Synchronization properties of bidirectionally coupled semiconductor lasers under asymmetric operating conditions

Jhon Fredy Martínez Ávila,[1] Raúl Vicente,[2,3] José R. Rios Leite,[1] and Claudio R. Mirasso[4]
[1]*Departamento de Física, Universidade Federal de Pernambuco, 50732-910 Recife PE, Brasil*
[2]*Departament de Física, Universitat de les Illes Balears, E-07122, Palma de Mallorca, Spain*
[3]*Frankfurt Institute for Advanced Studies (FIAS) Max-von-Lave-Str. 1 60438 Frankfurt am Main, Germany*
[4]*Instituto de Fisica Interdisciplinar y Sistemas Complejos, CSIC-UIB, Camput Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain*
(Received 27 October 2006; published 6 June 2007)

We study, both experimentally and numerically, a system of two coupled semiconductor lasers in an asymmetric configuration. A laser subject to optical feedback is bidirectionally coupled to a free running laser. While maintaining the coupling strength, we change the feedback rate and observe a transition from highly correlated low-frequency fluctuations to episodic synchronization between dropouts and jump-ups. Our results resemble those obtained recently in a unidirectionally coupled system [Buldú *et al.*, Phys. Rev. Lett. **96**, 024102 (2006)].

DOI: 10.1103/PhysRevE.75.066202 PACS number(s): 05.45.Xt, 42.55.Px, 42.65.Sf

I. INTRODUCTION

Oscillators that are coupled, either unidirectionally or mutually, often give rise to a collective dynamical behavior known as synchronization [1,2]. Synchronization has been found to play an important role in many complex systems [3–7]. In particular, the use of unidirectionally coupled chaotic oscillators has been proposed to improve privacy and security in information transmission, with its emphasis in the optical domain [8–12]. From a more fundamental point of view, it has been shown that unidirectionally coupled lasers can give rise to the interesting phenomenon of "anticipated synchronization" [13–18], in which the dynamics of a master laser can be predicted by a slave one.

Bidirectionally coupled lasers with long separation distances, on the other hand, have been deeply investigated from the fundamental point of view [19–30] although they have recently found practical applications in chaos-based communication systems [31,32]. Interesting features like spontaneous symmetry breaking and leader-laggard dynamics or long-range zero-lag synchronization have been reported [21,27,30]. Most of these studies consider symmetric operation, in which both lasers operate in the cw or oscillatory regime when uncoupled; asymmetric operation has been only briefly considered up to now.

Semiconductor lasers have proven to be excellent model systems to investigate the behavior of delay-coupled elements. These lasers can be well controlled, and their dynamical behavior can be accurately described by well-known models. In addition, delays in the coupling occur naturally even for short propagation distances, due to the fast dynamical time scales of semiconductor lasers. In this paper, we study, both experimentally and numerically, the dynamics of two mutually delay-coupled semiconductor lasers in an asymmetric configuration, where one of the lasers is subject to optical feedback. We focus on the change of dynamical regimes that can be found when the feedback strength is varied, while maintaining the rest of the parameters fixed. In particular, we concentrate on the transition between a typical low-frequency fluctuation regime with a defined leader-laggard dynamics to a regime of episodic synchronization where the role of the leader and laggard have changed.

The experimental setup is depicted in Fig. 1. The configuration consists of two semiconductor lasers bidirectionally coupled and separated by a distance of about 1 m ($T \approx 3.5$ ns is the flying time). We use two similar single-mode diode lasers (SDL-5401) emitting at 805 nm, with a solitary linewidth of 50 MHz. The laser operation temperatures are stabilized using thermoelectric controllers to a precision of 0.01 K. The intensity of the laser outputs is detected by 1.5 GHz bandwidth photodetectors and monitored using a digital oscilloscope (Tektronix TDS3032B, 2.5 GS/s) with a 300 MHz bandwidth. One of the lasers is subjected to optical feedback from an external mirror located at about 40 cm ($\tau_f \approx 2.8$ ns is the round-trip feedback time), and the feedback strength is controlled using a continuously variable neutral density filter. Thus this laser can exhibit complex dynamics even when uncoupled. The other laser operates in the cw regime when uncoupled. We choose for both lasers an injection current close to threshold (21.9 mA), and consequently the laser with feedback operates in the well-known regime of low-frequency fluctuations (LLFs) [33–37] when uncoupled. Low injection currents allow for better experimental resolution, which is more difficult to achieve when the laser operates at higher currents, as in the well-known coherence collapse regime.

II. EXPERIMENTAL RESULTS

Two semiconductor lasers that operate in the cw regime when uncoupled exhibit LFFs when coupled bidirectionally

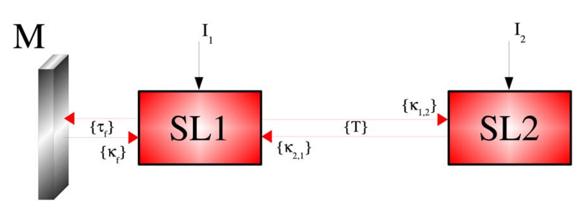

FIG. 1. (Color online) Experimental setup.

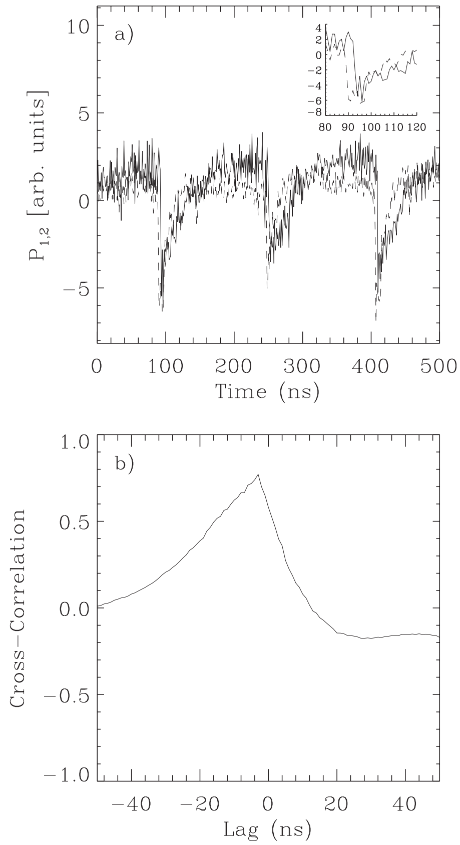

FIG. 2. Experimental time traces of the optical powers for a feedback strength yielding a ~2.7% threshold reduction (a). Solid line represents the optical power of the laser with feedback and dashed line represents the optical power of the laser without feedback. In (b) we plot the cross-correlation function.

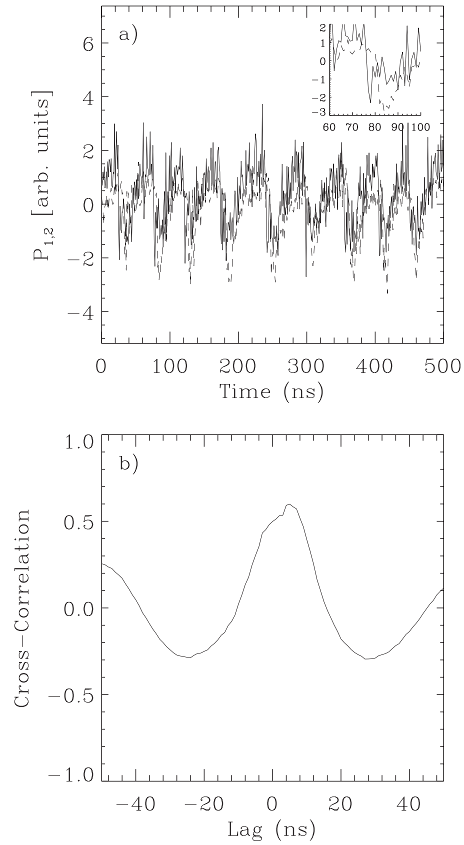

FIG. 3. Experimental time traces of the optical powers for a feedback strength yielding a 2.7% threshold reduction. Traces in (a) were obtained for the case in which the temperature of the laser without feedback was 2.3 °C higher than one subjected to feedback. (b) shows the cross-correlation function.

beyond a certain coupling strength and separation distance [21]. In this case, and when the devices are very similar to each other, the lasers develop the LFFs in a leader-laggard type of dynamics; the roles of the leader and laggard change in time, giving rise to a cross-correlation function with two high peaks located at $\pm T$, T being the time of flight between the lasers [21,27]. In our experiment, we selected two similar semiconductor lasers and slightly adjusted their injection currents and temperatures to compensate for small mismatches and obtain this symmetric regime.

The optical power of a semiconductor laser subject to feedback exhibits LFFs in a coarse-grained time scale [38] and fast subnanosecond pulsations underneath this slow motion when biased close to threshold. LFFs are well resolved in our experiments, but the limitations of our detection scheme do not allow us to follow the fast pulsations. Instead, we take advantage of numerical simulations with picosecond time resolution to study the full dynamics. It will be shown that the qualitative features of the experimentally observed dynamics is well reproduced by a simple model of rate equations. Moreover, numerical simulations allow us to understand and identify the role played by the fast dynamics.

We first study the situation in which one of the lasers is subject to low feedback. In Fig. 2(a) we plot the experimentally measured time series of the evolution of the optical power of the lasers for a low feedback level (2.7% threshold reduction). It can be clearly seen that the lasers develop highly synchronized LFFs in which the laser without feedback drops first and consequently leads in the dynamics. This fact is also reflected in the cross-correlation function depicted in Fig. 2(b) with a high peak located at the time $-T$, indicating that the laser without feedback drops first. The fact that the laser without feedback leads in the dynamics was interpreted as the occurrence of anticipated synchronization in Ref. [15]. In a subsequent paper [16] some of the authors studied theoretically and in more detail the configuration of Fig. 1. They found that an increase of the power of the laser (due to the feedback and a higher pump current) yields an asymmetric coupling and generates a regime in which the external cavity laser leads the dynamics. However, an alternative explanation can be given for this dynamical regime. The laser with feedback operates at a frequency that, although it changes in time, is always lower than the free-running laser frequency. According to Ref. [21] two bidirectionally coupled semiconductor lasers establish a well-defined leader-laggard dynamics when one of the lasers is detuned in frequency with respect to the other; the laser with higher frequency is always the leader. If this interpretation applies to our case, we could say that the laser without feedback becomes the leader due to the frequency reduction of its

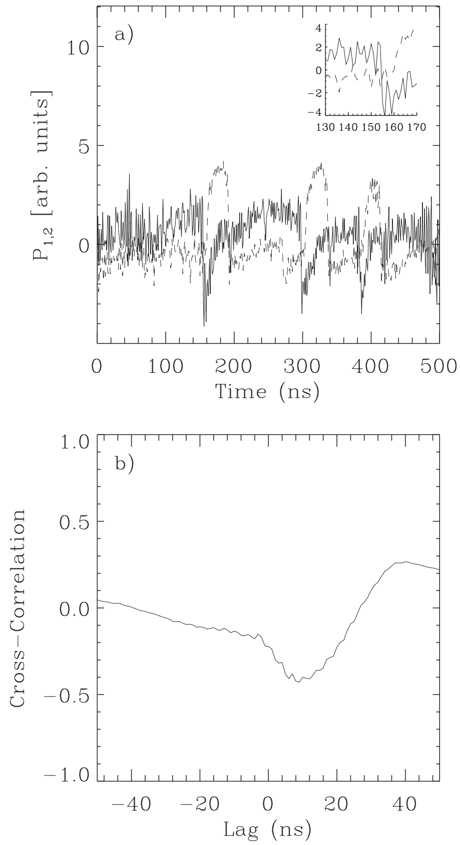

FIG. 4. Same as Fig. 2 but for a stronger feedback, ~13% threshold reduction.

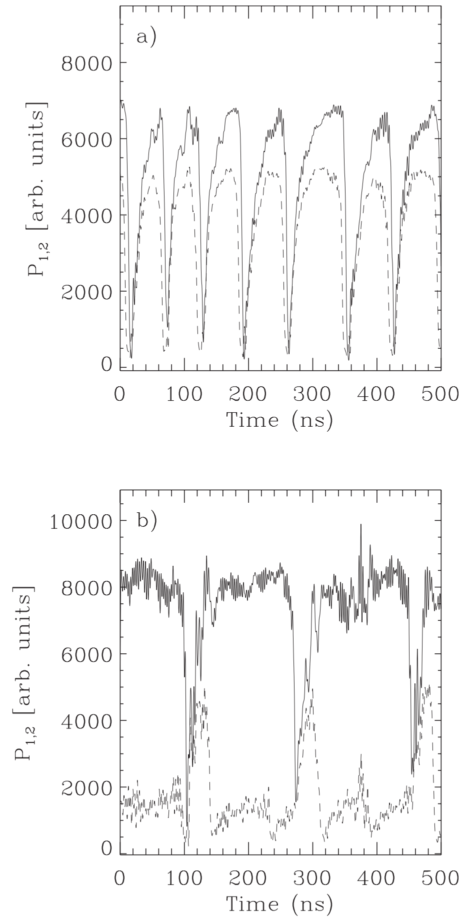

FIG. 5. Time traces of the optical powers, obtained from the numerical simulations, for a weak (a) and a strong feedback strength (b). The traces have been filtered with a fifth-order Butterworth filter of 300 MHz cutoff frequency.

counterpart. With this interpretation, and the fact that the maximum correlation occurs at $|T|$ and not at $|T-\tau_f|$ (see the numerical simulation below), as occurs in the anticipated synchronization case, we might not be in the presence of an anticipated identical synchronization but a generalized synchronization as found in Ref. [21]. To check our hypothesis we have induced a frequency detuning between the two lasers by increasing the temperature of the laser without feedback by about 2.3 °C (approximately −320 GHz detuning, as obtained from the laser data sheet). In Figs. 3(a) and 3(b) we plot two time series for low feedback (~2.7% threshold reduction as for Fig. 2) with detuning [Fig. 3(a)] and its corresponding cross-correlation function [Fig. 3(b)]. As can be clearly seen, the roles of leader and laggard are now interchanged, a fact that would reinforce our hypothesis. However, since the definition of the leader and laggard in a bidirectionally coupled dynamical system is subtle, a strict classification of this dynamics would require a more careful analysis [39].

We now turn our attention to a stronger feedback regime. In Fig. 4(a) we plot time series for the optical power of the two lasers for a feedback level yielding a threshold reduction of ~13%). Dropouts of the laser with feedback seem to be followed by jump-ups of the laser without feedback. The cross-correlation function depicted in Fig. 4(b) confirms this situation; a peak of anticorrelation for the filtered series appears. We highlight some features of this cross-correlation function. First, the peak is located at the positive side of the time axis, which indicates that the jump-ups in the laser without feedback occur after the dropouts in the laser with feedback develop. Second, this result seems to contradict the fact that the laser without feedback should be the leader, as it happens in the weak feedback regime. However, the extra power from the laser subject to optical feedback must be responsible for its leading action in inducing the delayed jump-ups of power in laser 2. This leading effect is manifest, as shown in our experiments and calculations, only at moderate feedback of laser 1. Thus the statement of Ref. [21] about the relative optical frequency condition to assign the leader in a coupled pair of lasers has some range of validity which has been touched by our contribution. It is worth mentioning that a consistent result is given in Ref. [41]. Therein the authors show a unidirectional coupled system where the leading role is independent of the relative optical frequency. Third, the correlation between dropouts and jump-ups is far from attaining a value of −1, although the recovery of the dropouts seems to match the rise of the upward jumps, which would indicate that we are not in the presence of antisynchronization [15,40]. What we probably observe is an episodic synchronization, similar to the one recently reported in Ref. [41] for unidirectionally coupled semiconductor lasers. Since our experimental measurements do not allow us to

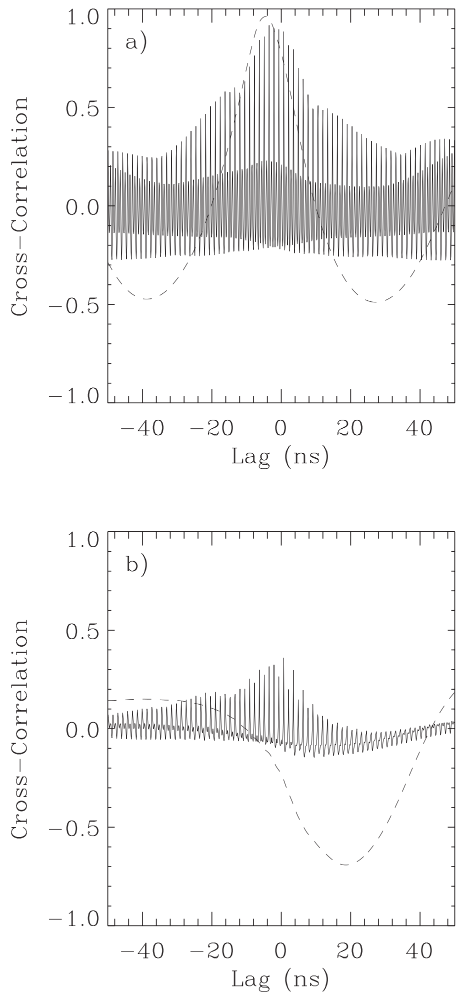

FIG. 6. Cross-correlation function obtained from the numerical simulations for the unfiltered (solid line) and filtered (dashed line) time series of the output powers. (a) is for the weak and (b) is for the strong feedback regime.

resolve the fast dynamics, we take advantage of numerical simulations to gain insight into this fast dynamics that occurs in our system.

III. NUMERICAL RESULTS

We performed numerical simulations using the model developed in Ref. [42] extended to include the optical feedback in laser 1. The model reads

$$\dot{E}_{1,2}(t) = \frac{1}{2}(1+i\alpha)[G_m - \gamma]E_{1,2}(t) + \kappa_{2,1}e^{-i\omega_0 T}E_{2,1}(t-T) + \kappa_f e^{-i\omega_0\tau}E_1(t-\tau), \quad (1)$$

$$\dot{N}_{1,2}(t) = \frac{I_{1,2}}{e} - \gamma_e N_{1,2} - G_m|E_{1,2}|^2, \quad (2)$$

where the indices 1 and 2 refer to lasers 1 and 2, respectively. The internal laser parameters are assumed identical for the two lasers, with linewidth enhancement factor α=3, differential gain g=1.2×10^{-5} ns^{-1}, transparency inversion N_0 =1.25×10^8, saturation coefficient ϵ=5×10^{-7}, photon decay rate γ=496 ns^{-1}, and carrier decay rate γ_e=0.651 ns^{-1}, ω_0 being the free-running frequency of the lasers and e the elementary charge. $\kappa_1=\kappa_2$=20 ns^{-1}, τ_f=2.8 ns, and T=3.5 ns. $\omega_0\tau=\omega_0 T=0 \pmod{2\pi}$. It is well known that this model qualitatively describes the kind of experiment we have performed. To start with, we first check the simple situation of weak feedback. In Fig. 5(a) we plot filtered time series assuming the weak feedback condition (k_f=15 ns^{-1}) to compare with Fig. 2(a). A qualitative agreement can be seen between the two figures. In Fig. 5(b) we plot the time series for the optical power of both lasers for stronger feedback coupling (k_f=35 ns^{-1}). Dropouts developed by the laser subject to feedback are followed by jump-ups of the laser without feedback. When checking the cross-correlation functions (Fig. 6) it can be seen that for the weak feedback case [Fig. 6(a)] the laser without feedback drops first (the maximum of the cross correlation is located at a negative time of approximately −4 ns), while in the strong feedback case there is a minimum of the cross-correlation function located at positive times, indicating that the laser with feedback now drops first. In Fig. 6(a) we also plot the cross-correlation function without filtering and with a time resolution of 5 ps. While the

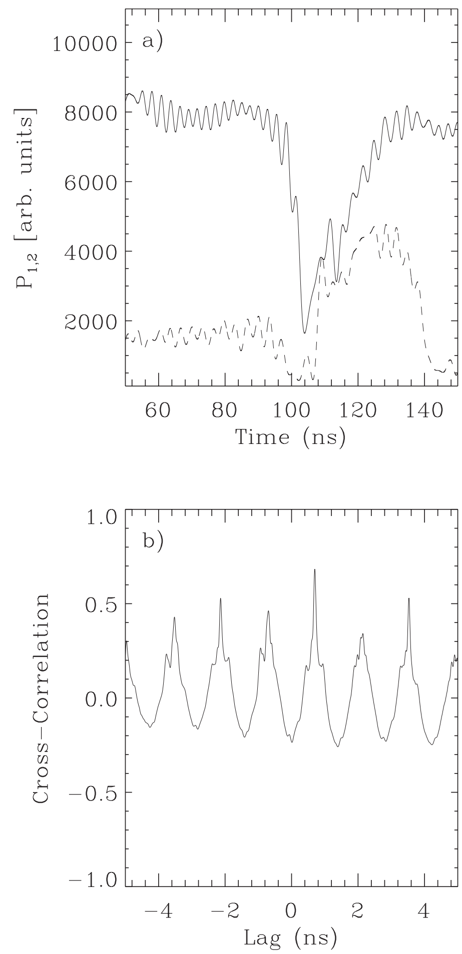

FIG. 7. (a) Filtered time series for the two output powers in a 100 ns time window. (b) Cross correlation between the unfiltered time series taken in the time interval 100–140 ns.

several peaks correspond to the fast dynamics (filtered in the other case) we have identified that the maximum is located exactly at 3.5 ns, which corresponds to the coupling time T, independently of the feedback time. When looking at the unfiltered cross-correlation function for the case of strong feedback we do not observe any indication of anticorrelation but a weak correlation between the two time series.

To check if we are in the presence of episodic synchronization we computed the cross-correlation function at different time intervals during one LFF cycle. In Fig. 7(a) we show a 100 ns time trace for the strong feedback case. When computing the cross correlation function in the time interval $[100\text{ ns},140\text{ ns}]$ we observe a high correlation between the series, which gives evidence that an episodic synchronization takes place. The position of the maximum reveals that the maximum correlation occurs at 0.7 ns, exactly the difference between T and τ_f. This result can be attributed to the fact that the laser with strong feedback has considerably more power than its counterpart and dominates in the dynamics. Under this situation an almost unidirectional link between the two lasers is established, and the appearance of such episodic synchronization can be explained following the arguments given in Ref. [41].

IV. SUMMARY AND CONCLUSION

To summarize, we have performed experimental and numerical analysis of an asymmetric system composed of two bidirectionally coupled semiconductor lasers, one subject to optical feedback and the other one not. When varying the feedback strength we find a transition from synchronized low-frequency fluctuations to a regime of low-frequency fluctuations in the laser with feedback, and jump-ups in the laser without feedback. In the latter case, although the correlation between the slow dynamics reveals a minimum at a positive time lag, an analysis of the fast oscillations reveals that the dropouts and jump-ups are well correlated. We have also found another plausible explanation for the well-established leader-laggard dynamics observed under weak feedback conditions. Our interpretation suggests that the laser without feedback becomes the leader because the frequency of the counterpart laser is reduced due to the feedback. This hypothesis is confirmed experimentally by reducing the frequency of the laser without feedback by increasing its temperature. Our explanation is alternative to the one given in Refs. [15,16].

ACKNOWLEDGMENTS

This work is supported by the MCyT Spain and Feder, Projects No. TEC–2006–1009/MIC and No. FIS2004-00953, the EC Project PICASSO No. IST-2005-34551, and by Brazilian Agencies MCT-FINEP, MCT-CNPQ, MEC-CAPES, and FACEPE, project Pronex INCA.

[1] S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares, and C. S. Zhou, Phys. Rep. **366**, 1 (2002).
[2] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University Press, Cambridge, U.K., 2003).
[3] K. Coffman, W. D. McCormick, and H. L. Swinney, Phys. Rev. Lett. **56**, 999 (1986).
[4] S. H. Strogatz and I. Stewart, Sci. Am. **269**(12),68 (1993).
[5] C. Schäfer, M. G. Rosenblum, J. Kurths, and H. H. Abel, Nature (London) **392**, 239 (1998).
[6] U. Ernst, K. Pawelzik, and T. Geisel, Phys. Rev. Lett. **74**, 1570 (1995).
[7] R. Roy and K. S. Thornburg, Jr., Phys. Rev. Lett. **72**, 2009 (1994).
[8] Y. Liu, P. C. de Oliveira, M. B. Danailov, and J. R. Rios Leite, Phys. Rev. A **50**, 3464 (1994).
[9] G. D. VanWiggeren and R. Roy, Science **279**, 1198 (1998).
[10] *Optical Chaos and Applications to Cryptography*, edited by S. Donati and C. Mirasso, IEEE J. Quantum Electron. **38**, 1138 (2002).
[11] *Criptography using Optical Chaos*, edited by L. Larger and J.-P. Goedgebuer, C. R. Phys. **5**, 609 (2004).
[12] A. Argyris *et al.*, Nature (London) **438**, 343 (2005).
[13] V. Ahlers, U. Parlitz, and W. Lauterborn, Phys. Rev. E **58**, 7208 (1998).
[14] C. Masoller, Phys. Rev. Lett. **86**, 2782 (2001).
[15] S. Sivaprakasam, E. M. Shahverdiev, P. S. Spencer, and K. A. Shore, Phys. Rev. Lett. **87**, 154101 (2001).
[16] P. Rees, P. S. Spencer, I. Pierce, S. Sivaprakasam, and K. A. Shore, Phys. Rev. A **68**, 033818 (2003).
[17] Y. Liu, Y. Takiguchi, P. Davis, T. Aida, S. Saito, and J. M. Liu, Appl. Phys. Lett. **80**, 4306 (2002).
[18] K. Kusumoto and J. Ohtsubo, IEEE J. Quantum Electron. **39**, 1531 (2004).
[19] Y. Liu and J. R. Leite, Phys. Lett. A **191**, 134 (1994).
[20] A. Hohl, A. Gavrielides, T. Erneux, and V. Kovanis, Phys. Rev. Lett. **78**, 4745 (1997).
[21] T. Heil, I. Fischer, W. Elsässer, J. Mulet, and C. R. Mirasso, Phys. Rev. Lett. **86**, 795 (2001).
[22] C. R. Mirasso, M. Kolesik, M. Matus, J. K. White, and J. V. Moloney, Phys. Rev. A **65**, 013805 (2001).
[23] J. M. Buldu *et al.*, Appl. Phys. Lett. **81**, 5105 (2002).
[24] J. Javaloyes, P. Mandel, and D. Pieroux, Phys. Rev. E **67**, 036201 (2003).
[25] S. Tang *et al.*, IEEE J. Sel. Top. Quantum Electron. **10**, 936 (2004).
[26] R. Vicente, S. Tang, J. Mulet, C. R. Mirasso, and J.-M. Liu, Phys. Rev. E **70**, 046216 (2004).
[27] J. Mulet, C. Mirasso, T. Heil, and I. Fischer, J. Opt. B: Quantum Semiclassical Opt. **6**, 97 (2004).
[28] E. Klein, N. Gross, M. Rosembluh, W. Keinzel, L. Khaykovich, and I. Kanter, Phys. Rev. E **73**, 066214 (2006).
[29] R. Vicente, S. Tang, J. Mulet, C. R. Mirasso, and J.-M. Liu, Phys. Rev. E **73**, 047201 (2006).
[30] I. Fischer, R. Vicente, J. M. Buldú, M. Peil, C. R. Mirasso, M. C. Torrent, and J. García-Ojalvo, Phys. Rev. Lett. **97**, 123902

(2006).
[31] E. Klein, R. Mislovaty, I. Kanter, and W. Kinzel Phys. Rev. E **72**, 016214 (2005).
[32] R. Vicente, I. Fischer and C. Mirasso, Opt. Lett. **32**, 403 (2007).
[33] Ch. Risch and C. Voumard, J. Appl. Phys. **48**, 2083 (1977).
[34] I. Fischer, G. H. M. van Tartwijk, A. M. Levine, W. Elsasser, E. Göbel, and D. Lenstra, Phys. Rev. Lett. **76**, 220 (1996).
[35] D. W. Sukow, J. R. Gardner, and D. J. Gauthier, Phys. Rev. A **56**, R3370 (1997).
[36] J. Mulet and C. R. Mirasso, Phys. Rev. E **59**, 5400 (1999).
[37] T. Heil, I. Fischer, W. Elsässer, J. Mulet, and C. Mirasso, Opt. Lett. **24**, 1275 (1999).
[38] J. F. M. Avila, H. L. D. de S. Cavalcante, and J. R. Rios Leite, Opt. Commun. **271**, 487 (2007).
[39] Z. Zheng, X. Wang, and M. C. Cross, Phys. Rev. E **65**, 056211 (2002).
[40] I. Wedekind and U. Parlitz, Phys. Rev. E **66**, 026218 (2002).
[41] J. M. Buldú, T. Heil, I. Fischer, M. C. Torrent and J. García-Ojalvo, Phys. Rev. Lett. **96**, 024102 (2006).
[42] J. Mulet, C. Masoller and C. R. Mirasso, Phys. Rev. A **65**, 063815 (2002).