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Experimental study of stochastic resonance in a Chua's circuit operating in a chaotic regime

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Abstract

We present results of an experimental study of stochastic resonance in an electronic Chua's circuit whose dynamics switches between two different stable chaotic attractors when it is driven by a periodic signal and a Gaussian white noise. Due to the internal dynamics of the attractors the minimum amplitude for the external forcing to induce jumps strongly depends on the external frequency. We determine from the Fourier transform of the output signal the amplification factor of the input signal and study its dependence on the external frequency and the noise intensity. We show that the envelope of the distribution of switching times follows a gamma distribution, typical from bistable systems, and that the mean switching time decays exponentially with the noise intensity. We propose a simple method for obtaining the optimal noise intensity from the residence and switching times probability distributions and show that it coincides with the value obtained from the maximum of the amplification factor.

Keywords: Stochastic resonance; Chaotic systems; Chua circuits

1. Introduction

Noise has traditionally been viewed as detrimental to signal detection and information propagation. However, it has also been found that in certain nonlinear systems noise can enhance the detection and transmission of weak signals. The phenomenon can occur e.g. in bistable systems forced by sub-threshold signals which are too weak to cause transitions between the stable states. In these systems the addition of noise can force the system to switch between the two states with a switching rate that depends on the noise intensity. While for low noise intensity switchings between the two states are very rare, for large enough noise intensity random switchings between the two states occur. There is instead an optimal amount of noise that conveys the maximal information about the forcing signal. This phenomenon is very similar to the resonance in deterministic dynamical systems and, since it is induced by the noise, it is called stochastic resonance. The study of stochastic resonance in bistable systems has also been extended to excitable systems having a single rest state and to threshold detectors (a pulse occurs whenever the sum of the signal and the noise at the input crosses a threshold). Since the conditions for the appearance of stochastic resonance do not depend on very specific model details, it has been observed in many different fields such as paleoclimatology, lasers, neurophysiology, electronic detectors, etc. and studies of this phenomenon cross disciplinary boundaries. Various reviews [1–4] address the recent and extensive work on this fascinating subject.

Stochastic resonance has also been studied in chaotic systems whose dynamical trajectories have different preferred regions in phase space called chaotic attractors. The chaotic system can thus be considered as a generalized bistable system although, contrary to the case of classical bistable systems, stochastic resonance can also be observed in the

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absence of external noise, because chaos can act as an internal noise source. This deterministic stochastic resonance has been observed, e.g., in the Lorenz model [5] and in the Rössler oscillator [6]. Stochastic resonance in systems generating their own internal noise is important in situations where the external noise is either absent or cannot be controlled. Anishchenko and collaborators [7,8] have reported numerical observations of stochastic resonance in an electronic Chua's circuit driven by a sinusoidal signal when two chaotic attractors are present (double-scroll attractor). These authors showed that the phenomenon can be controlled by varying either the external noise intensity or some system parameters which corresponds to a variation of the internal noise intensity. More interesting is the situation in which a chaotic system can jump between one attractor (single-scroll attractor) and its mirror image. The experimental observation of stochastic resonance subject to a high frequency periodic signal was partially reported in Refs. [9,10]. The stochastic resonance resulting from the interaction of two chaotic attractors and induced by external noise or by the variation of control parameters was also reported for a discrete one-dimensional cubic map and the Lorenz model [11]. The stochastic resonance generated only by the variation of the internal noise in the absence of external noise was demonstrated numerically for the periodically forced Duffing oscillator and a one-dimensional intermittent map [12].

The aim of this paper is to present a thorough study of the characteristic signatures of stochastic resonance following from its experimental observation in an electronic Chua's circuit operating in a chaotic regime. The circuit is forced by a periodic signal and a Gaussian white noise. Since the system moves between two single-scroll chaotic attractors the minimum amplitude of the external forcing to induce jumps turns out to be strongly dependent on the frequency a fact that is absent in the case of bistable system with fixed point attractors. Once this new situation is identified a complete analysis of the system is performed. For the external signal we consider a wide range of frequencies up to approximately twice the mean characteristic frequency of the chaotic circuit. In Section 2 we describe the circuit, its chaotic dynamical behavior, the mechanism of the observed stochastic resonance and the method of processing experimental data. In Section 3 we present the dependence of the amplification factor on the frequency of the input signal and the noise intensity, for both sub-threshold and supra-threshold input signals. In Section 4 we describe the characteristic features of the probability distributions of the residence and switching times and we propose a simple method, based on bistable systems, that allows us to obtain the optimal noise intensity from the residence and switching times probability distributions and show that it coincides with the value obtained from the maximum of the amplification factor. Finally, the main results are listed in the conclusions presented in Section 5.

2. Experimental procedure

The Chua's electronic circuit consists of one resistor, two capacitors, one inductor and one nonlinear element called Chua's diode. This is one of the simplest circuits that exhibits a large variety of bifurcations and chaotic phenomena and it can be constructed using standard electronic components at very low cost [13]. The equations describing the Chua's circuit (shown in Fig. 1) are:

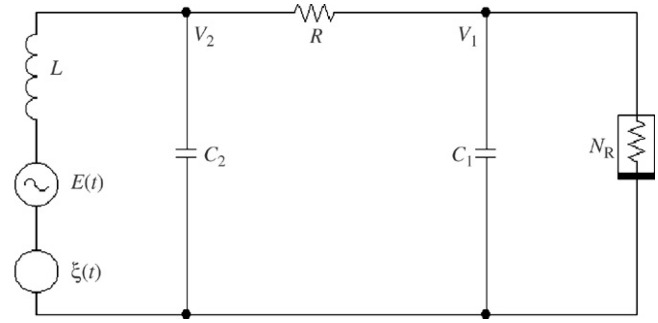

Fig. 1. Diagram of the electronic Chua's circuit with periodic signal voltage source $E(t)$, Gaussian white noise source $\xi(t)$, the resistor $R = 1682\ \Omega$, the inductor $L = 18$ mH and the capacitors $C_1 = 10$ nF and $C_2 = 100$ nF. The nonlinear Chua's diode N_R has a piecewise-linear current–voltage characteristic $I = f(V) = m_1 V + 0.5(m_0 - m_1)(|V + V_0| - |V - V_0|)$ with parameters $m_0 = -0.758$ mA/V, $m_1 = -0.409$ mA/V and $V_0 = 1.08$ V. V_1 is the voltage on the capacitor C_1.

$$
\begin{aligned}
L\,\frac{\mathrm{d}I}{\mathrm{d}t} &= -V_2 + E(t) + \xi(t)\\
C_1\,\frac{\mathrm{d}V_1}{\mathrm{d}t} &= \frac{(V_2 - V_1)}{R} - f(V_1)\\
C_2\,\frac{\mathrm{d}V_2}{\mathrm{d}t} &= I - \frac{(V_2 - V_1)}{R}\\
f(V_1) &= m_1 V_1 + \frac{m_0 - m_1}{2}(|V_1 + V_0| - |V_1 - V_0|)
\end{aligned}
\qquad (1)
$$

where R, C_1, C_2, L, m_0, m_1 and V_0 are constant of the circuit.

To study stochastic resonance we added, in series with the inductor, a sinusoidal signal voltage generator, $E(t) = E_0 \sin(2\pi f_0 t)$ of amplitude E_0 and frequency f_0, and a Gaussian white noise generator (Hewlett-Packard 33120) of zero mean and standard deviation σ. The experimental set-up is shown in Fig. 1 together with the parameters of the electronic components. These are such that, in the absence of sinusoidal and noise signals, the autonomous Chua's circuit exhibits a chaotic dynamical behavior characterized by two "single-scroll" symmetric attractors, having the same structure as Rössler bands [14]. The dynamics selects one of the two single scrolls depending on the (uncontrollable) initial conditions without the possibility to jump to the other attractor. When noise or a sinusoidal signal of high enough amplitude are present, there are numerous jumps between the attractors, as shown in Fig. 2 where we plot the time evolution of the voltage $V_1(t)$ on the capacitor C_1. In our experiments, $V_1(t)$ was digitized (using a digital acquisition board NI-DAQ from National Instruments) and recorded during at least 5 min with a sampling rate of 20 kHz for frequencies $f_0 \le 1.8$ kHz and with a sampling rate of 40 kHz for higher frequencies. The Fourier spectrum of the voltage $V_1(t)$ for the unforced system has a peak at a characteristic frequency $f_{\mathrm{ch}} \approx 2.745$ kHz superimposed with broad spectrum. As usual in other studies, most of our numerical analysis of the data replaces the trajectories $V_1(t)$ by a two-state dynamics,

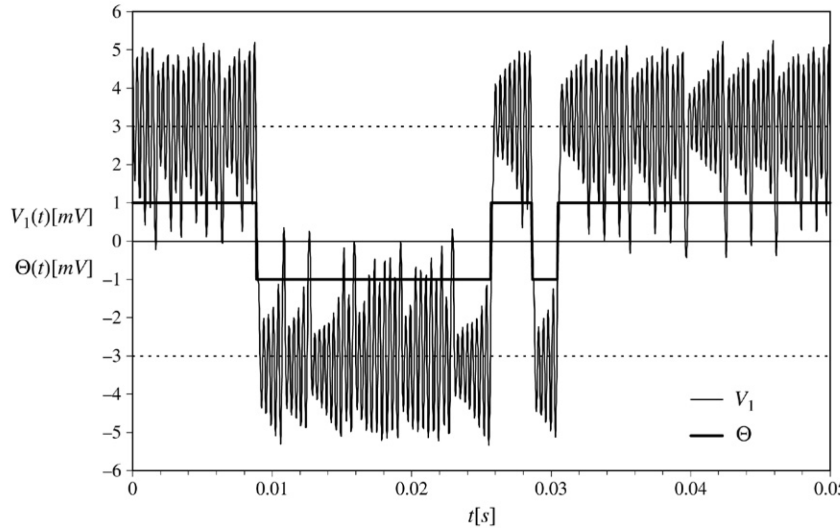

Fig. 2. Temporal evolution of the voltage V_1 on the capacitor C_1 and its representation by the step function $\Theta(t)$. The periodic signal voltage source has an amplitude of 36 mVpp and the frequency is 270 Hz, whereas the noise intensity is 312.5 mV. The dotted lines correspond to crossing levels ± 3 mV.

$\Theta(t)$, in which the detailed motion within each chaotic attractor is neglected: $\Theta(t) = \pm 1$ depending on the selected chaotic attractor. Usually, additional crossing levels, which in our case are $V_1^c = \pm 3$ mV, are used in this procedure [2]. They help to eliminate anomalous switching events, where e.g. $V_1(t)$ crosses the level $V_1 = 0$ but, without reaching V_1^c, switches back and crosses $V_1 = 0$ again. The step function $\Theta(t)$ and the crossing levels are shown in Fig. 2 where several anomalous switching events can be seen. We also calculated $\hat{\Theta}(f)$, the Fourier transform of $\Theta(t)$, using the Welch window and overlapping data segments with 2^{17} data points [15].

3. Amplification of deterministic signals

In the absence of external modulation the Chua system develops a coherence resonance effect when the amplitude of the noise is varied [16]. This effect, however, appears for large noise intensities. Both coherence and stochastic resonance reveal a nearly periodic character of the system. Since they can be present simultaneously in the same system [17] the differences in the dynamics associated with the two effects can be detected by studying, e.g., inter-spike histograms or two times correlation functions [5].

In the absence of external noise, a sinusoidal voltage source $E(t)$ can induce jumps between the two coexisting single-scroll attractors only if it is supra-threshold, i.e. if its amplitude E_0 is above a certain threshold value $E_0^{\max}$. While the usual studies of stochastic resonance consider the influence of noise in sub-threshold signals, there are also experiments in which the system is stimulated by both sub-threshold and supra-threshold periodic signals [18,19]. In these experiments it was demonstrated that the noise increases detectability of sub-threshold signals and decreases that of supra-threshold. In order to check whether this conclusion also holds for the Chua's circuit, we first determine the threshold value $E_0^{\max}$ as a function of input signal frequency f_0. This situation, in which the system moves between two single-scroll chaotic attractors, has important consequences for systems with internal

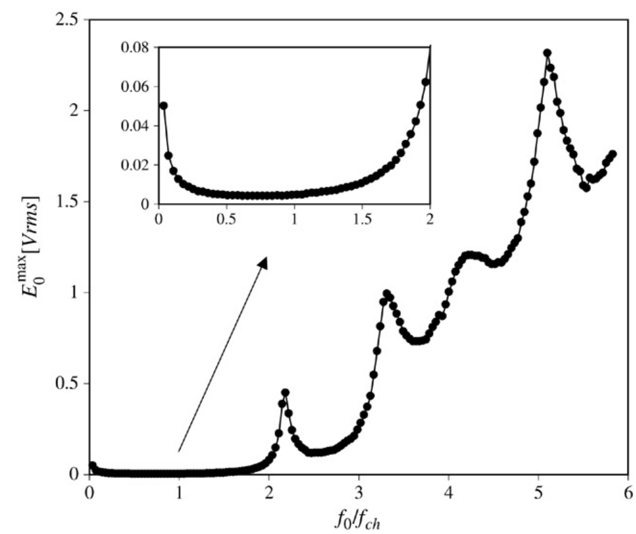

Fig. 3. Dependence of the threshold amplitude $E_0^{\max}$ of the input periodic signal on its frequency f_0. $f_{\mathrm{ch}} = 2.745$ kHz is the characteristic frequency of the autonomous Chua's circuit.

dynamics, as is shown below. The results in Fig. 3 indicate that for low frequencies the threshold is relatively low and quite independent of f_0 but it increases almost linearly, with some peaks superimposed, for frequencies above twice the natural frequency f_{ch}. Standard bistable systems also develop an increase of the forcing amplitude with increasing frequency. Actually, our system has some similarities with bistable systems since it moves between two states although these states are chaotic attractors and not fixed points. We attribute the peaks in Fig. 3 to the dynamics of the Chua's system around each attractor.

We now set the frequency of the voltage source at $f_0 = 1.8$ kHz, i.e. $f_0/f_{\mathrm{ch}} = 0.66$ which, according to Fig. 3, corresponds to the region around the lowest value of $E_0^{\max}$. We study the amplification of the input periodic signal as a function of the noise intensity σ for several amplitudes E_0, above and

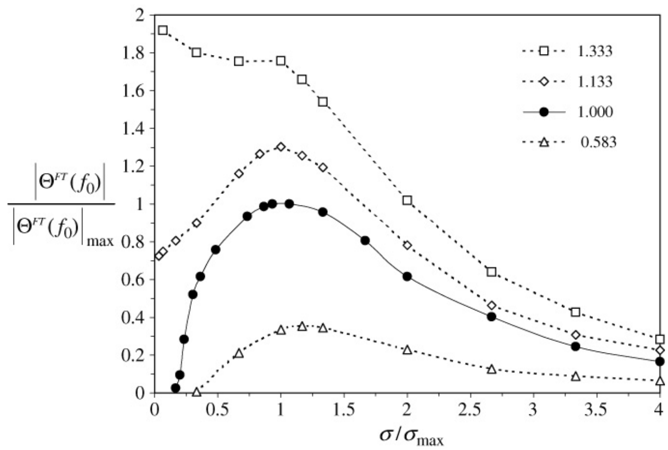

Fig. 4. Dependence of the normalized amplitude η versus the normalized noise intensity σ for different amplitudes E_0 of the input periodic signal and frequency $f_0 = 1.8$ kHz. The normalized values of the amplitude of the input periodic signal $E_0/E_0^{\max}$ are given in the figure.

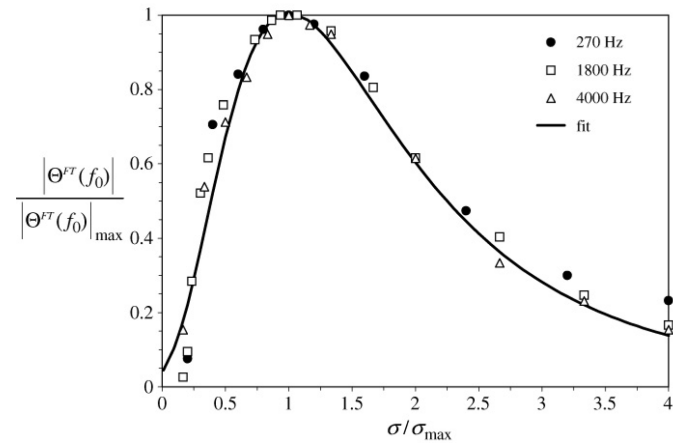

Fig. 5. Dependence of η versus the normalized noise intensity σ for different frequencies f_0 of the input periodic signal. The values of f_0 are given on the figure. The solid curve results from a fit of the function (3) given in the text.

below its threshold value, namely the values $E_0/E_0^{\max} = 1.333, 1.133, 1.000, 0.583$. We chose as a quantifier of this amplification the ratio

$$\eta = \frac{|\hat{\Theta}(f_0)|}{|\hat{\Theta}(f_0)|_{\max}} \tag{2}$$

where $|\hat{\Theta}(f_0)|_{\max}$ is the maximum value that $|\hat{\Theta}(f_0)|$ takes in the case $E_0/E_0^{\max} = 1$ at a noise intensity $\sigma = \sigma_{\max}$ (the value of $\sigma_{\max}$ depends on the external frequency f_0). We also normalize the noise intensity as $\sigma/\sigma_{\max}$. The results in Fig. 4 show that for $E_0 \sim E_0^{\max}$ the value of $|\hat{\Theta}(f_0)|$ presents a clear maximum as a function of the noise intensity σ. This is the usual stochastic resonance effect for sub-threshold and slightly over-threshold forcing, indicating that the maximum amplification appears at a non-null value of the noise intensity. In the same figure one can notice that for input signals whose amplitudes are close to the threshold amplitude $E_0^{\max}$ the stochastic resonance effect is the most pronounced i.e. the noise significantly enhances these input signals as compared to input signals with other amplitudes. In contrast, signals with amplitudes well above the threshold value, e.g. the case $E_0/E_0^{\max} = 1.333$, are instead degraded by the presence of noise. For those signals, $|\hat{\Theta}(f_0)|$ monotonically decreases with increasing noise intensity. We thus obtained that in the chaotic Chua's circuit the noise is a "negative masker" (i.e. enhancing the detectability) for weak sub-threshold or slightly over-threshold signals, and a "positive masker" (i.e. degrading the detectability) for strong supra-threshold signals. The same conclusion followed from the observation of noise-mediated enhancements and decrements in human tactile sensation [19].

We have extended this study to two additional frequencies $f_0 = 0.27$ kHz and $f_0 = 4$ kHz, much lower and higher, respectively, than the characteristic frequency f_{ch} of the autonomous Chua's circuit, and the results are qualitatively similar to those shown in Fig. 4 for $f_0 = 1.8$ kHz. Furthermore, the corresponding stochastic resonance curves at $E_0 = E_0^{\max}$, rescaled as previously explained, follow a master curve independent of the frequency f_0 of the input periodic signal as shown in Fig. 5. This master curve can be fitted by the function

$$\eta = x^4 e^{2(1-x^2)}, \quad x = \frac{2}{\frac{\sigma}{\sigma_{\max}} + 1} \tag{3}$$

which follows from the adiabatic theory [20]. Remarkably, this function that has been shown to fit the data in studies of stochastic resonance describing jumps between two stable attractors [11,12,21,22] also fits very well the data when the systems develops its dynamics between chaotic attractors.

4. The residence and switching times probability distributions

Besides the above discussed amplification factors, alternative characterizations of stochastic resonance use the residence and switching times probability distributions. In this section we analyze in detail both distributions. As we will see from the definitions, both times can be easily obtained from the time sequence $\Theta(t)$ [23]. In all experiments described in this section the amplitude of the input periodic signal is taken just sub-threshold, $E_0^{\max} \gtrsim E_0$.

The residence time T_r is defined as the time the system spends in one state, a single scroll in our case, and it has been often determined for the stochastic resonance observed in chaotic dynamical systems [7,8,11,12]. In Fig. 6 we plot the probability distribution of the residence times $P(T_r/T_0)$ (normalized as T_r/T_0; $T_0 = 1/f_0$) obtained from our experiments in the Chua's circuit. We considered an input periodic signal of frequency $f_0 = 270$ Hz and noise intensity σ equals to the optimum noise level, $\sigma_{\max} = 312.5$ mV for this frequency. As seen clearly from this and similar figures, the residence times probability distribution consists of peaks and a modulating envelope. The peaks, formed by several spikes, are located at odd-integer multiples of $T_0/2$. The spikes result from the presence of a characteristic time $T_{\mathrm{ch}} = 1/f_{\mathrm{ch}}$ equal to the mean duration of one rotation of the dynamical trajectory around the single-scroll chaotic attractor. The modulating

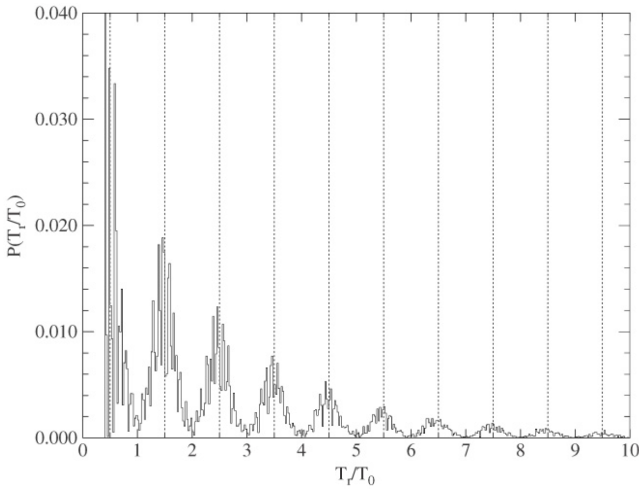

Fig. 6. Probability distribution of the residence times $P(T_r/T_0)$ for the frequency of input periodic signal $f_0 = 270$ Hz and the noise intensity $\sigma = 312.5$ mV. $T_0 = 1/f_0$ is the period of the input signal. The bin length is 0.025.

envelope has a maximum at $T_r/T_0 = 1/2$ and decays exponentially for larger times. This structure of the residence times probability distribution was also obtained analytically for a standard bistable system [24] and in the numerical studies of the stochastic resonance in Chua's circuit [7].

The switching time T_s is the time between two consecutive arrivals to one of the states or, alternatively, the sum of two consecutive residence times in the two different attractors. The probability distribution of switching times is usually determined for the stochastic resonance phenomenon in biological systems. It corresponds e.g. to an inter-spike interval histogram recorded from real periodically forced sensory neurons [25–27]. In Fig. 7 we show the switching times probability distribution $P(T_s/T_0)$ (normalized as T_s/T_0) obtained from our experiments in Chua's circuit for input periodic signal frequency $f_0 = 270$ Hz and for three values of the noise intensity σ. For very low noise intensities (Fig. 7 (upper panel)), the structure of $P(T_s/T_0)$ consists of very narrow peaks located at integer multiples of the period T_0 of the input periodic signal. This indicates that jumps of dynamical trajectory between single scrolls are well synchronized with the input signal. The envelope of the distribution has a maximum at $T_s = T_0$ and decays for large times. The peaks are widely spread, so switchings are very irregular. For near-optimal noise, $\sigma \approx \sigma_{\max}$ (Fig. 7 (middle panel)), the probability distribution has a multi-peaked structure, with peaks formed by spikes, and the envelope of the distribution has a maximum at $T_s = 3T_0$. Finally, the case of intense noise (Fig. 7 (lower panel)) consists of a few broad overlapping peaks reflecting that the noise can induce jumps of dynamical trajectory between single scrolls independently of the magnitude of the input signal. For even larger noise intensities the input signal is swamped and peaks in the switching times probability distribution disappear.

We now present the dependence on the forcing frequency. In Fig. 8 we plot the switching times probability distribution for the frequency of the input signal close (Fig. 8 (upper panel)) to the characteristic frequency f_{ch} and much higher (Fig. 8 (lower panel)) than f_{ch}. One can notice that both the multi-peaked structure of the probability distribution and the maximum in the envelope of the distribution are preserved.

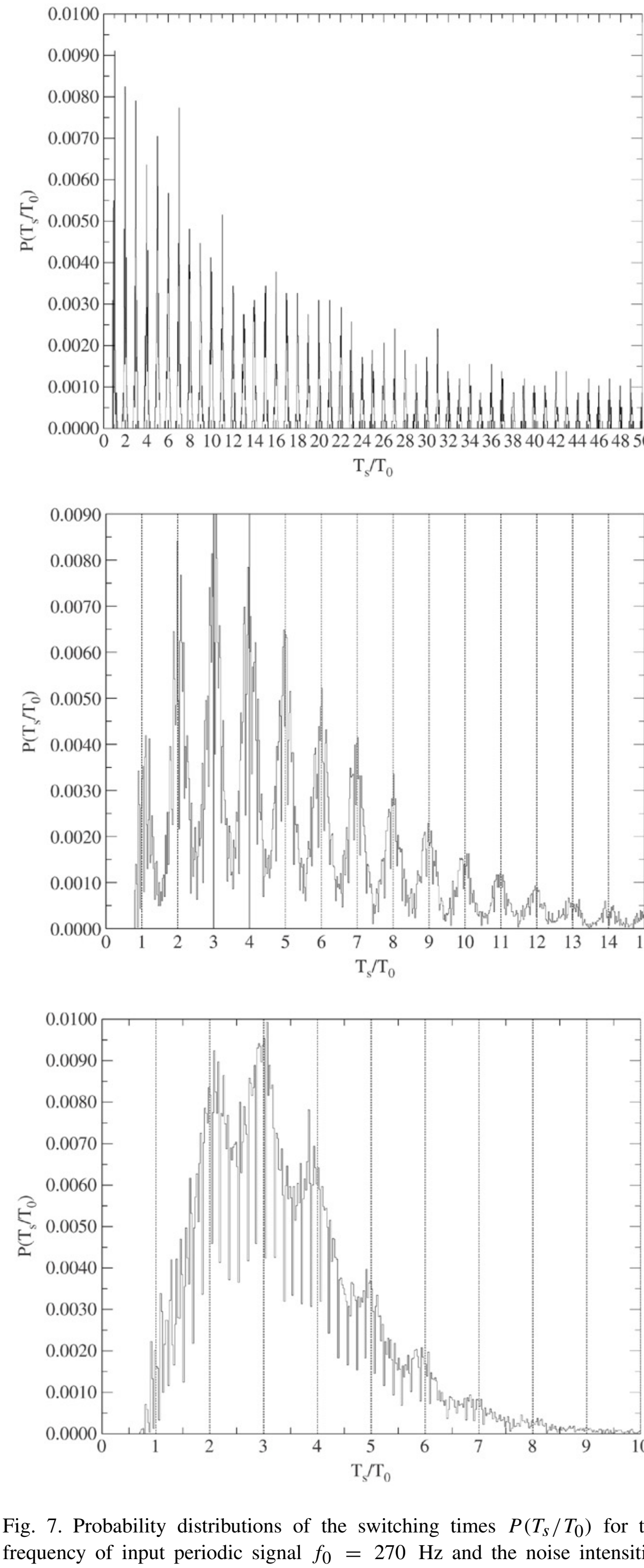

Fig. 7. Probability distributions of the switching times $P(T_s/T_0)$ for the frequency of input periodic signal $f_0 = 270$ Hz and the noise intensities $\sigma = 62.5$ mV (upper panel), $\sigma = 312.5$ mV (middle panel) and $\sigma = 750.0$ mV (lower panel). $T_0 = 1/f_0$ is the period of the input signal. The bin length is 0.025.

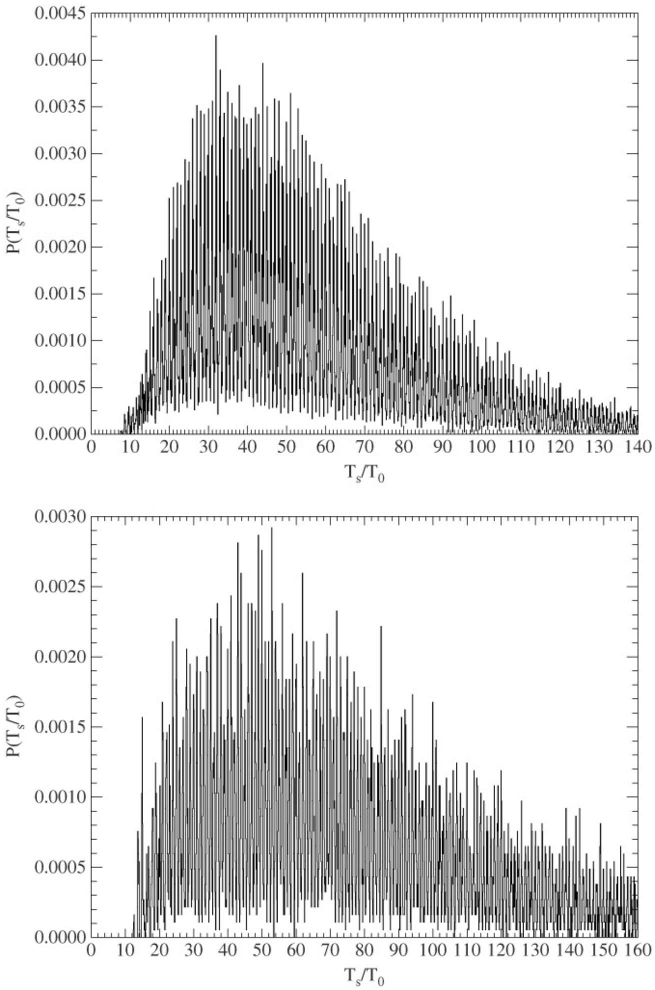

Fig. 8. Probability distributions of the switching times $P(T_s/T_0)$ for the noise intensity $\sigma = 312.5$ mV and the frequency of input periodic signal $f_0 = 2.728$ kHz (upper panel) and $f_0 = 4.0$ kHz (lower panel). $T_0 = 1/f_0$ is the period of the input signal. The bin length is 0.1.

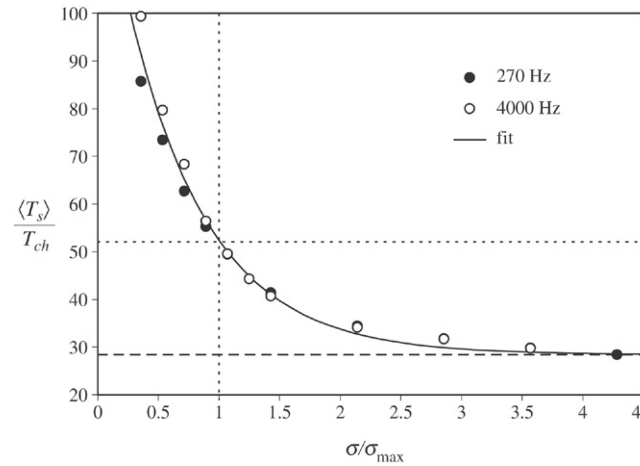

Fig. 9. Dependence of the normalized mean switching time $\langle T_s \rangle / T_{\mathrm{ch}}$ on the normalized noise intensity $\sigma/\sigma_{\max}$ for two frequencies of the input periodic signal $f_0 = 270$ Hz and $f_0 = 4000$ Hz. $T_{\mathrm{ch}} = 1/f_{\mathrm{ch}} = 1/(2.745$ kHz) and $\sigma_{\max}$ is the optimal noise intensity. The solid curve results from a fit by the formula (4) given in the text.

In experimental [28] and numerical [7,11] studies of stochastic resonance attention is also paid to the dependence of the mean switching frequency between different states of a system or chaotic attractors on the noise intensity. It was found that the mean switching frequency for a weak periodic forcing increases exponentially with the noise intensity [28]. As shown in Fig. 9, the dependence of the mean switching time, $\langle T_s \rangle$, on the noise intensity in our experiments can be fitted to an exponential law:

$$\frac{\langle T_s \rangle}{T_{\mathrm{ch}}} = C + A \exp\left(-B \frac{\sigma}{\sigma_{\max}}\right) \tag{4}$$

where $\sigma_{\max}$ is the optimal noise intensity and the fitting parameters $A = 107.3$, $B = 1.5$ and $C = 28.4$ are independent of the frequency f_0 of input periodic signal.

Finally we consider the overall shape, the envelope excluding the resonance peaks, of the switching times probability distribution functions. Numerically, we obtained this envelope by binning the data for T_s in bins $[(i-1/2)T_0, (i+1/2)T_0]$, $i = 1, 2, \ldots$. The corresponding histograms are plotted in Fig. 10 in terms of the normalized variable $t_s \equiv T_s/\langle T_s \rangle$ for small $f_0 = 270$ Hz (Fig. 10 (upper panel)) and large $f_0 = 4000$ Hz (Fig. 10 (lower panel)) frequencies of the input periodic signal and for several values of the noise intensity, below and above the optimal value $\sigma_{\max}$. It turns out that these figures can be fitted by a gamma distribution, namely:

$$P(t_s) = \frac{\beta^{-\alpha}}{\Gamma(\alpha)} (t_s + m)^{\alpha-1} \exp\left[-\frac{t_s + m}{\beta}\right] \tag{5}$$

where m and $\alpha > 0$ are the location and shape parameters, respectively, $\Gamma(\alpha)$ is the gamma function and $\beta > 0$ is the scale parameter. Since by definition $\langle t_s \rangle = 1$, it turns out that $\beta = (1 + m)/\alpha$ leaving α and m as the only parameters for fitting the data. A convenient way to do the fitting is to compute the standard deviation of the switching times σ_s and use $\alpha = (1 + m)^2/(\sigma_s/\langle T_s \rangle)^2$, leaving m as a free parameter. For the frequency $f_0 = 270$ Hz and for the following values of $\sigma/\sigma_{\max}$: 0.36, 0.71, 1.07, 1.43, 2.14, the corresponding values of α are: 1.54, 2.18, 2.80, 3.52, 5.13, whereas $m = 0$. For the frequency $f_0 = 4000$ Hz and for the following values of $\sigma/\sigma_{\max}$: 0.54, 0.89, 1.43, 2.14, the corresponding values of α are: 1.75, 1.88, 2.32, 2.97, whereas values of m are: -0.103, -0.146, -0.202 and -0.242. As shown in Fig. 10 (upper panel) and (lower panel) the fitting is quite good for all the values of the noise intensity and frequency considered and the gamma distribution given by the formula (5) describes well the experimental switching times probability distributions.

In applications of stochastic resonance, it is important to chose a noise intensity σ close to its optimal value $\sigma_{\max}$, such that the amplification factor η takes the maximum value. However, it is true that η is not easy to measure in many situations. Hence, one has to rely on the calculation of $\sigma_{\max}$ by other methods. Several alternative methods have been proposed for obtaining the optimal noise intensity from the residence and switching times probability distributions [2,25,29,30]. However the optimal noise intensities determined by these methods do not coincide with $\sigma_{\max}$ as determined from η. The form

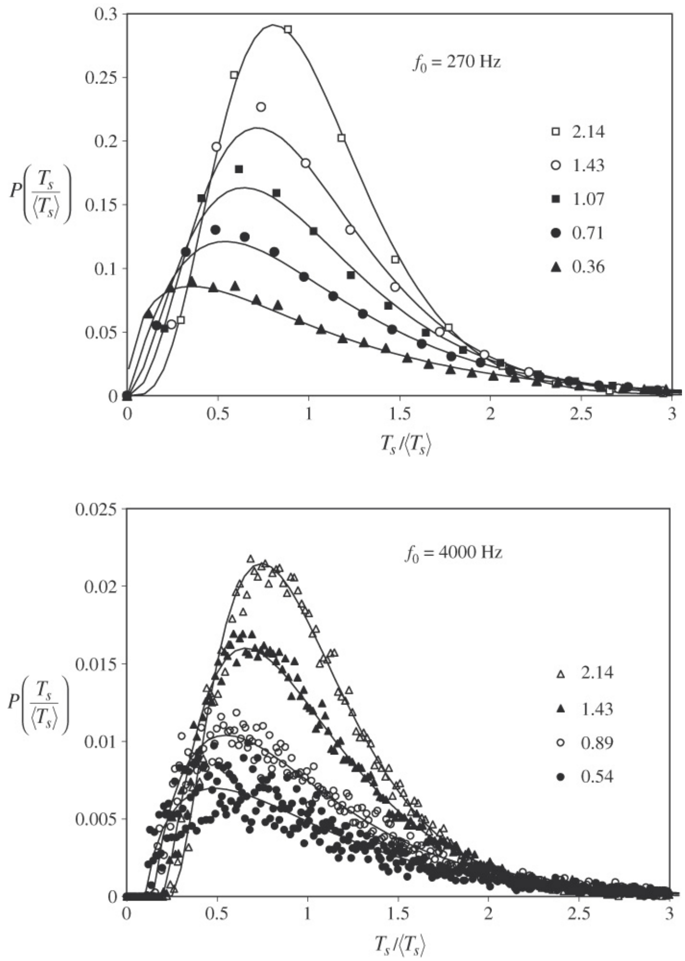

Fig. 10. Probability distributions of the normalized switching times $P(T_s/\langle T_s \rangle)$ for the frequency of input periodic signal $f_0 = 270$ Hz (upper panel) and $f_0 = 4000$ Hz (lower panel). The normalized noise intensity $\sigma/\sigma_{\max}$ for each distribution is given on the figure. $\sigma_{\max}$ is the optimal noise intensity and $\langle T_s \rangle$ is the mean switching time. The solid curves result from a fit by the gamma probability distribution (5) given in the text.

of the residence and switching times probability distributions which we obtained for the stochastic resonance in chaotic Chua's circuit for different noise intensities (shown e.g. in Fig. 7) suggests a method of determination of the optimal noise intensity from these distributions. The method is similar to the one proposed in Ref. [25], it can be applied to both the residence and switching times and gives the optimal noise intensity which coincides with $\sigma_{\max}$ as determined from η. The method goes as follows:

We count $N_{\max}$ as the number of residence times in all the intervals $[T_i - T_0/4, T_i + T_0/4]$, with $T_i = (i - 1/2)T_0$, $i = 1, 2, \ldots$, the location of the maxima of the probability distribution. In the case of switching times we take $T_i = iT_0$. The total number of residence or switching times is N. The dependence of the proportion $N_{\max}/N$ on the normalized noise standard deviation $\sigma/\sigma_{\max}$ is shown in Fig. 11 for two frequencies of the input periodic signal. At low noise intensities $N_{\max}/N \approx 1$, so residence and switching times are very well concentrated around values T_i. For large noise intensity the proportion $N_{\max}/N \approx 0.5$, so residence and switching times are almost uniformly distributed. At the optimal noise level $\sigma = \sigma_{\max}$ corresponding to the maximum of the amplification factor η the quantity $N_{\max}/N$ shows an inflection point both for the residence and switching times. Hence, we propose to define the optimal noise intensity as the value of σ at this inflection point.

It would be interesting to apply the method proposed here to determine the optimal noise intensity from the residence and switching times probability distributions obtained in other experiments of stochastic resonance and to compare this intensity with the optimal noise intensity determined from the amplification factor η.

5. Conclusions

We have presented a thorough experimental study of stochastic resonance in an electronic Chua's circuit, operating in a chaotic regime, forced by a periodic signal and a Gaussian white noise. The phenomenon results from the noise-induced switching process of the dynamical trajectory of the circuit between two single-scroll chaotic attractors. This situation in which the system moves between two chaotic states has important consequences for its dynamics.

We have found that, due to the residual dynamics on the attractors, the response of the system to a periodic perturbation strongly depends on the external frequency. The minimum

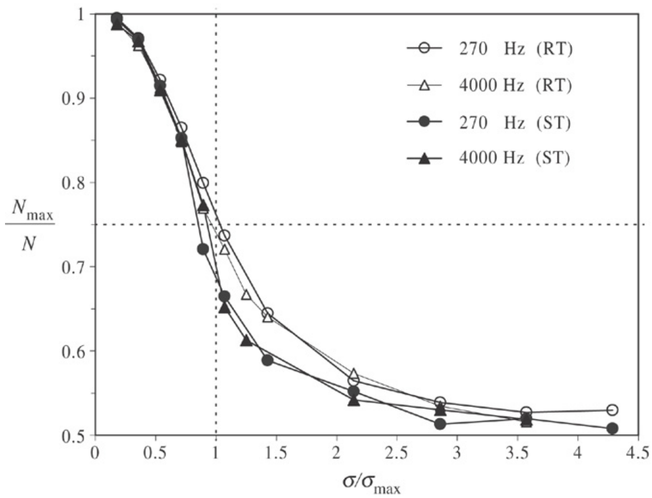

Fig. 11. Dependence of the proportion N_{max}/N for the residence (RT) and switching (ST) times on the normalized noise intensity σ/σ_{max} for two frequencies of the input periodic signal $f_0 = 270$ Hz and $f_0 = 4000$ Hz. N_{max} is the number of residence or switching times around peaks in their probability distributions and N is the total number of these times. σ_{max} is the optimal noise intensity corresponding to the maximum of the amplification factor determined by the Fourier transform of the step output signal.

threshold values for the amplitude of the external forcing required to induce jumps between these attractors in the absence of noise have been determined as a function of the frequency. While this threshold is very small for frequencies below approximately $2f_{ch}$, where $f_{ch} = 2.745$ kHz is the characteristic frequency of the autonomous Chua's circuit, it rapidly increases for higher frequencies and develops peaks at certain frequency values. As in the case of stochastic resonance between two stable states, we have also found that noise enhances the detectability for weak sub-threshold or slightly over-threshold signals, but degrades the detectability for strong supra-threshold signals. We have also observed that the stochastic resonance effect is maximum for input signals with amplitudes close to the threshold value. Furthermore, the dependence of the amplification factor of the input periodic signal on the noise intensity complies with the standard form of stochastic resonance in bistable systems. This dependence, after normalization, is the same for all frequencies of the input periodic signal and it is well described by the results of the adiabatic theory obtained for transitions between two stable states.

We have shown that the residence time probability distribution consists of a series of decreasing peaks at odd multiples of the half-period of the input periodic signal. Similarly, the switching time probability distribution consists of a sequence of peaks centered at all integer multiples of the input signal period and its envelope is well described by the gamma distribution with a shape parameter that increases with the noise intensity. The spikes of the residence and switching times probability distributions are associated with the characteristic time $1/f_{ch}$. The mean switching time decays exponentially with the noise intensity for sub-threshold amplitudes for all the frequencies of the input periodic signal.

Finally, we have shown that the optimal noise intensity corresponds to the inflection point in the dependence of the noise intensity on the proportion of the number of residence or switching times around peaks in their probability distributions to the total number of these times. This optimal noise intensity is the same as the one defined by the maximum of the amplification factor determined by the Fourier transform of the step output signal.

It is worth mentioning that some open questions remain although the phenomenon of stochastic resonance and the Chua system have been extensively studied. For example, how colored noise with a correlation time of the order of the inverse mean Chua frequency or the inverse of the mean frequency of jumps between the attractors affects the system dynamics is unknown. The response of the system to the effect of aperiodic input signals or a combination of periodic signals (ghost resonance) is yet to be considered.

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