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GHOST STOCHASTIC RESONANCE IN AN ELECTRONIC CIRCUIT

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We demonstrate experimentally the regime of ghost stochastic resonance in the response of a Monostable Schmit Trigger electronic circuit driven by noise and signals with N frequency components: $kf_0 + \Delta f$, $(k+1)f_0 + \Delta f, \ldots, k + nf_0 + \Delta f$ where k is an integer greater than one. It is verified that stochastic resonance occurs at the frequency $f_r = f_0 + (\Delta f/(k + (N-1)/2))$, as predicted in the theory. At the frequency for which the resonance is maximum there is no input energy, and thus this form is called "ghost" stochastic resonance.

Keywords: Stochastic resonance; noise; nonlinear systems.

1. Introduction

The dynamics emerging from the cooperative effects between noise, nonlinearity and weak periodic forces have attracted broad interest recently. This includes the celebrated case of stochastic resonance (SR) [Wiesenfeld & Moss, 1995; Bulsara & Gammaitoni, 1996; Gammaitoni *et al.*, 1998; Hanggi, 2002; Gammaitoni *et al.*, 1995]. In the regime of SR some quantifiable property of the input signal (signal-to-noise ratio, degrees of coherence, etc.) is optimally enhanced at the output for some optimal noise level. For example, in neurons SR is seen as a maximum coherence between the intervals between neuronal firings and the frequency of the signal driving the input. In contrast, a new form of stochastic resonance was introduced recently [Chialvo *et al.*, 2002] whereby the frequency which is enhanced is *absent* in the signals driving the system. This type of phenomenon is not possible within the framework of linear signal processing and deserves to be further explored experimentally. In this Letter we analyze the response of an nonlinear electronic circuit which emulates the system in [Chialvo *et al.*, 2002] when it is driven by noise and by weak signals composed of multiple periodic tones.

2. Threshold Device Implemented with an Electronic Schmit Trigger

The system considered here is a nondynamical threshold device [Gingl *et al.*, 1995] which compares the signal $x(t)$ with a fixed threshold, and emits a "spike", i.e. a rectangular pulse of relatively short fixed duration, when it is crossed from below. This emulates (as in [Chialvo *et al.*, 2002]) in a very simplified way the neuronal "firing". The signals considered are:

$$x(t) = \frac{A(\sin f_1 2\pi t + \sin f_2 2\pi t + \cdots \sin f_n 2\pi t)1}{n + \xi(t)} \tag{1}$$

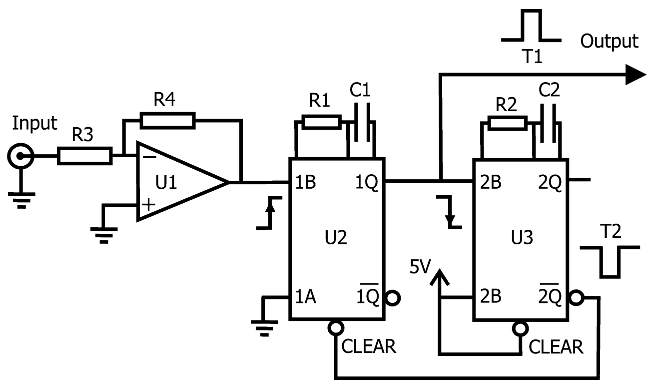


Fig. 1. Block diagram of the electronic circuit used ($R1 = 3.2\,\mathrm{k\Omega}, R2 = 64\,\mathrm{k\Omega}, R3 = R4 = 3.3\,\mathrm{k\Omega}, C1 = 95\,\mathrm{nF}, C2 = 4.2\,\mathrm{nF}$).

where $f_1 = kf_0, f_2 = (k+1)f_0, \ldots, f_n = (k+n)f_0$ and k and n are integers greater than one. The number of frequencies used is denoted as N. The term $\xi(t)$ is a zero mean Gaussian distributed white noise.

The circuit (Fig. 1) implementing the threshold device is comprised by two monostable Schmitt Triggers, (74LS123 from Texas Instruments) U2 and U3. The input signal (i.e. $x(t)$ in Eq. 1.) is amplified by the operational amplifier U1 and fed to the first monostable (input 1B) which will trigger or not depending on the amplitude of $x(t)$. When it triggers, a pulse is generated in U2 during a period of T1 (emulating a neuronal spike). The falling edge of T1 triggers the second monostable (input 2B). The complemented output of U3 ($\overline{2Q}$) is used to clear the first monostable inhibiting further triggering until the expiration of T2 (this emulates the neuronal refractory period). The circuit can be triggered again after the completion of T1 and T2, which are times fixed by the R and C values.

Signals [Eq. (1)] were generated on a personal computer within a Matlab routine and sent to the input of the circuit shown in Fig. 1 using the standard audio device of the computer. The Matlab signal generation code was implemented as a loop where Gaussian distributed noise was generated using the Matlab function *randn()* and played at the audio device with the Matlab function *sound()*. The noise intensity was increased in small steps, and held at each step for a fixed time interval. The steps were long enough to collect good statistics, even for low noise intensity levels where rate of spikes is lower. Up to four input frequency combinations [i.e. "N" in Eq. (1)] were explored: two, three, four and five frequencies.

The output of the circuit was digitized at 32 KHz using a National Instruments PCI data acquisition (Model Daq 6025) board controlled by LabView software and processed offline to compute intervals of time between triggering, from which inter-spike intervals (ISI) histogram (ISIH) was calculated. The signal to noise ratio (SNR) is computed as the ratio between two quantities: the number of spikes with ISI equal to (or near within $\pm 5\%$) the time scale of $1/f_0$, $1/f_1$ and $1/f_2$, and the total number of ISI (i.e. at all other intervals). SNR defined this way captures the temporal information encoded in the spikes train, as in the cases often described for some sensory neurons.

3. Experimental Results

3.1. *Signal to noise ratio of ISI for Δf equal to zero*

Figure 2 shows the results from the experiments using harmonic signals composed up to five periodic terms (i.e. $x(t)$ with $N = 2, 3, 4, 5$ and $f_0 = 200\,\mathrm{Hz}$ and $\Delta f = 0$). The amplitude of the deterministic terms are set at 90% triggering level, i.e. without noise there is no triggering, which is the case for classical SR. Depicted are the SNR of spikes spaced by intervals close to the periods of the terms comprising the driving signal as well as with $1/f_0$ for increasing noise intensity. Each of the three curves represent the probability of observing an interspike interval equal or near to $1/f_0$, $1/f_1$ and $1/f_2$ respectively, computed as the ratio between the number of spikes with intervals within the time scale of interest and all other intervals. Specifically, we count the number of spikes spaced by periods equal to or near by 5% of $1/f_n$ for $n = 1, 2, 3, 4, 5$. SNR for $n > 2$ are vanishingly small, thus are not plotted. The output is rather uncoherent with any of the input frequencies (empty circles and stars), however it is maximally coherent, at some range of noise intensity, with the period close to $1/f_0$ (filled circles). It is important to remark that f_0 is a frequency *absent* in the signals used to drive the system and for that reason we call it "ghost stochastic resonance". The system has nonlinearly detected this "missing fundamental" as further discussed in [Chialvo *et al.*, 2002; Chialvo, 2003].

We have verified that for signals composed of harmonic components, the frequency of the strongest resonance always correspond to the difference $f_n + 1 - f_n$, (independently of the relative phases of the components). However, we are about to see that the resonance at the different

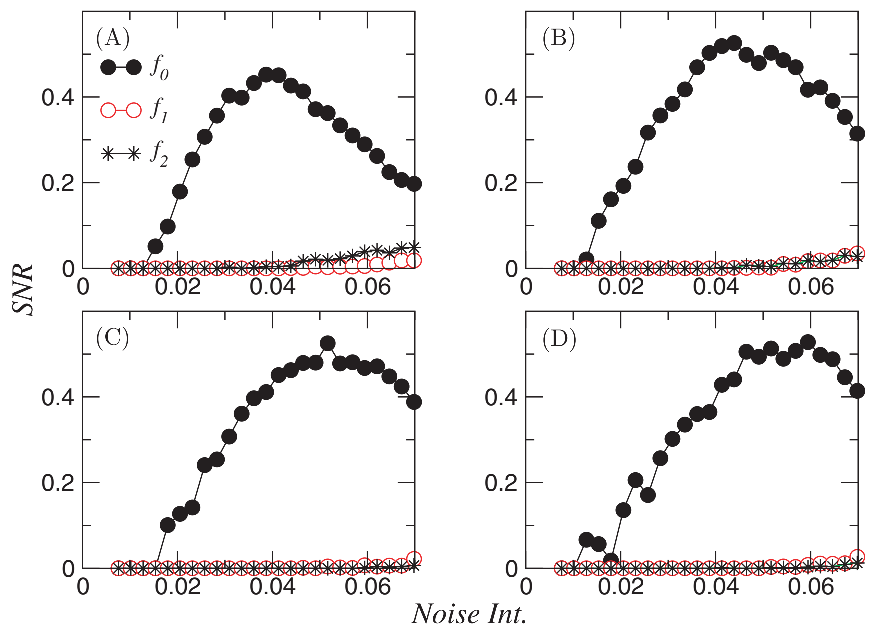


Fig. 2. Signal-to-noise ratios versus noise intensity for signals with two to five frequencies (panels A to D respectively). It is computed as the probability of observing an inter-spike interval close to the time scales (with a 5% tolerance) of the frequencies f_o (●), f_1 (○) and f_2 (∗). Notice that the largest resonance is always for the "ghost" f_0, while the others are negligible.

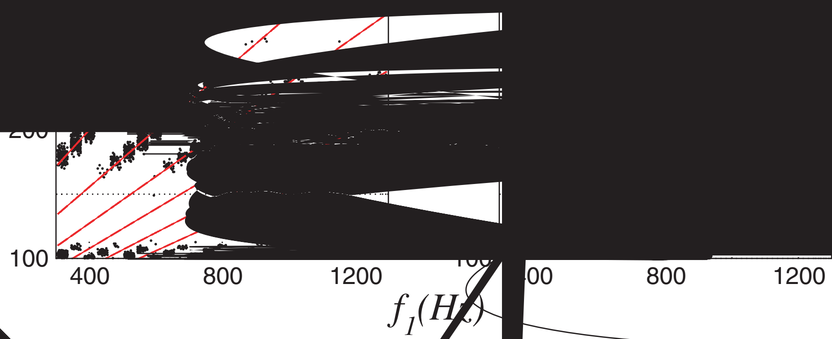
frequency is just a singular case of a more general phenomenon. Signals are often comprised of individual components (sometimes called partials) that are not integer multiples of a unique fundamental resulting in this case on a waveform which is aperiodic. This type of driving signal is dubbed "inharmonic" in music jargon and "anharmonic" in the physics literature. We can construct such a signal by shifting all components of an originally harmonic complex by the same amount Δf. In agreement with the theory in [Chialvo *et al.*, 2002], we find that the frequency of the main resonance shifts linearly despite the fact that that the frequency difference between successive partials remains constant. Specifically: the periodic terms are shifted multiples of f_0 (the absent fundamental) and partials are labeled: $f_1 = kf_0 + \Delta f$, $f_2 = (k+1)f_0 + \Delta f, \ldots, f_n = (k+n)f_0 + \Delta f$.

3.2. *Resonance for nonzero* Δf

Figure 3 shows the experimental results with Δf different from zero. Selecting the optimum noise intensity as revealed by the experiments shown in Fig. 2. (i.e. 0.04), Δf was increased in steps of 40 Hz, spanning values of f_1 from 300 to 1300 Hz. For each case, intervals between triggering were computed. For ease of presentation, the figure depicts the inverse of inter-spike intervals (dots) collected at each step of frequency, plotted as a function of f_1. It is immediately apparent that despite the fact that the spacing between the terms remains constant (at 200 Hz) since $[(k+n+1)f_0 + \Delta f] - [(k+n)f_0 + \Delta f] = f_0$, the results show a linear shift of the frequency maximally enhanced by the noise (i.e. the ghost SR) as a function of Δf. The quantitative aspects of this shift is fully accounted by the prediction in [Chialvo *et al.*, 2002] which are over-imposed (lines) in the figure. The argument described in [Chialvo *et al.*, 2002] shows that for inputs composed of N sinusoidal signals, the ghost resonances are expected at a frequency:

$$f_r = f_0 + \frac{\Delta f}{\frac{k + (N-1)}{2}} \qquad (2)$$

In summary, we have verified experimentally two of the most important quantitative features of this type of new resonance described in previous work. First, for the case of harmonic signals there is a robust resonance for f_0 regardless of the number of terms composing the signals (Fig. 2). Second, for the case of inharmonic signals, the general expression derived previously predicts precisely the location of the resonances observed experimentally (Fig. 3). Given that the robustness of the phenomena is expected, the main findings reported here and in [Chialvo *et al.*, 2002] can be observed in a



Main resonance … signals with two to five frequencies (panel A to D respectively). In each panel, the intervals … d as its inverse f_r) be… triggered pulses $1/T$ are plotted as a function of f_1, which was varied in steps of 40 Hz. Fam… ver-imposed lines are th… oretical expectation (i.e. Eq. (2) with $N = 2, 3, 4$ or 5 in panels A through D respectively) … reasing $k = 2 - 7$.

…ity of nonlinear systems such … semiconduc… asers with optical feedback in th… excitable …e [Buldú *et al.*, 2003], propagating n… ral dis… es [Chapeau-Blondeau *et al.*, 1996], muscle …ors [Fallon *et al.*, 2004], spinal and cortical …potentials evoked to tactile stimuli [Manjarrez … 2003], and vision [Kingdom & Simmons, 1998; …*et al.*, 2000], areas which deserve to be further …igated.

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