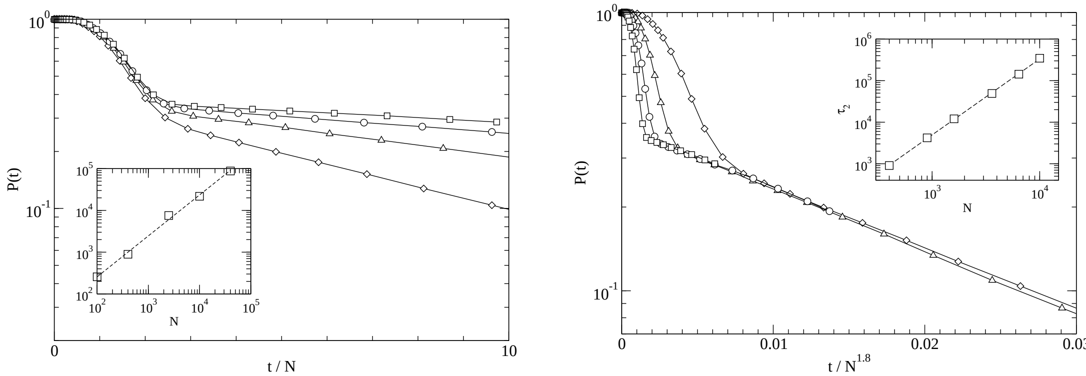

Figure 3.8: Time evolution of the fraction of alive runs, $P(t)$, for the AB-model in a 2-dimensional regular lattice for different system sizes. Averages are calculated over 5000-20000 realisations depending on the system size. ***Left***: the time has been rescaled by N, in order to observe the scaling for the first type of realisations that approach the absorbing state after the coarsening stage. From bottom to top: 40^2 ($\diamond$), 60^2 ($\triangle$), 80^2 ($\circ$), 100^2 ($\square$). Inset: dependence on the system size of the characteristic time to reach an absorbing state τ for these first type of realisations: $\tau \sim N$. ***Right***: the time has been rescaled by $N^{1.8}$, in order to observe the scaling for the second type of realisations that get trapped in stripe-like metastable states. From right to left: 40^2 ($\diamond$), 60^2 ($\triangle$), 80^2 ($\circ$), 100^2 ($\square$). Inset: dependence on the system size of the characteristic time to reach an absorbing state τ for these second type of realisations: $\tau \sim N^{1.8}$.

lived metastable states, the characteristic time to reach an absorbing state can be estimated to scale as $\tau_1 \sim N$ since the coarsening is described by $\langle\rho\rangle \sim t^{-\gamma}$, with $\gamma \simeq 0.5$, and at the time of reaching consensus $\langle\rho\rangle \sim (1/N)^{1/d}$ (d is the dimensionality of the lattice). This has been confirmed by numerical simulations which consider only such kind of realisations. For the second type, $p_2(t) \sim e^{-\alpha(N)t}$, where $\alpha(N)$ is a constant depending on system size N. It is straightforward to show that for large enough systems, the second term scales as $\tau_2 \sim 1/\alpha(N)$. Therefore, to obtain the dependence of τ_2 with system size, we are just interested in the exponent $\alpha(N)$. In order to reduce the fluctuations observed in the tail for the distribution $p(t)$, we analyse instead the fraction of alive runs, $P(t)$, i.e., the fraction of simulations which have not reached consensus yet. They are related in the following way: $P(t) = 1 - \int_0^t p(t')dt'$. As $p_2(t)$ is exponential, we can obtain $\alpha(N)$ from the fraction of alive runs: the tails of $P(t)$ also decay as $P(t) \sim e^{-\alpha(N)t}$. We show in Figure 3.7 the fraction of alive runs for different system sizes. The first fast decay corresponds to the first type of simulations, while the exponential tail describes the approach to the absorbing state for simulations which get trapped in metastable states. Analysing the exponential tails for different system sizes, i.e., plotting $1/\alpha(N)$, we obtain $\tau_2 \sim N^{\alpha}$, with $\alpha \simeq 1.8$. The

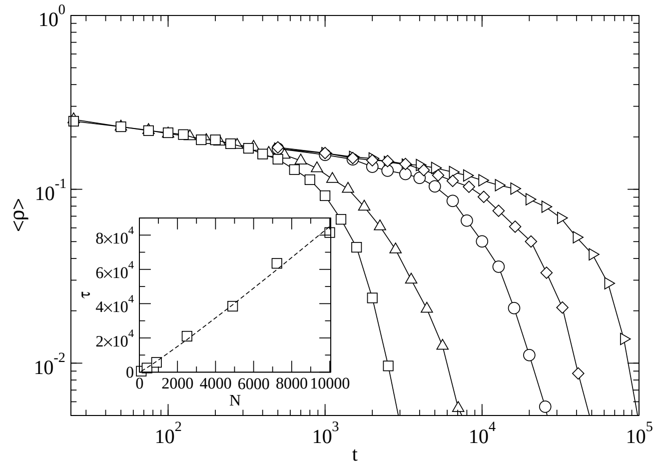

Figure 3.9: Time evolution of the average interface density $\langle\rho\rangle$ for the voter model in a 2-dimensional regular lattice for different system sizes. From left to right: $N = 20^2$ (□), 30^2 (△), 50^2 (○), 70^2 (◇), 100^2 (▷). Inset: dependence of the characteristic time to reach an absorbing state τ with the system size: $\tau \sim N^{1.08}$; compatible with the theoretical $\tau \sim Nln(N)$.

scalings regarding the two types of realisations are shown in Figure 3.8.

When taking into account all realisations, the global characteristic time to reach an absorbing state for large system sizes is dominated by the persistence of the dynamical metastable states, so that $\tau \sim N^{\alpha}$, with $\alpha \simeq 1.8$.

The AB-model analysed here is a modification of the two state voter model. For the voter model coarsening in a $d = 2$ square lattice occurs by a different mechanism, interfacial noise, such that $\langle\rho\rangle \sim (\ln t)^{-1}$ [48, 49]. In Figure 3.9, we can observe that for a finite system the characteristic time to reach an absorbing state scales as $\tau \sim N \ln N$ [33, 72]. Therefore, the introduction of the AB-state is identified as a mechanism to modify the interface dynamics from interfacial noise to curvature driven dynamics. In spite of the small number of AB agents that survive in the dynamical process, they cause a nontrivial modification of the dynamics. Indeed, in our simulations we observe the formation of well defined interfaces between A and B domains, populated by AB agents, that evolve by a curvature driven mechanism. The different nature of the coarsening process is illustrated comparing Figure 3.10-Top (AB-model) and Figure 3.10-bottom (voter model), for initial conditions with a closed single-option domain surrounded by a domain in the opposite opinion and Figure 3.11-Top (AB-model) and Figure 3.11-bottom (voter model) for random initial conditions.

On the qualitative side, the inclusion of the AB agents gives rise to a much faster coarsening process, but due to the existence of dynamical metastable

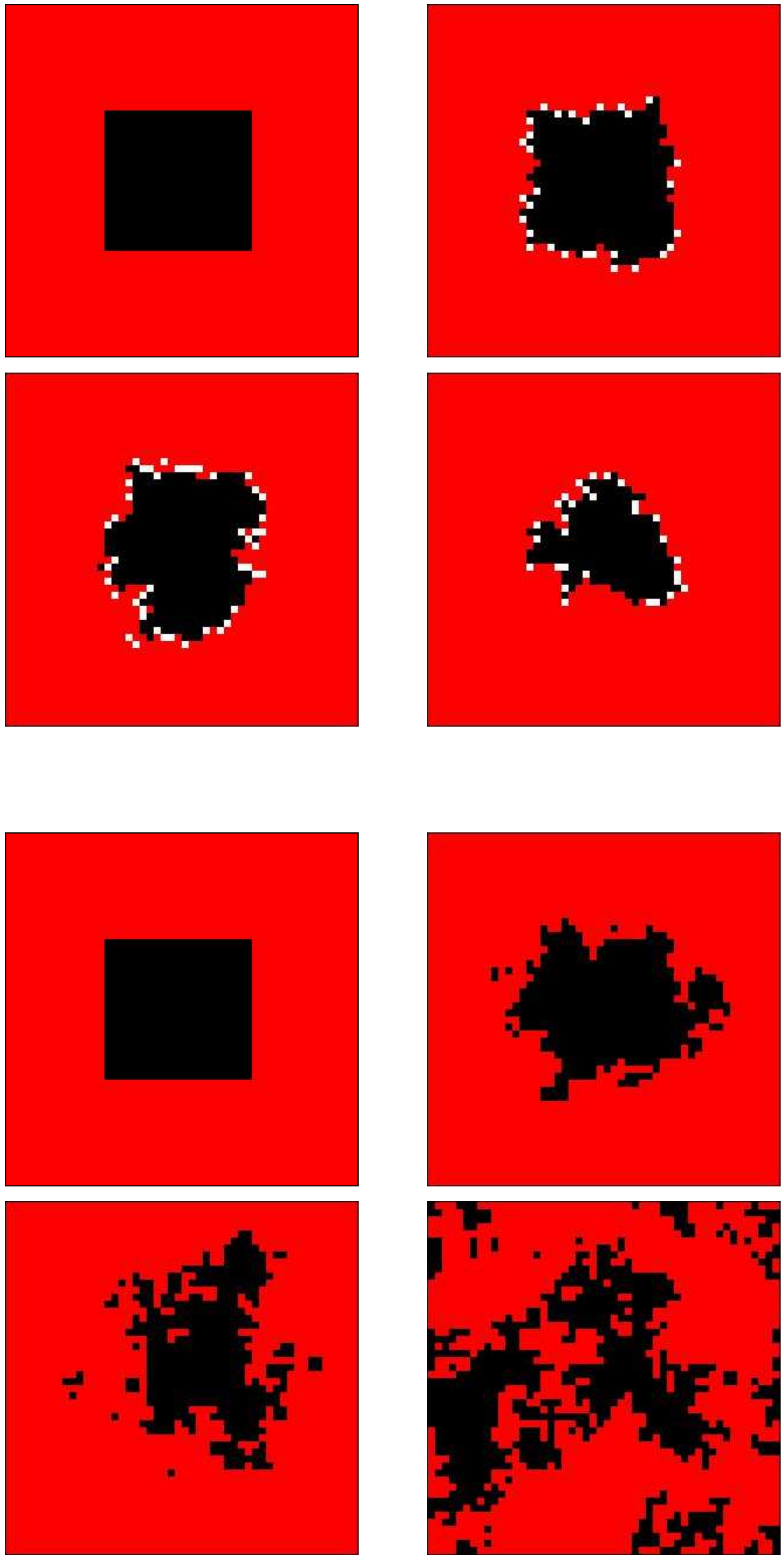

Figure 3.10: Comparison of the interface dynamics. Initial conditions with a single-option domain surrounded by a domain in the opposite option. Regular lattice of 2500 individuals. t=0, 40, 200, 1000 from left to right. *Top*: AB-model. Curvature driven interface dynamics: closed domain shrinks due to surface tension. *Bottom*: voter model. Noisy interface dynamics: closed domain diffuses throughout the lattice. Grey: monolinguals A, black: monolinguals B, white: bilinguals, AB.

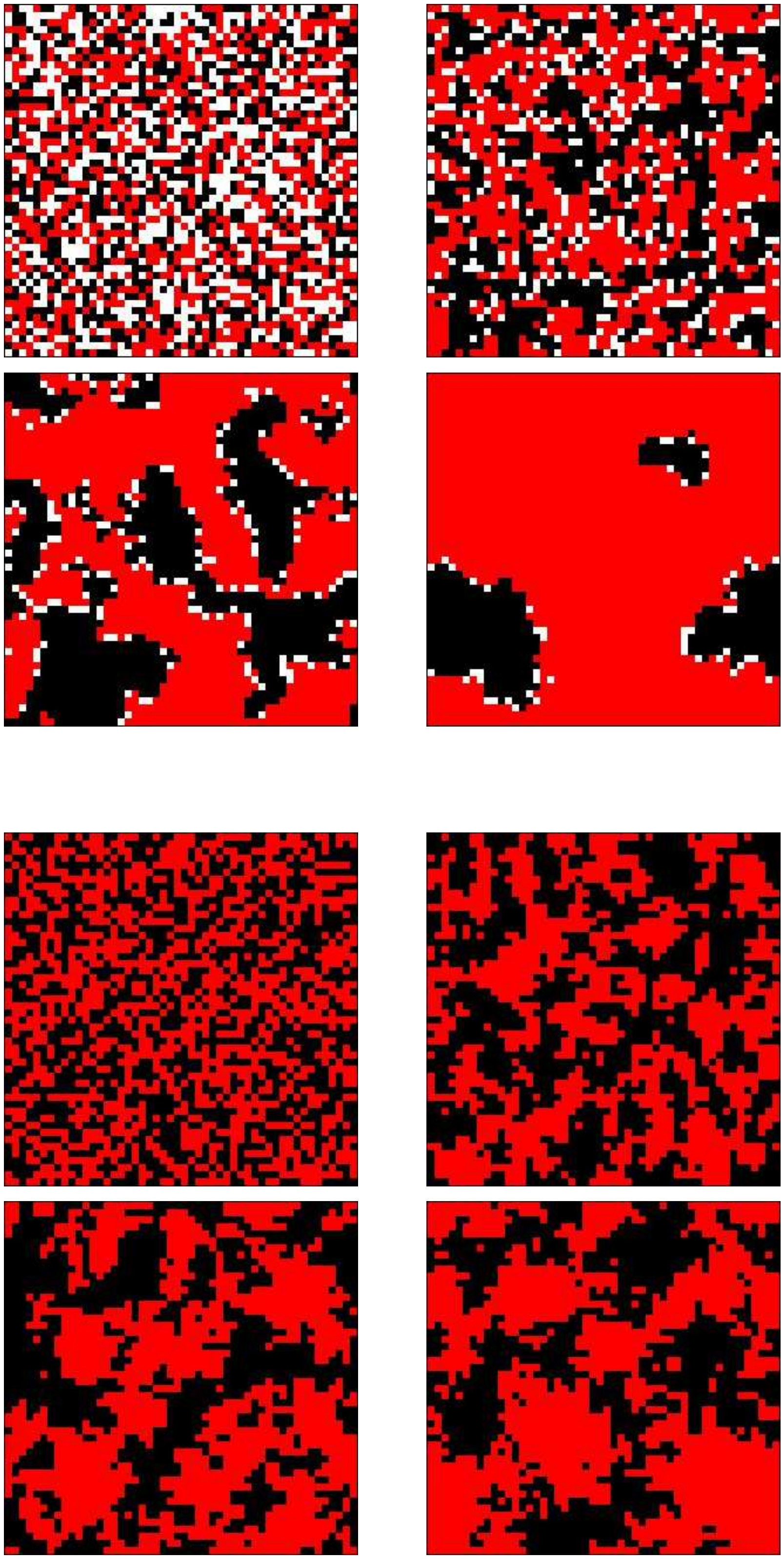

Figure 3.11: Comparison of the interface dynamics. Random initial conditions: snapshots of a typical simulation of the dynamics in a regular lattice of 2500 individuals. t=0, 8, 80, 800 from left to right. *Top*: AB-model: coarsening leading to the formation of single-option domains that evolve by curvature reduction. *Bottom*: voter model: slower coarsening leading to non-localised domains evolving by interfacial noise. Grey: monolinguals A, black: monolinguals B, white: bilinguals, AB.

states, on the average it also favours a longer dynamical transient in which domains of the two competing options coexist $\frac{\tau_{AB}}{\tau_{voter}} \sim \frac{N^{0.8}}{ln(N)}$ (larger lifetime before reaching the absorbing state for large fixed N).

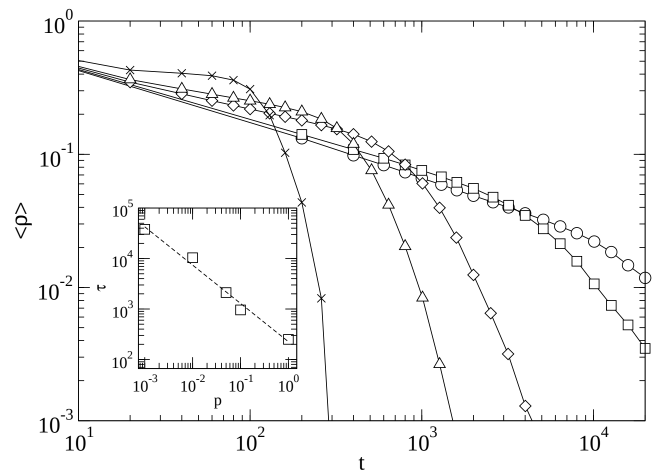

Figure 3.12: Time evolution of the average interface density $\langle\rho\rangle$ for the AB-model in small world networks with different values of the rewiring parameter p. From left to right: p=1.0 ($\times$), 0.1 ($\triangle$), 0.05 ($\diamond$), 0.01 ($\Box$), 0.0 ($\circ$). For comparison the case $p = 0$ for a regular network and the case $p = 1$ corresponding to a random network are also shown. The inset shows the dependence of the characteristic time to reach an absorbing state τ with the rewiring parameter p. The dashed line corresponds to the power law fit $\tau \sim p^{-0.76}$. Population of $N = 100^2$ agents. Averages taken over 500 realisations.

3.3 Small world networks

Up to now, we have considered finite size effects and a regular spatial distribution of the agents. However, social networks display complex features like the small world effect: short average path length and high clustering [73]. This is a consequence of the existence in the network of long range social interactions. To study the effect of such interactions in the network, we next consider the dynamics of the AB-model on a small world network [46] constructed following the algorithm of Watts & Strogatz [6]: starting from a two dimensional regular lattice with four neighbours per node, we rewire with probability *p* each of the links at random, getting in this way a partially disordered network with long range interactions throughout it.

In Figure 3.12 we show the evolution of the average interface density for different values of p. As we found in the regular lattice, we also observe here a dynamical stage of coarsening with a power law decrease of $\langle\rho\rangle$ followed by a fast decay to the A or B absorbing states caused by a finite size fluctuation. During the dynamical stage of coarsening, the A and B communities have similar size, while the total density of AB agents is much smaller. In the range of intermediate values of p properly corresponding to a small world network, increasing the rewiring parameter p has two main effects: i) the

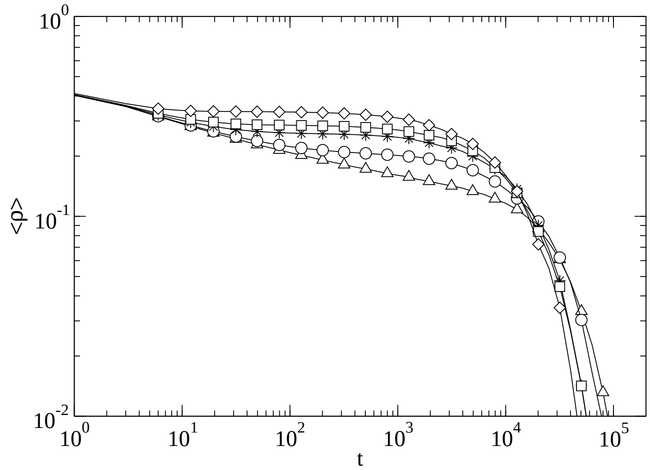

Figure 3.13: Time evolution of the average interface density $\langle\rho\rangle$ for the voter model in a small world network with different values of p. From up to bottom, p=1.0 (◇), 0.1 (□), 0.05 ($*$), 0.01 (○), 0.0 (△). Population of $N = 100^2$ agents. Averages taken over 900 realisations.

coarsening process is notably slower; ii) the characteristic time to reach an absorbing state τ, which can be computed here as the time when $\langle\rho\rangle$ sinks below a given small value, drops following a power law (inset of Figure 3.12): $\tau \sim p^{-0.76}$, so that the absorbing state is reached much faster as the network becomes disordered.

To understand the role of the AB-state in the ordering dynamics in a small world network, the results of Figure 3.12 should be compared with the ones in Figure 3.13 for the two state voter model in the same small world network [1]. In contrast with the AB-model, moderate values of p stop the coarsening process of a two-state voter model leading to dynamical metastable states [2] characterised by a plateau regime for the average interface density [50, 51]. The plateau height is larger for increasing p, indicating that the domains become smaller. However, the lifetime of these states is not very sensitive to the value of p, with the characteristic time to reach an absorbing state being just slightly smaller than the one obtained in a regular lattice ($p = 0$). This is a different effect than the strong dependence on p found for the characteristic time to reach an absorbing state when AB agents are included in the dynamics. Comparing the results of Figures 3.12 and 3.13 for a fixed intermediate

[1]Note that the small world network considered in [50] is obtained by a rewiring process of a $d = 1$ regular lattice.

[2]In the AB-model, a metastable state is reached, but only after the coarsening stage. As we show at the end of this section, $\tau \sim \ln N$; and therefore, this can be seen only for very large system sizes.

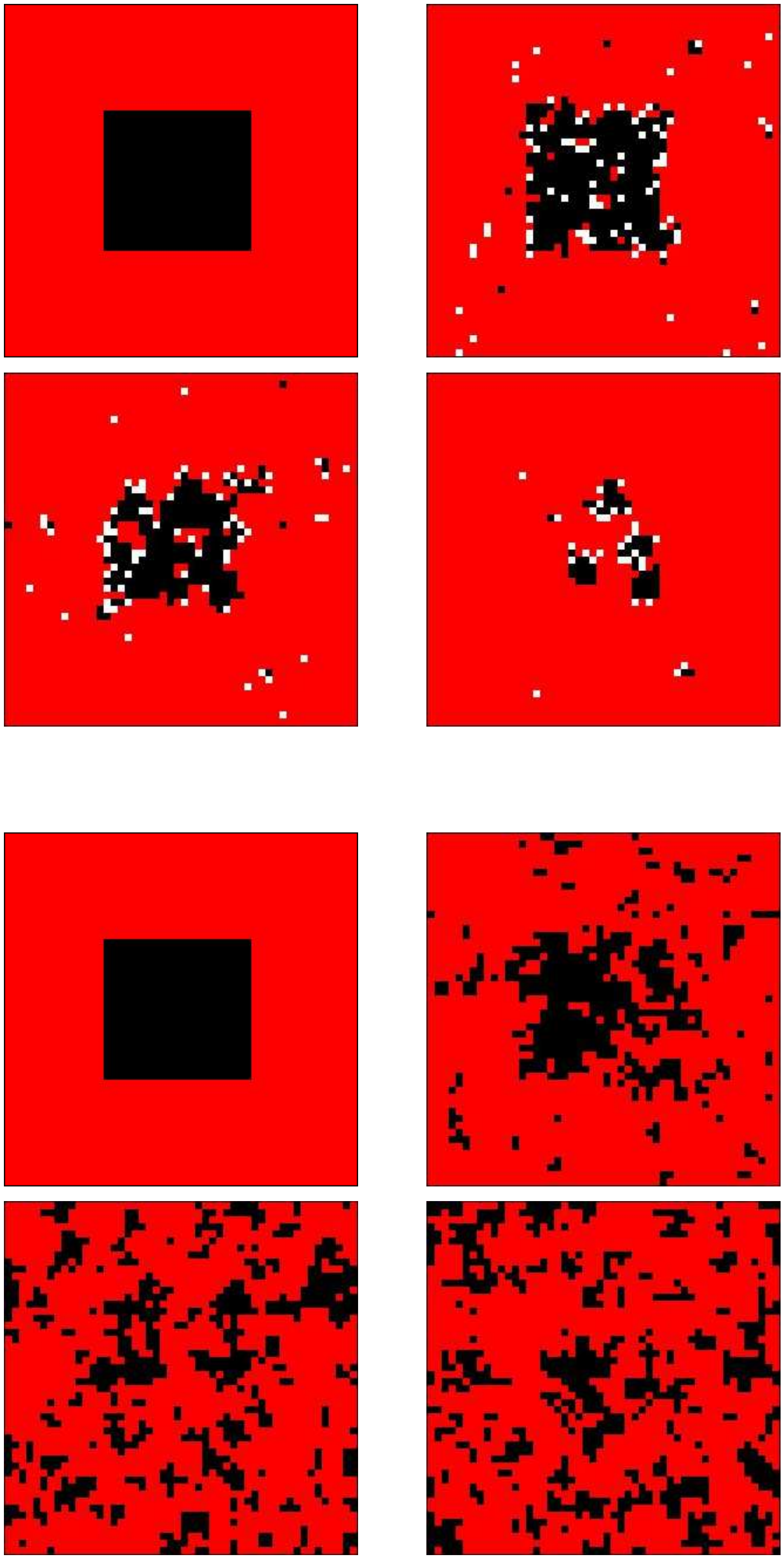

Figure 3.14: Initial conditions with a single-option domain surrounded by a domain in the opposite state: Small world network with $p = 0.1$ projected in two dimensions of 2500 individuals. t=0, 20, 60, 140 from left to right. *Top*: AB-model: a domain shrinks much faster due to the long range interactions that connect it to the rest of the network, which fragment the initial domain accelerating the approach to consensus. *Bottom*: voter model: long range connections do not make a qualitative different behaviour than the one observed in a regular lattice (Figure 3.10). Grey: monolinguals A, black: monolinguals B, white: bilinguals, AB.

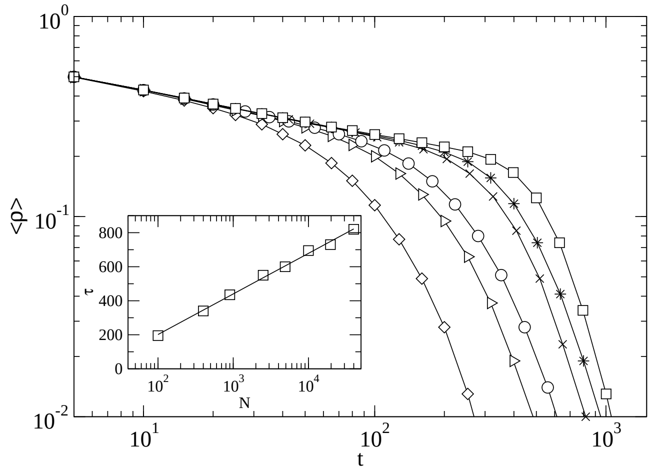

Figure 3.15: Time evolution of the average interface density, $\langle\rho\rangle$, for different values of the population size, N, in a small world network with $p = 0.1$. $N = 10^2$ ($\diamond$), 20^2 ($\triangleright$), 30^2 ($\circ$), 70^2 ($\times$), 100^2 ($*$), 200^2 ($\Box$); from left to right. Averaged over 1000 realisations in 10 different networks. Inset: dependence of the characteristic time to reach an absorbing state τ with the system size: $\tau \sim \ln N$.

value of p, we observe that including AB agents in the dynamics on a small world network of interactions allows the coarsening process to take place, and it also produces an earlier decay to the absorbing state. To illustrate qualitatively the different effect of the long range interactions on the two models, we show in Figure 3.14 snapshots of the two dynamics in a small world network with $p = 0.1$: a curvature driven dynamics (*majority rule*) is very sensitive to long range links, while noisy interface dynamics (*imitation*) is barely affected by them.

System size dependence for a fixed value of the rewiring parameter p is analysed in Figure 3.15. We observe that the initial stage of the coarsening process is grossly independent of system size, but the characteristic time to reach an absorbing state scales with the system size N as $\tau \sim \ln(N)$. This results to be the same scaling law obtained in fully connected networks (see Figure 3.1). For the two state voter model $\tau \sim N$ [50]. Therefore the faster decay to the absorbing state caused by the presence of AB agents in a large system interacting through a small world network is measured by the ratio $\frac{\tau_{AB}}{\tau_{voter}}|_{SW} \sim \frac{\ln(N)}{N}$. We note that this faster decay is qualitatively the opposite result than the one found in a regular lattice where $\tau_{AB} \sim N^{1.8} > \tau_{voter} \sim N\ln(N)$. In the case of a regular lattice, on the average the AB agents slow down the decay towards the absorbing state due to the dominance of the dynamical metastable states described in the previous

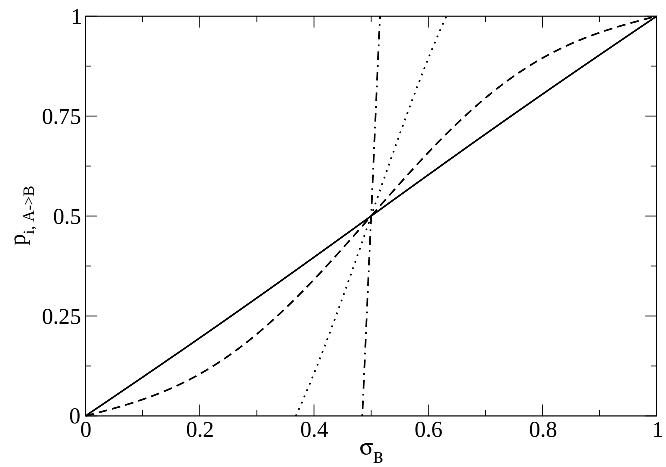

Figure 3.16: Transition probabilities (Equation (3.2)) for the ϵ-model for different values of ϵ. When $\epsilon > 1/(2\pi)$ the transition probability for such a given ϵ is defined as follows: $p_{i,A\to B}$ as given by Equation (3.2) for values of σ such that $0 \leq p_{i,A\to B} \leq 1$; $p_{i,A\to B} = 0[1]$ for values of σ such that Equation (3.2) gives $p_{i,A\to B} < 0[> 1]$. The limit $\epsilon \to \infty$, corresponds to the step-function transition probability of the SFKI model at zero temperature. ϵ= 0.01 (solid line), 0.2 (dashed), 1.0 (dotted), 10.0 (dot-dashed).

section.

3.4 Modification of the voter model: the ϵ - model

In the previous sections, we have shown how the extension of the voter model dynamics by the introduction of a third AB-state of coexisting options at the individual level (AB-model), leads to a radical change in the interface dynamics. A natural question that these results pose is if the crossover from interfacial noise dynamics of the voter model to curvature driven dynamics is generic for any structural modification of the voter model. In order to interpolate from the voter model dynamics towards the majority model represented by the zero-temperature SFKI model where the dynamics is curvature driven, we have considered the coarsening process in a 2-dimensional lattice in which agents can choose between two excluding options (states A and B) and the dynamic rules are as defined in Chapter 2 but with transition probabilities (see Figure 3.16) [46]:

$$p_{i,A\to B} = \sigma_{i,B} - \epsilon \sin(2\pi\sigma_{i,B}), \quad p_{i,B\to A} = \sigma_{i,A} - \epsilon \sin(2\pi\sigma_{i,A}), \quad \epsilon \leq \frac{1}{2\pi} \qquad (3.2)$$

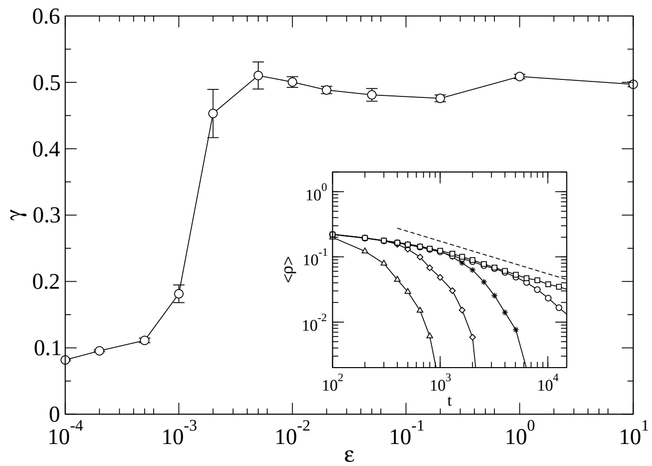

Figure 3.17: Characteristic coarsening exponent γ ($\langle\rho\rangle \sim t^{-\gamma}$) for the ϵ-model as a function of the perturbation parameter ϵ. $N = 400^2$ agents. Averages taken over 75 realisations. Inset: time evolution of the average interface density. From left to right: $N = 20^2$ ($\triangle$), 50^2 ($\diamond$), 100^2 ($*$), 200^2 ($\circ$), 400^2 ($\square$) agents. Averages taken over 100 realisations. Given a value of ϵ ($\epsilon = 0.01$ in this figure), a power law for the average interface density decay is found for large enough system sizes. Dashed line for reference: $\langle\rho\rangle \sim t^{-0.5}$.

In the following, we will call this modification of the voter model the ϵ-*model*. The parameter ϵ measures the strength of the term that perturbs the interaction rules of the voter model ($\epsilon = 0$). This perturbation of the voter model implies that the probability of changing option is no longer a linear function of the density of neighboring agents in the option to be adopted. With the perturbation term chosen here there is a nonlinear reinforcing (of order ϵ) of the effect of the local majority: the probability to make the change $A \to B$ is larger [smaller] than σ_B when $\sigma_B > 1/2$ [$\sigma_B < 1/2$]. In particular, we note that for $\epsilon \neq 0$, the conservation law of the ensemble average magnetisation [47, 68], a characteristic symmetry of the voter model, is no longer fulfilled. For later comparison we recall that in the zero-temperature SFKI model the local majority determines, with probability one, the change of option: $p_{A\to B} = 1$ [0] if $\sigma_B > 1/2$ [$\sigma_B < 1/2$].

Our results for the exponent γ in a power law fitting $\langle\rho\rangle \sim t^{-\gamma}$ for the ϵ-model are shown in Figure 3.17 for different values of ϵ. In these simulations we have considered a 2-dimensional lattice with eight neighbours per node so that more values are allowed for the perturbation term in Equation (3.2). For very small values of ϵ we observe an exponent $\gamma \simeq 0.1$ compatible with the logarithmic decay ($\langle\rho\rangle \sim (\ln t)^{-1}$) of the voter model, as obtained in [33]. However, for small but significant values of ϵ there is a crossover to a value $\gamma \simeq 0.5$ associated with curvature driven coarsening. Dynamical metastable states analogous to the ones found in the AB-model are also found (Figure 3.6-Right) with probability $\sim 1/3$ (for large enough systems) for values of ϵ for which $\gamma \simeq 0.5$. The $1/3$ fraction of realisations corresponds to the probability to reach a frozen configuration in the SFKI at zero temperature [71]. The distribution of survival times of these dynamical metastable states also displays an exponential tail analogous to the one found in Figure 3.7 for the AB-model. These realisations have been removed to calculate the value of γ.

We conclude that a small perturbation on the linear transition probabilities of the voter model dynamics, such that there is a nonlinear reinforcing of the effect of the local majority, leads to a new interface dynamics equivalent to the one found for the AB-model by including a third state where options are non-excluding. This illustrates the fact that the voter model dynamics is very sensitive to perturbations of its dynamical rules.

Chapter 4

Conclusions

4.1 Summary and conclusions

The Abrams-Strogatz model for the competition between two languages reduces to the voter model in the case of two socially equivalent norms or options (languages), s = 0.5, and volatility a = 1. In the same way and for the same parameter values, the Minett-Wang model, an extension of the AS-model which introduces a third state modelling bilingualism, becomes the AB-model. In the global context of consensus dynamics and motivated by these studies of language competition, we have studied the nonequilibrium transient dynamics of approach to the absorbing state for the AB-model, which can be seen as an extension of the voter model in which the interacting agents can be in either of two equivalent states (A or B) or in a third mixed state of coexistence of two options at the individual level (AB). We have analysed the ordinary differential equations which describe the model in the thermodynamic limit (case of mixed population and infinite system size), and the role of finite size effects by considering the dynamics in fully connected networks. We have studied in detail the effects of the following topologies: (i) two-dimensional lattices, to account for the effect of a spatial distribution of agents, and (ii) small world networks, to model the effect of long range interactions throughout the network. Along our work, we have compared the AB-model with the voter model in order to analyse the role of the AB agents in the dynamics.

The mean field analysis shows that in the thermodynamic limit a global consensus state (A or B) is reached with probability one, except for initial conditions lying on the stable manifold ($\Sigma_A = \Sigma_B$) of the saddle fixed point corresponding to unstable coexistence of the three states. However, when considering a fully connected network to account for the finite size effects, fluctuations drive the system out from the unstable fixed point and consensus is always reached, with an average time to consensus that scales with the system size as $\tau \sim ln(N)$.

We have analysed exhaustively the AB-model dynamics in two-dimensional lattices. A domain of agents in the AB-state is not stable and the density of AB agents becomes very small after an initial fast transient, with AB agents placing themselves in the interfaces between single-option domains. In spite of these facts, the AB agents produce an essential modification of the processes of coarsening and domain growth, changing the interfacial noise dynamics of the voter model into a curvature driven interface dynamics characteristic of two-option models with updating rules based on local majorities like SFKI dynamics [18]. In this way, the typical growth of the size of such single-option domains, $\langle\xi\rangle$, changes from $\langle\xi\rangle \sim ln(t)$ to $\langle\xi\rangle \sim t^{\alpha}$, with $\alpha \simeq 0.5$. This change in the coarsening mechanism is also found for small perturbations of the random imitation dynamics of the voter model that modify the linear dependence of the transition probabilities on the local density (ϵ-model). This result indicates that the effect might be generic for small structural modifications of the voter model dynamical rules. We have also shown that in a two dimensional regular lattice, the system reaches stripe-like dynamical metastable states with a probability $\sim 1/3$ in both, the AB-model and the ϵ-model, as observed in the majority rule dynamics based on group interaction. The average time to consensus for the AB-model has proven to scale with the system size as $\tau \sim \ N^{1.8}$. This dependence with system size is dominated by the presence of the dynamical metastable states mentioned above. Compared to the $\tau \sim \ Nln(N)$ for the voter model, the AB agents produce a a faster coarsening, but also longer times for extinction due to the presence of these metastable states.

The effect of complex features in the topology of interactions such as the role of long range connections throughout the network, has been addressed considering a small world network. While for the original two-state voter model the small world topology results in long lived metastable states in which coarsening has become to a halt [50, 51], the AB agents restore the processes of coarsening and domain growth. Additionally, they speed-up the decay to the absorbing state by a finite size fluctuation: while in the voter model the times to consensus are essentially not affected by the parameter of rewiring p, in the AB-model we obtain $\tau \sim p^{-0.76}$, indicating a strong dependence of the times to consensus on the parameter of rewiring. Moreover, we obtain a characteristic time to reach an absorbing state that scales with system size as $\tau \sim \ln N$ to be compared with the result $\tau \sim N$ for the voter model: the decay to the absorbing state is much faster in small world networks when AB agents are present.

From the point of view of recent studies of linguistic dynamics, our extension of the voter model allowing for two non-excluding options, the AB-model, is a generalisation of the microscopic version of the Abrams-Strogatz model for two socially equivalent languages, to include the effects of bilingualism

(AB agents) and social structure. Within the assumptions and limitations of our model, our results imply that bilingualism is not an efficient mechanism to stabilise language diversity when a social structure of interactions such as the small world network is taken into account. On the contrary, bilingual agents are generally found to ease the approach to absorbing monolingual states by smoothing the communication across linguistic borders.

The studies of language competition from the point of view of statistical physics and complex systems give a new perspective and formalism in sociolinguistic problems. These models, although still limited, might help to face the challenging question of the mechanisms involved in the processes of language contact.

4.2 Further research

Different lines of further research can be considered as next steps in giving a more detailed analysis of the dynamics of the AB-model, in order to move forward in modelling consensus problems, and in particular, language competition:

In the first place, the dynamics could be analysed in other complex networks such as scale-free networks [7], and specially in topologies of increasing complexity which account for other characteristics observed in real social networks beyond the small world effect: community structure, assortativity, hubs, broad degree distributions with a cutoff, etc. Several models have been designed to capture some of the characteristics of social networks, based on mechanisms such as geographical proximity [74], social similarity [75, 76], and local search [77, 78, 79]. This would lead to a wider perspective on the dynamical properties of the model in comparison to the voter model depending on the topological characteristics of the network [80]. The existence of other types of metastable states in complex topologies is an important question for physicists which, from the point of view of sociolinguistic dynamics, is of broad interest: which network structures would lead to a dynamical coexistence of two competing languages during large periods of time? Moreover, the analysis made for the AB-model could be extended to the whole parameter space (a,s), giving a more complete picture on the modelling of ordering dynamics with two non-excluding options.

In the second place, one could consider the AS-model and the MW-model within a viability theory framework [81]. In a viability problem, a constraint set is defined: a region in the variable space inside which your system is viable. A control parameter can be tuned in order to favour the viability of the system. This leads to the identification of the viability kernel, which includes all the states of the system from which, given a bounded action on the control parameter, an indefinitely viable evolution exists. In the case of modelling language competition, the constraint set can be defined by imposing a threshold in the densities of speakers below which the system is not viable, and the control parameter taken as the relative prestige of the languages [82]. Essentially the idea is to analyse under which conditions the system is viable, i.e., the two languages can coexist indefinitely above the given threshold, given that the prestige of an endangered language can be enhanced by public policies.

In the last place and from the point of view of modelling language competition, there are several open research lines that can be addressed in the near future: (i) an extension of the models accounting for learning and use dynamics, which would distinguish clearly between competence and language use; (ii) an extension of the concept of network of interactions which could model different contexts of use, and therefore take into account the phenomenon of diglossia; (iii) build up a model where the language is modelled as a property of the interaction (link) rather than a property of the agent (node), which gives rise naturally to different degrees of bilingualism, and specially, (iv) exploring dynamics which would allow for the emergence of a new language during processes of language contact such as code-switching or creolization [83].

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- *"Dynamics of language competition: bilingualism and social structure effects"*, IMEDEA , Mediterranean Institute of Advanced Studies (CSIC-UIB) (Palma Mallorca, May 2006).

Oral communications in congresses

- *Modelling language competition: bilingualism and complex social networks*, 7th International Conference Evolution of Language (EVOLANG7), Barcelona (March 2008).

- *The effects of complex social structure in the dynamics of language competition*, Dynamics and evolution of biological and social networks, Palma Mallorca (February 2008).

- *Modelling bilingualism in language competition: the effects of complex social structure*, The Fourth European Social Simulation Association Conference, Toulouse (France, September 2007).

- *Ordering dynamics with two non-excluding options: Bilingualism in language competition*, Complex systems: from physics to biology and the social sciences, Lisboa (Portugal, November 2006).

- *Dynamics of Language Competition: Effects of Bilingualism and Social Structure*, WEHIA 2006: 1st International Conference on Economic Sciences with Heterogeneous Interacting Agents; Bologna (Italy, June 2006).

Posters presented in congresses

- *Language competition as an example of the consensus problem*, FISES '08, Salamanca (March 2008).
- *Social structure effects on the dynamics of language competition*, International School on complexity. Statistical physics of social dynamics: opinions, semiotic dynamics and language. Erice (Italy. Juliol 2007).
- *Dynamics of language competition: bilingualism and social structure effects*, Annual meeting of the DPG, Deutsche Physikalische Gesellschaft, Physics of Socio-Economic Systems, Dresden (Germany. March 2006).

Participation in workshops

- *1st International Meeting on Social Simulation*, Universitat Autònoma de Barcelona (UAB), Barcelona (May 2007).
- *Workshop on Language Simulations* (GIACS); University of Warsaw, Warsaw (Poland, September 2006).
- *Dynamics on complex networks and applications*, Max-Planck-Institut fur Physik Komplexer Systeme Dresden (Germany. February 2006).
- *Nonlinear Dynamics of Spatiotemporal Self-Organization, 4th Network meeting*, UPC (Universitat Politècnica de Barcelona), Barcelona (February 2006).

Stays at other research centers

- 23/09/2007 until 15/11/2007. Dipartimento de Fisica, Università di Roma "La Sapienza", Italy; under the supervision of Professor Vittorio Loreto.
- 10/12/2007 until 21/12/2007. LCE (Laboratotry of Computational Engineering). Helsinki University of Technology, Finland; under the supervision of Professor Kimmo Kaski.

Computer experience and languages

- Programming: Fortran, Java, IDL, Maple.
- Operative systems: UNIX, Windows.
- Languages: Catalan (native), Spanish (fluent), English (fluent), Italian (good), French (basic), Swedish (basic).