Theoretical Analysis of a New Technique for Inertial Rotation Sensing Using a Semiconductor Ring Laser

Antonio Pérez S. and Alessandro Scirè

***Abstract*—We introduce a new technique for measurement of inertial rotations using a semiconductor ring laser, whithin the so-called *locking band*. The rotation affects the detuning on the optical frequency of the two counterpropagating waves, which are coupled and can be locked by intracavity backscattering making the direct measurement impossible. Here we propose to measure inertial rotation whitin the *locking band* taking advantage of a phase/amplitude coupling mechanism typical of semiconductor lasers. The dephasing accumulated by the two counterpropagating waves unbalances the fields amplitudes within the locking region. We analytically derive the Responsivity quantifying the two-mode power unbalance versus rotation rate, by means of a two-mode rate equations model. Rotations of the order of 10^{-3} Hz can be measured for 1 mW of output power.**

***Index Terms*—Semiconductor lasers, ring lasers, metrological applications, laser dynamics**

I. INTRODUCTION

SEMICONDUCTOR Ring Lasers (SRLs) [1] raised interest because they do not require claved facet mirrors or gratings for optical feedback, thus monolithic integration is easily achievable [2]. SRLs are used in numerous applications like wavelength filtering, unidirectional travelling wave operation [3], multiplexing/demultiplexing applications, and data storage-gating as optical bistables [4], [5], [6]. One interesting potential application of a SRL is a Semiconductor Ring Laser Gyroscope (S-RLG) [7], which could represent a compact, simple and low cost optical device for inertial rotation sensing.

The idea of using a ring interferometer as a rotation rate sensor was introduced by Sagnac [8], and became a technical reality after the assessment of a reliable laser technology. Optical gyroscopes are currently used in satellites, aircrafts and remote control devices. RLGs are based on the measurement of the detuning induced by rotation onto the optical frequency of the two counterpropagating modes in a Ring-Laser. However, any source of intracavity backscattering couples and can lock the counterpropagating waves [9], [10], [11], representing a major source of error in practical devices [12].

The advantages of a S-RLG are monolithic integration potential, potential low cost and small size. However, integrated S-RLGs show a wide locking band of the order of 10-100 MHz [13] hindering the Sagnac effect for practical rotation rates.

In this Letter, we theoretically show that a SRL can be used to measure inertial rotation within the so called locking band, i.e. without the need to un-lock the two counterpropagating waves. Indeed, the dephasing accumulated by the two counterpropagating waves due to rotation within the locking region is coupled to the field amplitudes via alpha factor. This in turn unbalances the field amplitudes by a quantity proportional to the rotation angular velocity. We provide analytical expression for the *Responsivity* that would characterize a possible rotation sensor. Assuming 1 mW of output power rotations of the order of 10^{-3} Hz can be measured. This technique can be exported to other ring lasers than SRLs, if an intracavity mechanism of phase-amplitude coupling is provided.

II. THE MODEL

The theoretical analysis is based on a set of dimensionless dynamical equations [13], [14] for the two (slowly varying) complex amplitudes of the counter-propagating fields E_+ and E_- modified to account for inertial rotation effects [9], [15] and a Bloch equation for the carrier density N. The equations read:

$$\left\{ \begin{array}{l} \dot{E}_\pm(t) = \mathcal{G}_\pm(N(t), |E_\pm(t)|^2)\ E_\pm(t) - \eta\ E_\mp(t) \pm i\Delta E_\pm(t) \\ \dot{N}(t) = \gamma\ \mathcal{F}(N(t), |E_\pm(t)|^2), \end{array} \right. \tag{1}$$

where

$$\mathcal{G}_\pm(N(t), |E_\pm(t)|^2) = \frac{1}{2}(1+i\alpha)\{N(t)\ \sigma_\pm - 1\},$$

$$\begin{array}{c} \mathcal{F}(N(t), |E_\pm(t)|^2) = \mu - N(t) - N(t)\ \sigma_+\ |E_+(t)|^2 \\ -N(t)\ \sigma_-\ |E_-(t)|^2, \end{array}$$

$$\sigma_\pm = 1 - s\ |E_\pm(t)|^2 - c\ |E_\mp(t)|^2,$$

and

$$\eta = k_d + ik_c,$$

the parameter α accounts for phase-amplitude coupling, the self and cross saturation coefficients are given by s and c respectively; the parameters k_d and k_c represent the dissipative and conservative components of the backscattering, respectively and 2Δ is the rotation induced detuning. The carrier density N obeys the Bloch equation for semiconductor lasers, where μ is the dimensionless pump ($\mu = 1$ at laser threshold). In the set (1) the dimensionless time is rescaled by the photon lifetime τ_p. The parameter γ is the ratio of the carrier lifetime

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τ_s over τ_p.

Consistently with the standard theory [15], the emission frequency of the two modes (referred to a common optical carrier set to zero) is shifted by the inertial rotation of an amount equal to 2Δ, when the rotation vector is orthogonal to the cavity plane,

$$\Delta = \frac{2\pi R \tau_p}{\lambda} \Omega_{rot}\,, \tag{2}$$

where R is the ring radius, λ is the laser wavelength, and Ω_{rot} is the rotation angular velocity. We seek for a solution of the following form for the set (1)

$$E_{\pm}(t) = Q_{\pm} e^{i(\omega t \pm \phi)},\ N(t) = \bar{N}. \tag{3}$$

By substitution of (3) in the set (1) setting $\Delta = 0$ we find [14] symmetric steady state solutions for the carrier density and the amplitudes of the fields,

$$\bar{N} = \frac{\mu}{1 + 2Q^2 - 2(c+s)Q^4}, \tag{4}$$

$$Q^2 = \frac{\bar{N} - 1 + k_d}{(c+s)\bar{N}}. \tag{5}$$

There are two possible cases, the *in phase* case:

$$\phi = 0 \rightarrow \omega_{in} = \alpha k_d - k_c, \tag{6}$$

and the *out of phase* case:

$$\phi = \frac{\pi}{2} \rightarrow \omega_{out} = -\alpha k_d + k_c. \tag{7}$$

Depending of the sign of the backscattering parameters one of the solutions is stable and the other unstable, if $k_d > 0$ the *out of phase* case is stable, and the *in phase* case is stable for $k_d < 0$. The set (1) presents other solutions (i.e. alternate oscillations, bistability), in this work we focus on the effect of a Sagnac detuning on the symmetric solution. From now on we focus on the *out of phase* case without loosing generality. If Δ is small, the stationary solution can be written as

$$Q_{\pm} = Q \pm \delta, \tag{8}$$

$$\psi = 2\phi + \theta, \tag{9}$$

where δ and θ are perturbative deviations of the stationary solution (4)-(7). By substituting eq. (8) and (9) with (3) in the set (1), at first order in δ and θ, separating the real and the imaginary part we obtain

$$\left\{ \begin{array}{l} \Delta Q + k_d \theta Q + 2k_c \delta - \alpha Q^2 \delta \bar{N}(s-c) = 0 \\ 2k_d \delta - k_c \theta Q - \bar{N} Q^2 \delta (s-c) = 0 \end{array} \right. . \tag{10}$$

We define χ as the difference between the fields intensities divided by the total intensity,

$$\chi = \frac{|E_-|^2 - |E_+|^2}{|E_+|^2 + |E_-|^2} = \frac{2}{Q}\delta = \mathcal{R}\Omega_{rot} \tag{11}$$

where $\mathcal{R}$ is the Responsivity of the system to the dimensional inertial rotation, solving (10) and using (11), we find the analytical expression for the Responsivity

$$\mathcal{R} = \frac{4\pi k_c R \tau_p}{\lambda(2k_d^2 + 2k_c^2 - (k_d + \alpha k_c) Q^2 \bar{N}(s-c))}. \tag{12}$$

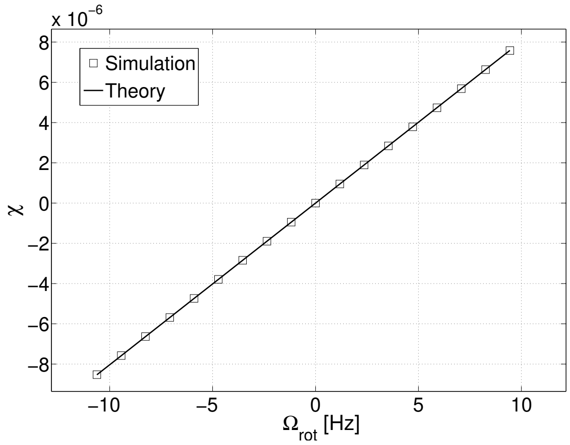

Fig. 1. System response χ versus inertial rotation Ω_{rot}. The slope is the Responsivity $\mathcal{R}$. The ring radius is $R = 600\ \mu m$ and the pump is $\mu = 1.2$, $\alpha = 3.5$, $s = 0.005$, $c = 0.01$, $k_d = -3.27\ 10^{-4}$, $k_c = 4.4\ 10^{-3}$ and $\tau_p = 1\ ps$.

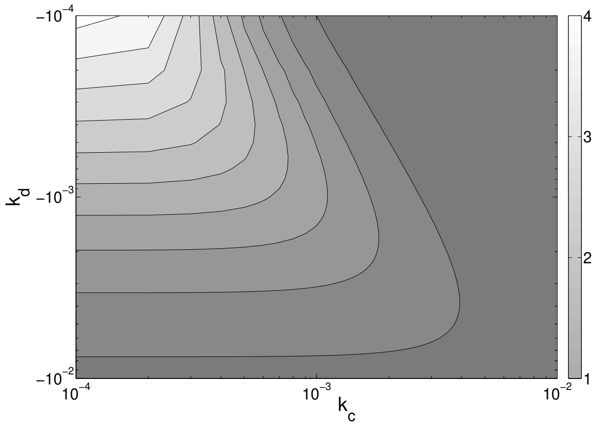

Fig. 2. Responsivity contour plot (x $10^{-5}\ s$) versus backscattering coefficients, $\alpha = 3.5$, $s = 0.005$, $c = 0.01$, $\tau_p = 1\ ps$, $R = 600\ \mu m$ and $\mu = 1.2$.

The Responsivity $\mathcal{R}$ quantifies how the rotation unbalances the counterpropagating fields intensities. In figure 1, χ is plotted versus the inertial rotation Ω_{rot} using the analytical expression (11), and compared to numerical simulations.

Fig. 2 shows how Responsivity behaves versus backscattering coefficients. Fig. 3 shows Responsivity versus backscattering dissipative coefficient k_d and pump current μ. As a general trend, the Responsivity decreases for increasing values for the dissipative backscattering coefficient and decreasing values for the conservative backscattering coefficient. Physically, the dissipative backscattering stabilizes the locking by damping perturbations of the phase difference between the two modes. On the other side, conservative backscattering enhances the relative phase dynamics, thus making the system more sensitive to sources of dephasing like the Sagnac effect. To characterize the response of the device to a rotation variation, we simulate the equations set (1) with a time dependent rotation. The Results are shown in Fig. 4, the response time of the device is a few ns for our parameters choice.

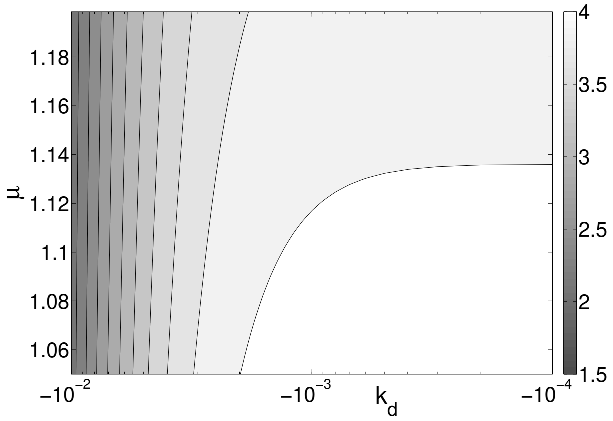

Fig. 3. Responsivity contour plot (x 10^{-6} s) versus backscattering coefficient k_d and pump current μ, $\alpha = 3.5$, $s = 0.005$, $c = 0.01$, $\tau_p = 1$ ps, $R = 600$ μm and $k_c = 10^{-2}$.

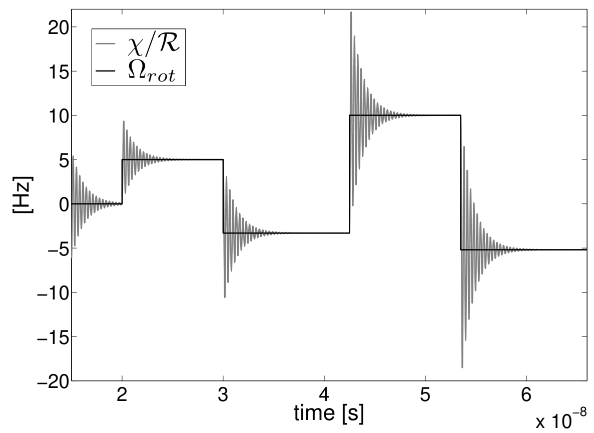

Fig. 4. Numerical simulations of the response of the SRL to a rotation depending on time, $R = 600$ μm, $\mu = 1.2$, $\alpha = 3.5$, $s = 0.005$, $c = 0.01$, $k_d = -3.27$ 10^{-4}, $k_c = 4.4$ 10^{-3} and $\tau_p = 1$ ps.

III. Conclusion

In conclusion, we have introduced a theory for a new technique to measure inertial rotation using a semiconductor ring laser. We have derived an analytical expression for the Responsivity of a possible rotation sensor using a two-mode rate equation model. We have investigated the effect of the backscattering coefficients and the pump current on the Responsivity, and the dynamic response of the device when a time dependent rotation rate is applied. Our conclusion is that the proposal of using a semiconductor ring laser as a rotation sensor is viable, as it is not necessary limited by locking effects. A good agreement between theoretical and numerical results is found. Noise characteristics are also important to characterize the possible implementation of this technique in real gyroscopes, and will be the object of future investigations.

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