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SEMICONDUCTORS AND DIELECTRICS

Dynamics of a Nonlinear Ring Cavity: Excitability and Coherent Resonance

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Abstract—The phenomena of excitability and coherent resonance in a nonlinear ring cavity in the presence of excitons and biexcitons are considered. A bifurcation analysis of the dynamics of the nonlinear ring cavity indicates that both self-pulsation and excitability can exist in the system. It is demonstrated that coherent resonance can be observed in an excitable exciton–biexciton system in the ring cavity. The optimum conditions for the manifestation of these phenomena are investigated, and their possible applications are discussed.

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1. INTRODUCTION

The phenomena of excitability and coherent resonance are associated with new, rapidly developing trends in optics. Originally, these phenomena received much attention in biology [1–3] and chemistry [4]. In terms of biology, the phenomenon of excitability can be illustrated by neuron behavior: pulses with amplitudes below a threshold value excite local nonpropagating responses, whereas pulses with amplitudes above a threshold value give rise to an intense response of the system. Recently, it was predicted that excitability can occur in optical systems. Under specific conditions, the phenomenon of excitability can manifest itself in a nonlinear ring cavity [5, 6], lasers with a saturable absorber [7], and semiconductor lasers with a delayed optical feedback [8]. In the case of lasers with dispersive reflectors, this phenomenon has been investigated theoretically and experimentally [9, 10].

The phenomenon of excitability exhibited by optical systems based on semiconductors has attracted particular research attention, because it holds considerable promise for practical applications in optoelectronic devices. Investigation of this phenomenon is based on the analysis of two possible mechanisms [11]. The so-called first-order excitability occurs when a singularity of the saddle type in the corresponding phase space is located in the vicinity of the stable point, whereas the second-order excitability arises from the stability loss associated with the Andronov–Hopf bifurcations.

The inclusion of noise effects in the analysis of nonlinear systems is of special interest. The influence of noise on oscillatory, excitable, and bistable systems can bring about various effects [12]. However, in this paper, we will restrict our consideration to the specific case of coherent resonance in an excitable ring cavity. Coherent resonance arises when there exists a nearly periodic response of the system to noise and this response can be associated with a nearly periodic trajectory in the phase space of the perturbed system. This behavior can be explained in terms of the different time dependences of the activation and return times (the activation time is the time it takes for the given system to be excited from the state corresponding to the stable point, and the return time is the time it takes for the given system to return from the excited state to the stable state). The FitzHugh–Nagumo model [3] is a simplified version of the Hodgkin–Huxley model, which, however, makes it possible to perform analytical treatment. In the framework of the FitzHugh–Nagumo model, it has been demonstrated that coherent oscillations are strongly enhanced at specific amplitudes of external noise. Dubbeldam et al. [7] theoretically demonstrated that coherent resonance can occur in lasers with saturable absorbers. Buldü et al. [13] carried out a theoretical investigation of this phenomenon within the Lang–Kobayashi model for lasers with an optical feedback. Recently, Marino et al. [14] experimentally observed coherent resonance in a diode feedback laser and showed that the noise affects the laser intensity.

The possibility of observing the excitability phenomenon in a system of excitons and biexcitons in a nonlinear ring cavity was predicted in our previous study [6]. In the present paper, we generalize the results obtained in [6] and, for the first time, demonstrate theoretically that coherent resonance can be observed in this system. We derive the equations describing the model system and discuss possible bifurcations in the dynamic behavior of the system. It is shown that coher-

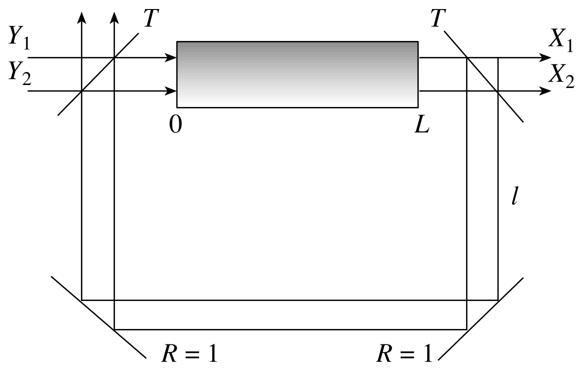

Fig. 1. Schematic drawing of the ring cavity. Designations: Y_1 and Y_2 are the amplitudes of incident fields, X_1 and X_2 are the amplitudes of transmitted fields, T stands for the transmission coefficients of the mirrors at the input and output of the ring cavity ($T = 0.01$), and L is the distance between the mirrors.

ent resonance occurs in an excitable nonlinear ring cavity.

2. DYNAMICS OF EXCITONS AND BIEXCITONS IN A RING CAVITY

We consider a system of excitons and biexcitons in a nonlinear ring cavity. Let a sample of length L be placed between the input and output mirrors of the cavity. These mirrors are characterized by the reflection coefficient T. The other two mirrors of the cavity are assumed to be total-reflection mirrors (Fig. 1). It is assumed that the optical nonlinearity is associated with the creation of biexcitons due to the exciton–photon interaction [15]. For simplicity, the microscopic dynamics will be described in terms of the three-level model and a giant oscillator strength [16] with allowance made for the response of the system to two simultaneous independent optical pulses. Photons of the first pulse Y_1 are in resonance with a quantum transition in the exciton region of the spectrum. Photons of the second pulse Y_2 induce an exciton–biexciton conversion. The dynamics of this system can be described by the following system of dimensionless equations [6]:

$$\frac{dX_1}{d\tau} = \sigma(-X_1 + 2CU + Y_1), \quad (1)$$

$$\frac{dX_2}{d\tau} = \sigma(-X_2 + 2CV + Y_2), \quad (2)$$

$$\frac{dU}{d\tau} = -\gamma U - \gamma(X_1 + X_2 V), \quad (3)$$

$$\frac{dV}{d\tau} = -V + X_2 U. \quad (4)$$

Here, X_j and Y_j are the normalized amplitudes of the fields shown in Fig. 1; U and V are the normalized

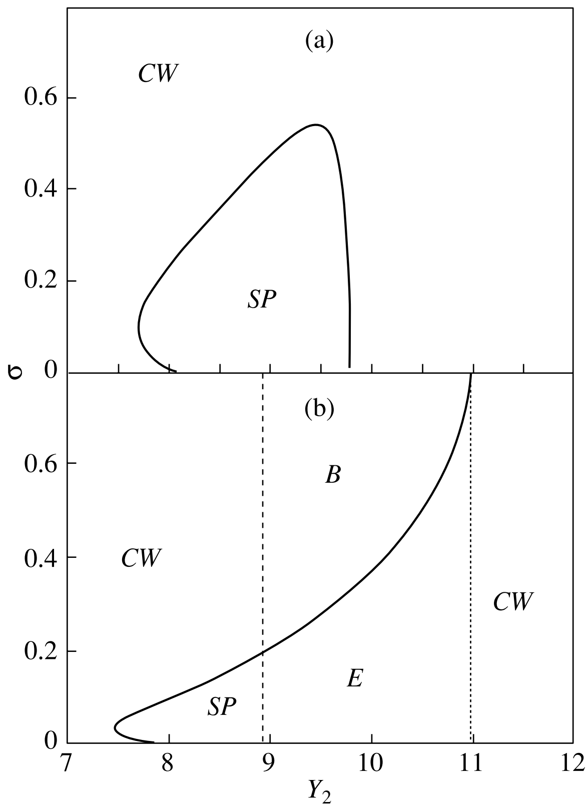

Fig. 2. Bifurcation diagrams for the system of equations (1)–(4) for the field amplitude $Y_1 = 10$, the rate ratio $\gamma = 0.1$, and constants C = (a) 5 (excitability is absent) and (b) 10 (excitability and resonances are possible). Designations: CW is the stable stationary region, B is the bistability region, SP is the self-pulsation region, and E is the excitability region.

amplitudes of excitons and biexcitons, respectively; $\tau = \gamma_{ex} t$ is dimensionless time; σ is the parameter describing the decay of the electric field amplitude in the cavity; $\gamma = \gamma_{ex}/\gamma_{biex}$ (where γ_{ex} and γ_{biex} are the exciton and biexciton decay rates determining the rate of transformation of the corresponding quasiparticles from coherent modes into incoherent modes); and C is the so-called optical hysteresis constant. It should be noted that, in our previous works [17, 18], we investigated the optical bistability, switching, and self-pulsation of excitons and biexcitons in a similar system with due regard for its bifurcations in the absence of external noise. In [17], we studied the stationary and nonstationary bistability and multistability and, moreover, predicted the possibility of spatial turbulence arising in a system of coherent excitons, photons, and biexcitons in different crystals. In [18], we proposed and validated a scenario of the crossover to the dynamical chaos regime.

Figure 2 illustrates different scenarios of the behavior of the system in the σ–Y_2 plane for two values of the constant C. As can be seen from Fig. 2a, the bifurcation diagram at $C = 5$ involves only the regions corresponding to self-pulsations and stable stationary states. The

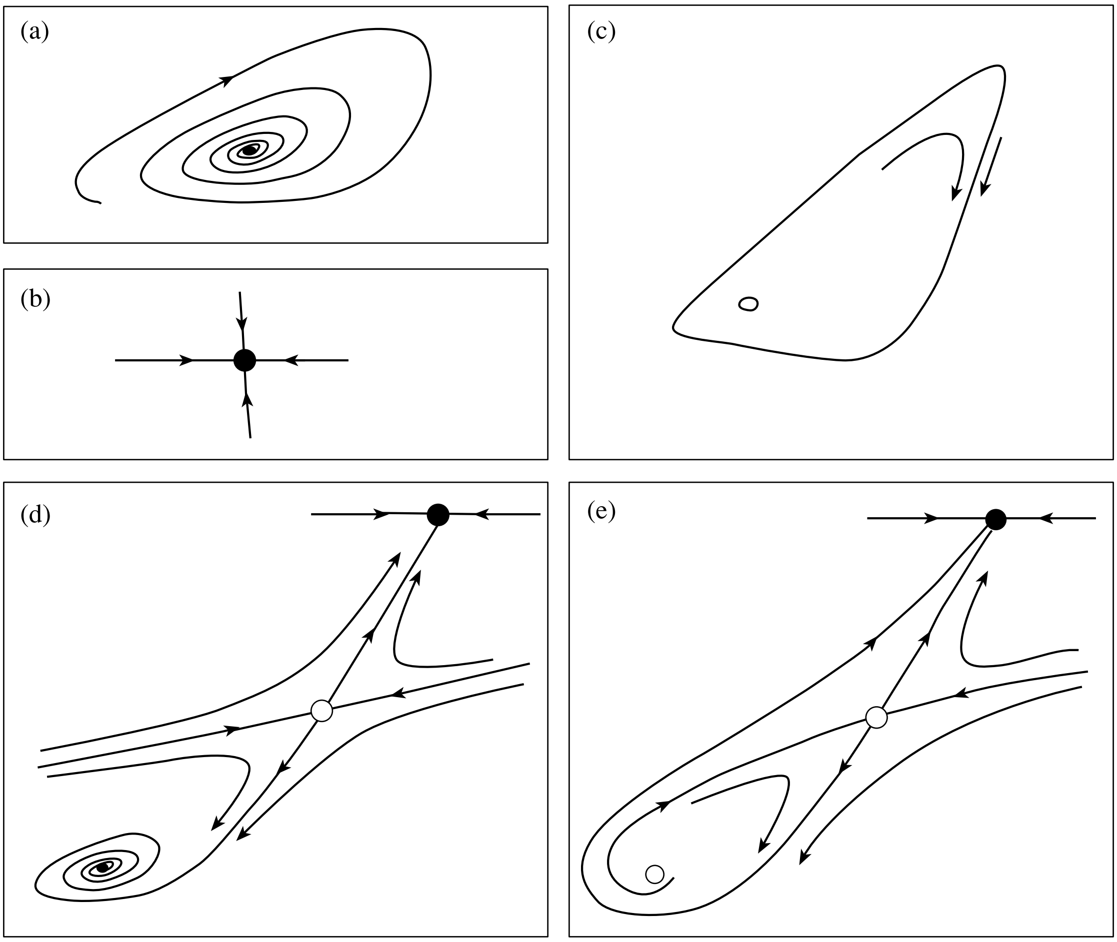

Fig. 3. Phase portraits of the system of equations (1)–(4) for different regions shown in Fig. 2b. Closed and open circles indicate stable stationary and unstable states, respectively. (a, b) Stationary regions for (a) small (stable focus) and (b) large (stable node) normalized amplitudes Y_2. (c) Self-pulsation region *SP* with a limit cycle. (d) Bistability region *B* with a saddle-type singularity, a stable node, and a stable focus. (e) Excitability region *E* with a saddle-type singularity, a stable node, and an unstable focus.

results of the calculations performed for $C = 10$ are presented in Fig. 2b. It can be seen from this figure that, apart from the aforementioned regions, there are regions corresponding to bistability and excitability of the stationary states. An analysis of the stability of the stationary states in the linear approximation demonstrates that, in the excitability region, there exists a saddle-type singularity located in the vicinity of the stable node. As was noted above, this singularity determines the mechanism responsible for excitability of the system. The phase portraits of the aforementioned regions are shown in Fig. 3. It can be seen from Figs. 3a and 3b that the stable stationary region contains a stable focus at small normalized amplitudes Y_2 and a stable node at large normalized amplitudes Y_2. Figure 3c depicts a typical phase portrait in the self-pulsation region, in which trajectories tend toward a limit cycle. In the bistability region, there coexist saddle-type singularities, a stable node, and a stable focus (Fig. 3d). In the excitability region (Fig. 3e), the focus becomes unstable and the phase portrait, as well as the response of the system, exhibits properties characteristic of excitability.

It is also of interest to investigate the influence of noise on the dynamics of excitons and biexcitons in a ring cavity. We performed a computer experiment in which the system of equations (1)–(4) was integrated with the following parameters: $Y_1 = 10$, $\gamma = 0.1$, $\sigma = 0.1$, and $Y_2 = 9.0$. These parameters were chosen so that the operating point was located in the excitability region shown in Fig. 2b. In the absence of noise, the amplitudes of the transmitted fields are constant and, after several relaxation oscillations, the trajectories tend toward the stable states.

Now, we take into account the external noise. In this case, relationship (2) can be rewritten in the following form [19]:

$$\frac{dX_2}{d\tau} = \sigma(-X_2 + 2CUV + Y_2 + Z), \qquad (5)$$

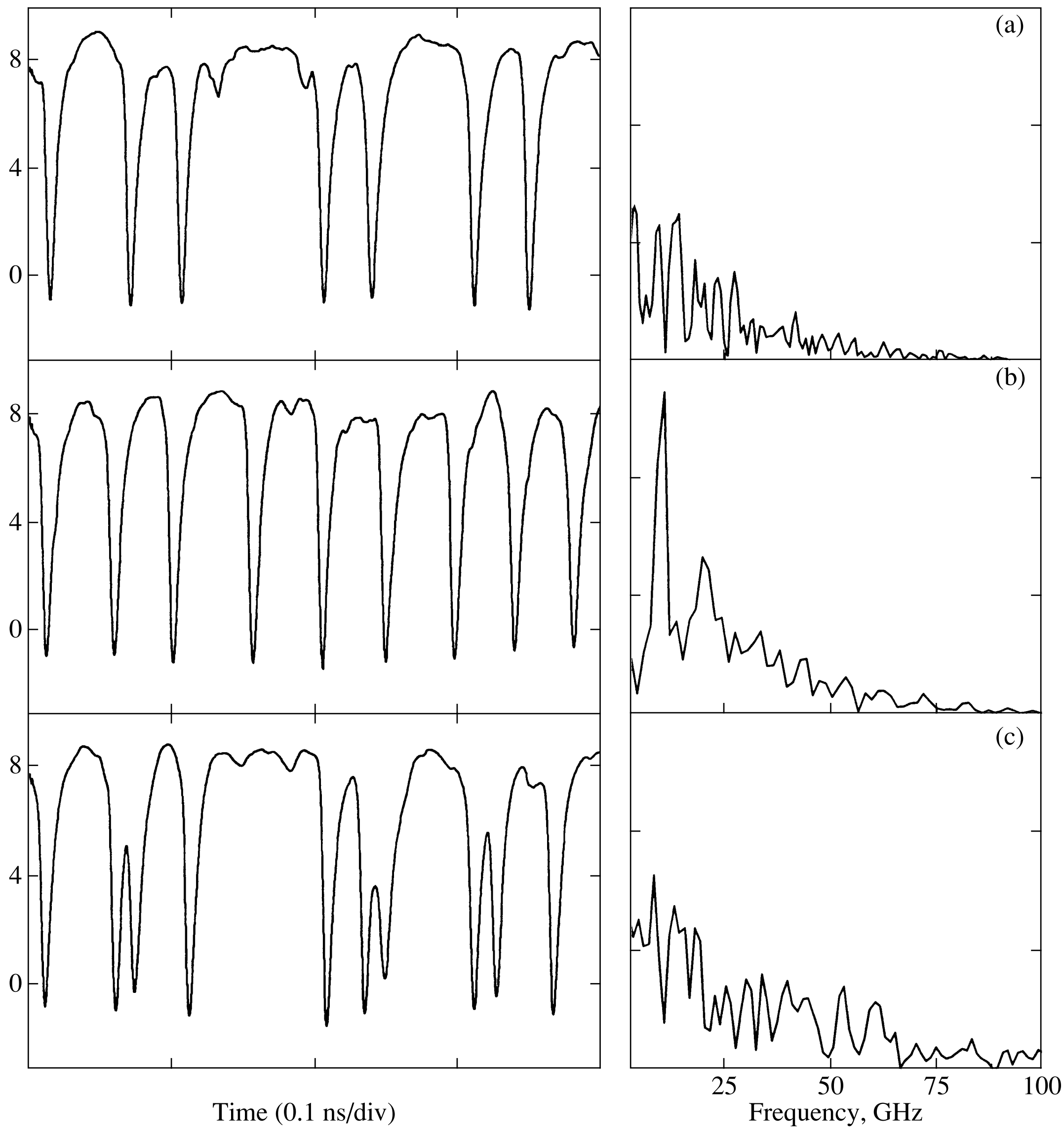

Fig. 4. Time dependences of the amplitude X_1 (at the left) and the corresponding energy spectra (at the right) at different levels of external noise D = (a) 0.7, (b) 0.9, and (c) 2.0. The operating point lies in excitability region E. Conditions: $Y_1 = 10$, $\gamma = 0.1$, $\sigma = 0.1$, and $Y_2 = 9.0$.

$$\frac{dZ}{d\tau} = -\frac{1}{\tau_c}Z + \frac{(2D)^{1/2}}{\tau_c}\xi(\tau), \tag{6}$$

where Z is the external colored noise characterized by the autocorrelation function

$$\langle\xi(\tau)\xi(\tau')\rangle = \frac{D}{\tau_c}e^{-(\tau - t')/\tau_c}$$

with the correlation time τ_c and the Gaussian distribution of white noise $\xi(\tau)$ with a zero mean and a standard deviation equal to unity.

For external noise with a level $D = 0.7$ and correlation time $\tau_c = 0.1$, there arises a random sequence of pulses (similar to that shown in Fig. 4a) at the output of the cavity. An increase in the noise level leads to an increase in the pulse frequency at the output of the cavity, which, in turn, affects not only the pulse duration and pulse separation but also the pulse amplitudes. In particular, the response of the system at the noise level $D = 0.9$ becomes regular. This behavior is characteristic of coherent resonance (Fig. 4b). However, a further increase in the noise level to $D = 2.0$ brings about the appearance of pulses that are irregular in shape at the output of the cavity (Fig. 4c). Therefore, we can make the inference that the response of the system to the noise level is more regular in the vicinity of the critical value $D = 0.9$; i.e., this noise level is optimum for the observation of coherent resonance. This behavior of the system is confirmed by the energy spectra of the field X_1 (shown in the right-hand side of Fig. 4).

Thus, it was demonstrated that coherent resonances can occur in a system of excitons and biexcitons in a nonlinear ring cavity. If this phenomenon is confirmed experimentally, it will be possible to use it as a source of regular pulses in communication systems. These pulses can be generated as a result of the interaction of a ring cavity with a suitable noise source.

3. CONCLUSIONS

In this paper, we showed that an exciton–biexciton system in a nonlinear ring cavity can exhibit coherent resonances. A bifurcation analysis allowed us to determine the range of optimum parameters at which the system becomes excitable. It was established that excitability occurs when singularities of the saddle type arise in the immediate vicinity of the stable node in the corresponding phase space. It was demonstrated that low-level noise can generate a sequence of arbitrary pulses at the output of the excitable system. However, when the noise level reaches a particular value, the response of the excitable system becomes nearly periodic. This phenomenon is characteristic of coherent resonances to some extent. A further increase in the noise level leads to the appearance of an irregular signal at the output of the excitable system. Thus, the results obtained in this study allowed us to draw the conclusion that the excitable system under consideration can find application as a functional optoelectronic element.

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