Turn-on Jitter of External-Cavity Semiconductor Lasers

E. Hernández-García, C. R. Mirasso, K. A. Shore, and M. San Miguel

***Abstract*—Analytical expressions, validated by numerical simulations, are obtained for the turn-on delay jitter of semiconductor lasers subjected to weak optical feedback in short external cavities. The results show explicitly that displacement of the external reflector on optical wavelength scales causes significant changes in the switch-on dynamics of the laser. It is found that more than a 400% increase of jitter can occur under certain circumstances. The demonstrated sensitivity of laser switch-on dynamics to reflector location is considered to be particularly relevant to the performance of packaged laser diodes.**

I. Introduction

TURN-ON delay jitter is of considerable importance for practical applications of semiconductor lasers. It causes a degradation of the temporal resolution and it acts as a limiting factor in the performance of high-bit rate optical communication systems. Jitter properties have been extensively studied both experimentally and numerically [1]–[4]. Most recently, the dependence of jitter properties on bias level and modulation frequency of the injection current have been considered [5] showing that biasing below threshold can be advantageous to reduce the jitter in a situation of signal transmission at high speed (Gb/s). The connection of pattern effects with the randomness of the turn-on time have also been evidentiated [6].

Optical feedback is a well-known effect to take into account when considering the performance of a laser diode in an optical communication system [7], [8]. Small amounts of feedback are known to be useful for linewidth reduction, but feedback intensities likely to occur in optical communication systems degrade its performance through the occurrence of the "coherence collapse" [9], [10] giving rise to linewidths of several GHz and a chaotic intensity signal [11]. A general classification of the effect of feedback on semiconductor laser spectral and dynamic properties has been reported [7], [12]. That classification is concerned with the CW operation of the device. However, few studies are available of external-cavity lasers under direct-modulation conditions [13]–[16] or in the transient regime following a gain-switching [17]. Only recently the degradation in the performance for a digital intensity modulated direct detection system caused by optical feedback has been considered in some detail [13]. In this context, the turn-on dynamics and associated jitter properties of semiconductor lasers affected by optical feedback have been examined [14]–[16].

The characterization of jitter properties associated with the turn-on dynamics of semiconductor lasers in the presence of optical feedback seems a worthwhile task given its practical implications in optical communication systems. The results of previous work are [15], [16] purely numerical, consider a fixed value of the external round-trip delay time and study the turn-on dynamics from a bias above threshold. In these situations, it has been shown that the jitter can be largely increased with increasing amounts of feedback power. This is true both in situations of largely separated gain-switching events and under fast pseudorandom-word modulation as appropriate for signal transmission. In this paper, we explore further these questions studying the dependence of jitter properties on the external-cavity round-trip time. We address the problem through an analytical calculation that allows us to capture in a general way some of the basic issues of the problem. In addition, numerical calculations are reported for parameter values for which our analytical approximations break down. We consider turn-on dynamics from a bias below threshold. This choice of bias is made in view of the results of reference [5] for rapidly modulated lasers. Our results are obtained for weak feedback conditions. Such conditions are met, for example, in laser diode modules where unwanted reflections from lenses, fiber facets, and other package components may affect the laser behavior. It has been shown that axial displacement of reflecting planes in laser diode modules can cause optical output power fluctuations as well as shifts in the laser emission spectrum [18].

Of additional significance for the present work is the fact that the weak optical feedback obtained in such laser packages would be associated with relatively short external cavities and consequently short feedback-delay times. In such a regime of operation, the accuracy of analytical expressions as reported in this paper have been confirmed numerically. Very weak reflections from distant reflectors, say from nominally reflectionless facets of semiconductor optical amplifiers or optical fibers, may also be described within the formalism presented here.

The basic physical effect which will be shown here to be of great significance in the transient regime is the change of

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laser threshold when the laser is subject to optical feedback. It is important to observe that this change in threshold is dependent upon the phase of the light returned to the laser cavity after a round trip of the external cavity. In consequence to this phase dependence, there is a phase dependence (or equivalently external-cavity length dependence) of the laser threshold in the presence of feedback. It is shown here that such phase dependence gives rise to (damped) oscillations in the laser turn-on time and jitter as a function of external-cavity length. The amplitude of these of oscillations is of significance for short external cavities as appropriate to packaged laser diodes as noted above. The frequency of these oscillations is determined by the optical frequency so that extremely small variations of the external-cavity length result in large variations of the turn-on times. This effect can give rise to an effective large jitter associated with noise sources which randomly modify the external-cavity length in tiny amounts. The sensitivity to small variations of the external-cavity length has also been addressed in the context of CW-operation for situations of stronger feedback [19].

The outline of the paper is as follows. In Section II, we present the analytical calculation of jitter properties and its main consequences. The calculation is substantiated by numerical results presented in Section III. Some general conclusions are summarized in Section IV. The Appendix contains the mathematical details of our calculation.

TABLE I
MEANINGS AND VALUES OF THE PARAMETERS IN (1) AND (2)

Parameter	Meaning	Value	Units
g	Gain parameter	$5.6 \cdot 10^{-8}$	ps^{-1}
γ	Inverse photon lifetime	0.4	ps^{-1}
γ_e	Inverse carrier lifetime	$5 \cdot 10^{-4}$	ps^{-1}
α	Linewidth enhancement factor	5.5	adimensional
β	Spontaneous emission rate	$1.1 \cdot 10^{-8}$	ps^{-1}
ω_0	Optical angular frequency	$1.216 \cdot 10^{3}$	ps^{-1}
N_0	Carrier number at transparency	$6.8 \cdot 10^{7}$	adimensional
C_{bias}	Bias current	$3.4 \cdot 10^{16}$	carriers/s
C_{th}	Threshold current	$3.76 \cdot 10^{16}$	carriers/s
C_{on}	Injection current after gain switching	$1.316 \cdot 10^{17}$	carriers/s
κ	Feedback coupling parameter	0.1, 0.15	ps^{-1}.
τ	Delay feedback time	variable	ps.

II. Calculation of Jitter Properties for Weak Feedback

A. Dynamical Model

We consider a situation in which a single-mode semiconductor laser is coupled to an external cavity. The delay-differential equations appropriate to describe this configuration are the Lang–Kobayashi equations [20] which describe the coupled time evolution of the complex amplitude of the electric field E of the laser (in the slowly varying envelope approximation) and the carrier number N inside the laser cavity. Supplemented with Langevin noise terms [7] and neglecting gain saturation effects, these equations can be written as follows:

$$\dot{E}(t) = \frac{1+i\alpha}{2}[g(N(t)-N_0)-\gamma]E(t) + K(\tau)E(t-\tau) + \sqrt{2\beta N(t)}\xi(t), \quad (1)$$

$$\dot{N} = C - \gamma_e N(t) - g(N(t)-N_0)|E(t)|^2 - \sqrt{8\beta N(t)}(E(t)^*\xi(t) + E(t)\xi(t)^*) + \sqrt{2\gamma_e N(t)}\xi_N(t). \quad (2)$$

The equation for the complex field E is written in the frame of reference in which the electric field is constant except for phase diffusion when the laser is on and in CW operation in the absence of feedback. The meaning and values used for numerical calculations of the parameters in (1)–(2) are given in Table I. Spontaneous emission noise is modeled through the last term in (1), and random nonradiative carrier decay is included through the term containing ξ_N in (2). The complex random process $\xi(t)$ and the real random process $\xi_N(t)$ are taken to be Gaussian processes of zero mean, and of correlations given by the following:

$$\langle \xi(t)\xi(t')^* \rangle = 2\delta(t-t') \quad (3)$$

$$\langle \xi(t)\xi_N(t') \rangle = 0 \quad (4)$$

$$\langle \xi_N(t)\xi_N(t') \rangle = \delta(t-t'). \quad (5)$$

$K(\tau)$ in (1) is given by

$$K(\tau) = \kappa e^{-i\omega_0 \tau} \quad (6)$$

where the feedback coupling parameter κ is determined by the quotient between the external reflectivity and the internal cavity roundtrip time [7]. We will consider here values of κ appropriate for weak external feedback situations while we will leave the delay feedback time τ measuring the length of the external cavity as a free parameter of our study.

B. Analytical Solution

The intention of the present paper is to obtain a description of the transient dynamics of the external-cavity laser in a relatively simple form which captures the significant physical effects associated with the dependence on the phase of the light reentering the laser after a roundtrip in the external cavity. We consider the dynamical evolution after the injection current C is suddenly changed from a value C_{bias} below its threshold value to a value C_{on} such that the laser switches on. In the absence of feedback, the threshold value of the current below which no laser amplification occurs is given by $C_{\text{th}} \equiv \gamma_e(N_0 + \gamma/g)$. The approach taken here to obtain analytical information from the coupled equations (1) and (2) is to exploit the linear nature of the laser dynamics in the turn-on regime. Within this approach, we obtain tractable expressions which clarify the essential features of the device behavior under transient conditions. These essential features will be substantiated by numerical simulations of the dynamical equations (1) and (2) as described in the next section.

In order to calculate the turn-on time, we consider the initial stage of evolution, when the intensity is very small, so the term containing $|E(t)|^2$ in (2) is negligible. In addition, the noise terms in that equation are much smaller than the deterministic drift and they can also be neglected [21]–[23]. Within this approximation, (2) can be solved as

$$N(t) = N_{\rm on} + (N_{\rm th} - N_{\rm on})e^{-\gamma_e(t-t_{\rm th})}. \qquad (7)$$

The early evolution of the electric field is described by

$$\dot{E}(t) = q(t)E(t) + K(\tau)E(t-\tau) + \sqrt{2\beta N(t)}\xi(t) \qquad (8)$$

where

$$q(t) \equiv \frac{1+i\alpha}{2}[g(N(t) - N_0) - \gamma]. \qquad (9)$$

In (7), $N_{\rm on} = C_{\rm on}/\gamma_e, N_{\rm th} = C_{\rm th}/\gamma_e$, and $t_{\rm th}$ is the threshold time at which $N(t_{\rm th}) = N_{\rm th}$.

The above equation for the field is a linear equation with a delayed term and time-dependent coefficients. We find in the Appendix an approximate solution to (8) and show that the statistics of switch-on times can be calculated from the known results in the absence of feedback but with the inverse photon lifetime γ and the spontaneous emission intensity β replaced by

$$\gamma_{\rm eff} = \gamma - 2\,{\rm Re}\,(z_0) \qquad (10)$$

$$\beta_{\rm eff} = \frac{\beta}{|1+\tau z_0|^2} \qquad (11)$$

with z_0 given by

$$z_0 = \kappa \exp\left(-i\omega_0\tau + \frac{1+i\alpha}{2}g \cdot \left[\frac{N_{\rm on} - N_{\rm th}}{\gamma_e}(1 - e^{\gamma_e\tau}) + (N_{\rm on} - N_{\rm th})\tau\right]\right). \qquad (12)$$

The main approximation needed to obtain this result is, as discussed in the Appendix, a single pole approximation (SPA). It consists in neglecting all poles except one in the calculation of an inverse Laplace transformation. The pole which is kept z_0 is such that $z_0 = 0$ in the absence of feedback and remains close to zero for small κ. The other poles are related to external-cavity modes of the laser. Numerical calculations of the position of these other poles in the complex plane show that they are strongly damped if $\kappa\tau \lesssim 0.1$. Then the reduction of the laser dynamics to the case with no external cavity but with the effective parameters given in (10), (11), and (12) will be correct in that range of parameters. We stress that the condition of weak feedback considered here for the transient properties needs not to coincide with the condition of weak feedback for steady-state properties which are the basis of the classification in [12]. For example, for the values of $\kappa\tau$ considered here one can still find frequency multistability in CW operation if the linewidth enhancement factor α is large enough.

C. Laser Threshold Variation

The reduction of the feedback problem to the one in the absence of feedback but with modified parameters, (10) and (11), has important consequences in the identification of the laser threshold. The laser threshold is here defined as the value of the injection current above which the laser switch-on will be triggered by any amount of noise. The replacement of γ by $\gamma_{\rm eff}$ in the expression for $C_{\rm th}$ leads to an effective threshold current $C_{\rm th}^{\rm eff}$:

$$C_{\rm th}^{\rm eff} = \gamma_e\left(N_0 + \frac{\gamma_{\rm eff}}{g}\right). \qquad (13)$$

This effective threshold is plotted in Fig. 1 as a function of τ. Its main characteristics, easily extracted from (10)–(12) are: a) an oscillatory behavior of frequency given by $\omega_0 + \mathcal{O}(\tau)$ for small τ; b) the oscillations are damped so that they are unimportant for $\tau \gtrsim (g(C_{\rm on} - C_{\rm th})/2)^{-1/2}$; c) for large $\tau, C_{\rm th}^{\rm eff} \to C_{\rm th}$, the value in the absence of feedback. The physical origin of these three characteristics can be easily understood. If the reinjected field is in phase with the field in the cavity, then the delayed term in (8) acts as an effective increase of $q(t)$ for all times or equivalently, a reduction in cavity losses, so that the field instability occurs at a reduced value $C_{\rm th}^{\rm eff}$ of the injection current. This is specially clear for $\tau \to 0$, where this effect is obvious from the form of the equation. When the feedback field reenters the cavity in phase opposition with respect to the field in the cavity, the reverse effect occurs; the current needed for the beginning of amplification is higher. The evolution of the phase during the external-cavity roundtrip is essentially given by the instantaneous value of the optical frequency of the laser at the time when carrier number crosses threshold, which turns out to coincide with ω_0, which is the frequency of the laser when it is on and in the absence of feedback. This determines the frequency of the oscillations of $C_{\rm th}^{\rm eff}$ as a function of τ. Small corrections to ω_0 arise from (12) and they are due to the variations in the instantaneous frequency during the switch-on process [23], [24]. These corrections are only noticeable for large τ, for which the oscillations are already damped. The damping of the oscillations at large τ is clearly expected; if τ is larger than the switch-on time of the laser in the absence of feedback, the laser will switch-on before feeling the effect of feedback (the feedback field is also present during times smaller than τ, but its intensity is given by spontaneous emission noise in the off-state which is of negligible importance compared with the nondelayed terms in the evolution equation). Then, the oscillation disappears and $C_{\rm th}^{\rm eff} \to C_{\rm th}$ for large τ.

It is important to note that the effective threshold of Fig. 1 is of different nature than other definitions of threshold often used in this context. For example, the existence of lasing steady states in the presence of feedback is frequently discussed in terms of another effective threshold [7]. Its value is also determined by the consideration of the phase of the feedback field with respect to the field on the cavity, *but* in the lasing state. Such threshold is determined as the condition for the existence of a nonvanishing steady-state solution for the field intensity and does not coincide with the $C_{\rm th}^{\rm eff}$ discussed by

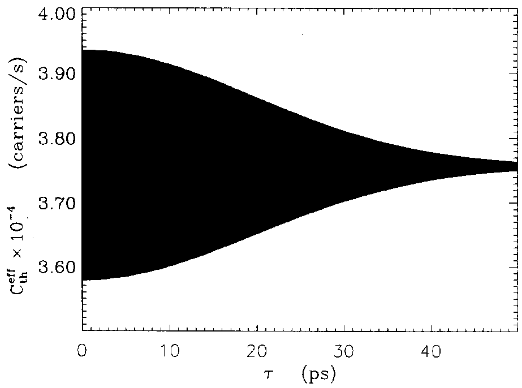

Fig. 1. Effective threshold for $\kappa = 0.1\,\text{ps}^{-1}$ as a function of τ. Individual oscillations are too fast to be appreciated in the scale of the plot.

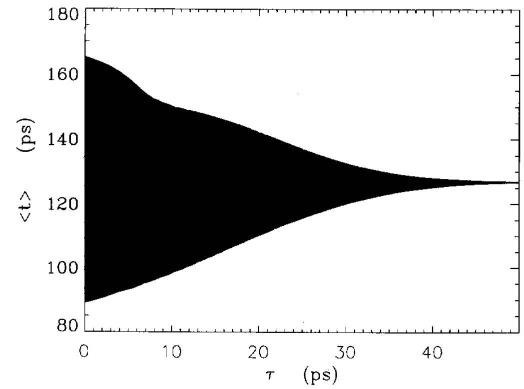

Fig. 2. Average turn-on time for $\kappa = 0.1\,\text{ps}^{-1}$ obtained from (14) as a function of τ.

us here. The threshold defined here (13) is associated with the value of the current for which a linear instability of (8) occurs. The difference between both kinds of threshold can have interesting consequences. In particular, the oscillations of the threshold determined in [7] are not damped as a function of τ. Then at large τ, it is possible to find values of the injection current and κ such that there exist lasing states while the off state is linearly stable. This implies a kind of bistability in which a finite perturbation is needed to switch-on the laser, and hysteresis can occur by increasing and decreasing the injection current.

D. Turn-on Jitter Calculations

Once the problem has been reduced to that in the absence of feedback with the effective parameters in (10) and (11), the calculation of the mean turn-on time $\langle t\rangle$ and its standard deviation σ around the mean can be done following the same procedure as in [22] and [23] to find

$$\langle t\rangle = \frac{1}{\gamma_e}\ln\frac{C_{\text{on}}-C_{\text{bias}}}{C_{\text{on}}-C_{\text{th}}^{\text{eff}}}+\sqrt{\frac{2f}{a}}\left[1-\frac{\Psi(1)}{2f}\right], \quad (14)$$

$$\sigma^2 = \langle(t-\langle t\rangle)^2\rangle = \frac{\Psi'(1)}{2af} \quad (15)$$

where we have introduced:

$$f = \ln\frac{I_r}{b} \quad (16)$$

$$a = g(C_{\text{on}}-C_{\text{th}}^{\text{eff}}) \quad (17)$$

$$b = 4\beta_{\text{eff}}\left(\frac{C_{\text{th}}^{\text{eff}}}{\gamma_e}\sqrt{\frac{2\pi}{a}}+\frac{1}{g}\right). \quad (18)$$

and $\Psi(1) = -0.577\ldots$ is the digamma function of one, and $\Psi'(1) = 1.638\ldots$ is its derivative at the same point [25]. I_r is the intensity of reference used to define the turn-on time as the time at which $|E|^2 = I_r$. In our numerical calculations, I_r is taken as $I_r = 3\cdot 10^4$, which corresponds to a 13% of the steady-state intensity in the absence of feedback.

Although the general picture just stated is physically sound, and its general predictions will be confirmed by numerical simulations, it should be stressed that the explicit formulas (10), (11), (14), (15) rely on the SPA, so that they are expected to be precise only for small $\kappa\tau$. In addition, (11) predicts an increase of the effective noise intensity when z_0 is near $-1/\tau$. For the parameters that we use in the numerical calculations, this increase is dramatic for $\tau \approx 10$ ps and 30 ps, and reflects the proximity to z_0 of some of the poles neglected by the SPA and associated with external-cavity resonances. Equation (15), which relies on a small noise intensity approximation, shows artificial divergencies for those values of τ, accompanied by "bumps" in $\langle t\rangle$. This is again a confirmation that (14) and (15) are quantitatively valid only for $\kappa\tau \lesssim 0.1$. However, we show in Section III that their qualitative predictions have a broader range of validity.

Figure 2 shows $\langle t\rangle$ as a function of τ from (14). To avoid the unphysical "bumps," the value of b has been fixed to the 5% of I_r when the value calculated from (18) exceeded that range. Thus, we expect this plot to be quantitatively correct for small τ and qualitatively for larger τ. The oscillations of $\langle t\rangle$ as a function of the feedback delay time τ, of frequency ω_0, are too fast to be distinguished in the scale of the plot. For τ of the order of 50 ps the effect of the feedback on the turn-on time disappears and the mean turn-on time approaches a constant value which coincides with the one in the absence of feedback.

The oscillations of the mean turn-on time with τ as obtained from (14) are clearly shown in an expanded scale in Fig. 3. The expression for $\langle t\rangle$ has two different terms which have been plotted separately. The dashed line corresponds to the first term in (14), which is the time $t_{\text{th}}^{\text{eff}}$ needed for the carrier number $N(t)$ to reach the modified threshold $C_{\text{th}}^{\text{eff}}/\gamma_e$. The difference between $\langle t\rangle$ and $t_{\text{th}}^{\text{eff}}$, is shown as the dashed-dotted line. This last contribution, the average time needed for the laser to switch-on after the current has reached threshold, is also oscillatory, but the amplitude of the oscillation is too small to be seen on the scale of the plot. The main contribution to the amplitude of the oscillations of $\langle t\rangle$ comes from the first term.

Our analytical result for the jitter given by the standard deviation σ of the turn-on time distribution also predicts an oscillatory behavior with τ and with the same frequency ω_0. The amplitude of the oscillations grows with τ for small τ.

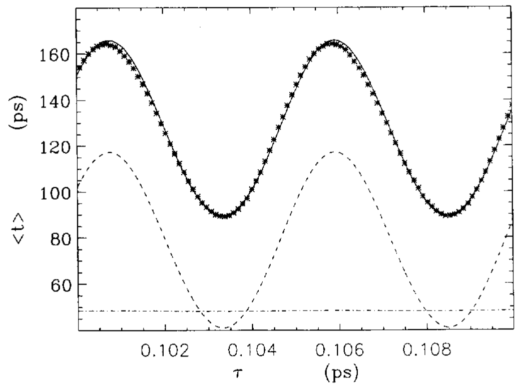

Fig. 3. Average turn-on time $\langle t\rangle$ for $\kappa = 0.1\,\mathrm{ps}^{-1}$ around $\tau = 0.1$ ps. Solid line is the result obtained from (14), which is the sum of contributions indicated by dashed and dash-dotted lines (see text). Stars are the results of numerical simulations.

At intermediate τ, it shows artificial divergencies, due to the breakdown of our approximations, but since this amplitude should go zero for large enough τ (recall the discussion on the shape of $C_{\rm th}^{\rm eff}$) it is clear that there is an intermediate τ value for which the envelope of σ has at least one maximum. The formula for the variance is intended to give a guide of the gross qualitative features just mentioned. However, a good quantitative description is not expected since this quantity is rather sensible to small variations in the injected signal [26] which are not properly taken into account in the SPA.

III. Numerical Solutions

As indicated above, a number of conditions need to be fulfilled in order to derive our analytical results for average turn-on time and associated variance. A very important requirement is a situation of weak feedback with a small delay time τ. The expressions obtained do demonstrate the main physical effects which may be anticipated in such a situation. However, it is considered valuable to compare the predictions of the formulas with results obtained from direct computer simulation of turn-on effects. Such results are given here. The numerical results also allow the exploration of the statistics of turn-on properties for a wider range of parameters beyond the range for which (10)–(12) are valid. In particular, we present results for larger values of τ. Prior to a discussion of those results, it is indicated that the analytical formulas are almost a prerequisite for undertaking the simulations. It is contended that direct simulations may not yield all the significant physical features if, without the basis of the analytical formulas, care is not taken to work to adequate accuracy. In particular, trial and error simulations without an analytical guide would be of rather little help to find the oscillatory behavior of turn-on properties with such a high frequency as ω_0.

The results of our numerical simulations are shown in Figs. 3 and 4. For a fixed value of τ and κ and the rest of parameters as given in Table I, we perform a direct numerical integration of (1) and (2) with the same method as in [23] with a time-integration step $\Delta t = 0.01$ ps. This integration step is much smaller than the one used in previous work [16] and it is required here to obtain an adequate accuracy. Each turn-on event is associated with a given sequence of random numbers needed to model the noise terms. The initial condition for each turn-on event is chosen as the value obtained from the numerical solution of the stochastic laser equations with an

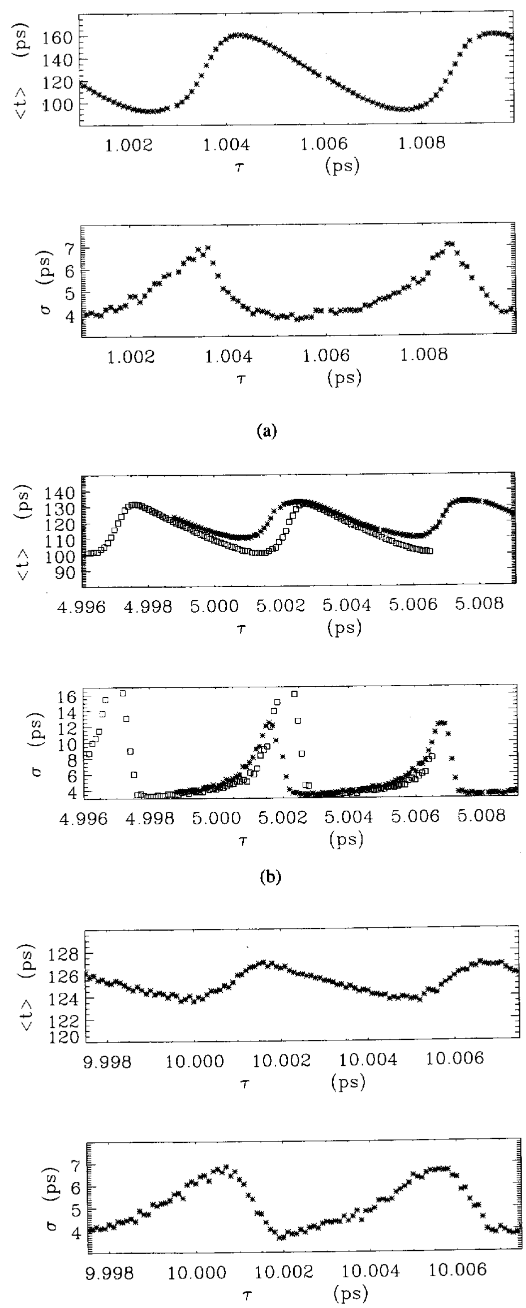

Fig. 4. Average turn-on time $\langle t\rangle$ and its associated dispersion σ from numerical simulations in three ranges of τ. The stars are for $\kappa = 0.1\,\mathrm{ps}^{-1}$, and in (b) the squares are for $\kappa = 0.15\,\mathrm{ps}^{-1}$.

injection current $C = C_{\text{bias}}$ during 500 ps. As stated before, the turn-on time is defined as the time at which the light intensity $I = |E|^2$ matches a value $I_r = 3 \cdot 10^4$.

Given the very fast oscillation predicted for the statistical properties of the turn-on times, we have looked at the variation with τ of the mean turn-on time $\langle t \rangle$ and its variance σ at intervals $\Delta\tau = 1.033 \cdot 10^{-4}$ ps, corresponding to 50 values of τ in a period $2\pi/\omega_0$ of the oscillations seen in Figs. 3 and 4. Each point in Figs. 3 and 4 corresponds to an average over 1000 turn-on events for fixed τ. Thus, we have considered $5 \cdot 10^4$ turn-on events within a period of the oscillations. This analysis has been performed at four regions of feedback delay times, namely around $\tau = 0.1, 1, 5$, and 10 ps.

Figure 3 contains a comparison of analytical and simulation results for the mean turn-on time for τ close to $\tau = 0.1$. These very small values of τ are here considered for a check of the SPA approximation used in our analytical calculation. For such values of τ, and with $\kappa = 0.1$ ps^{-1}, the approximation is well justified and the agreement is very satisfactory both in the amplitude and in the frequency of the oscillations. However, it is already noticeable that the simulation results do not follow a pure sine-oscillation, showing some degree of asymmetry around the extreme value of $\langle t \rangle$. Such asymmetry indicates a small deviation from the SPA which becomes apparent for larger values of τ (see Fig. 4) for which the analytical approximation breaks-down.

The numerical results for $\langle t \rangle$ in Fig. 4 indicate that the main qualitative predictions of (14) still hold beyond the values of τ for which quantitative agreement is found. Indeed, we find an oscillation behavior with τ of frequency ω_0 and amplitude decreasing with τ. For $\tau = 0.1$ the amplitude of the oscillations (from maximum to minimum value) is of 59% with respect to the value of $\langle t \rangle$ in the absence of feedback. This amplitude decreases to less than 4% for $\tau = 10$. The asymmetry of the oscillation with τ is seen to become more pronounced as τ increases.

The numerical results for σ (Fig. 4) are also in agreement with two general predictions of (15), namely, they also show an oscillating behavior with τ of frequency ω_0, and the amplitude of the oscillation first grows with τ going through a maximum before going to zero for very large τ. The effect of the feedback in the jitter, measured by σ, is relatively more important than for $\langle t \rangle$ as seen in the very large amplitude of the oscillations. We have found oscillation amplitudes in σ of a 55% for both $\tau = 1$ ps, and 10 ps. An additional important feature borne out by our simulations is that for intermediate τ, σ shows rather sharp maxima for values of τ for which $\langle t \rangle$ increases sharply with τ. The maximum values of $\langle t \rangle$ and σ do not occur for the same values of τ. The largest value of σ for $\kappa = 0.1$ ps^{-1} occurs for $\tau = 5$ ps, with an amplitude of oscillations of a 350% with respect to its value in the absence of feedback.

We have also examined the dependence of turn-on statistics on the feedback parameter κ, by considering $\kappa = 0.1$ and 0.15 ps^{-1}. It is seen that for both values of κ the dependence on τ of the mean turn-on time is qualitatively similar, but the amplitude of the oscillations increases with κ, as predicted by the analytic formulas. For $\kappa = 0.15$ ps^{-1} the amplitude of the oscillations of the variance at the range of τ displayed is close to the 470% of its value in the absence of feedback.

IV. Conclusions and Discussion

We have presented an analytical calculation and numerical simulations of the turn-on delay time statistics of a laser-diode gain-switched from below threshold in the presence of weak optical feedback. Our results indicate that when the optical feedback reenters the active medium of the laser before it turns on, the turn-on time becomes extremely sensitive to very small variations in the position of the external mirror. Variations in this position in a length scale determined by the optical frequency of the laser yield an increase in the jitter up to more than a 400% under certain circumstances. The position of the mirror can be adjusted for a compromise between minimization average turn-on time and maximum jitter. However, mechanical disturbances causing small variations of mirror position will result in a largely unpredictable value of the turn-on time which can be thought of as an effective large jitter. The results are of specific relevance to laser diode modules within which laser diodes may be subject to weak external reflections from a variety of optical elements including lenses, fiber endfaces, and optical isolators.

V. Appendix

In this appendix, we obtain an approximate solution to (8) with N given by (7). Equation (8) can be rewritten by introducing a new variable $h(t)$ defined from

$$E(t) \equiv h(t) e^{\int_{t_{\text{th}}}^{t} q(t')dt'}. \tag{19}$$

In terms of it, (8) reads

$$\dot{h}(t) = K(\tau) e^{-Q(t,\tau)} h(t-\tau) + R(t) \tag{20}$$

$$Q(t,\tau) \equiv \int_{t-\tau}^{t} q(t')dt' = -\frac{1+i\alpha}{2} g \cdot \left[\frac{N_{\text{on}} - N_{\text{th}}}{\gamma_e} e^{-\gamma_e (t-t_{\text{th}})} (1 - e^{\gamma_e \tau}) + (N_{\text{on}} - N_{\text{th}})\tau \right] \tag{21}$$

$$R(t) \equiv \sqrt{\beta N(t)} e^{-\int_{t_{\text{th}}}^{t} q(t')dt'} \xi(t). \tag{22}$$

These equations should be simplified further in order to obtain closed expressions for the moments of the passage time. To this end, we note that for times smaller than the switch-on time, and for typical values of the parameters $e^{-\gamma_e (t-t_{\text{th}})} \approx 1$. So that

$$Q(t,\tau) \approx Q(\tau) = -\frac{1+i\alpha}{2} g \cdot \left[\frac{N_{\text{on}} - N_{\text{th}}}{\gamma_e} (1 - e^{\gamma_e \tau}) + (N_{\text{on}} - N_{\text{th}})\tau \right]. \tag{23}$$

After this approximation, the Laplace transform of (20) can be readily obtained

$$z\tilde{h}(z) - h(t=0) = K(\tau) e^{-Q(\tau) - \tau z} h(z) + \tilde{R}(z) + M(z) \tag{24}$$

where $\tilde{h}(z)$ and $\tilde{R}(z)$ are the Laplace transforms of $h(t)$ and $R(t)$, respectively, and

$$M(z) \equiv e^{-\tau z} \int_{-\tau}^{0} h(t)e^{-zt}dt. \quad (25)$$

The solution of (24) is

$$\tilde{h}(z) = \frac{h(0) + M(z) + \tilde{R}(z)}{z - \hat{K}(\tau)e^{-z\tau}} \quad (26)$$

where we have introduced

$$\hat{K}(\tau) \equiv K(\tau)e^{-Q(\tau)}. \quad (27)$$

The inverse Laplace transformation should be applied to (26) to obtain an expression for the electric field. As long as the poles of $\tilde{h}(z)$ are simple, the Laplace inversion gives $h(t)$ as

$$h(t) = \sum_i \mu_i e^{z_i t} \quad (28)$$

where z_i are the zeros of the denominator in (26)

$$z_i - \hat{K}(\tau)e^{-z_i\tau} = 0 \quad (29)$$

and μ_i are the residua of (26) at these points. Equation (29) has two kinds of solutions: one of the zeros z_0 exists also in the absence of feedback ($z_0 = 0$) and in the presence of feedback can be obtained perturbatively in $\hat{K}(\tau)$:

$$z_0 = \hat{K}(\tau)(1 - \tau\hat{K}(\tau) + \mathcal{O}((\tau\hat{K}(\tau))^2)). \quad (30)$$

The other solutions of (29) have a very large and negative real part when $\tau\hat{K}(\tau)$ is small, so that their contribution to (28) will be negligible. When $\tau\hat{K}(\tau) \sim 0.1$, the real part of these zeros begins to be comparable to the real part of z_0. For large $\tau\hat{K}(\tau)$, all the zeros are very close, some of them even merge with z_0, and also the series (30) looses convergence. A good approximation for small $\tau\hat{K}(\tau)$ consists in replacing the series (28) by the first term, containing z_0. We call this the single pole approximation (SPA). The associated residue μ_0 can be calculated as

$$\mu_0 = \lim_{z\to z_0} (z - z_0)\tilde{h}(z) = \frac{h(0) + M(z_0) + \tilde{R}(z_0)}{1 + \tau z_0}. \quad (31)$$

By using definition (19), we obtain the following approximation for the evolution of the electric field:

$$E(t) \approx \frac{h(0) + M(z_0) + \tilde{R}(z_0)}{1 + \tau z_0} e^{z_0 t + \int_{t_{th}}^{t} q(t')dt'} \quad (32)$$

from which we will calculate the passage-time statistics. To this end, we compare this expression with the linear dynamics of the laser in absence of feedback

$$E(t) \approx (h(0) + \tilde{R}(0))e^{\int_{t_{th}}^{t} q(t')dt'}. \quad (33)$$

We see that one of the effects of the feedback is to replace the initial condition $h(0)$ by the quantity $h(0)+M(z_0)$, containing the memory of the field during the feedback cycle previous to the switching. Since the laser was off in that period, $M(z_0)$ is very small. In fact, it was shown in [22], [23] in the absence of feedback, that for bias not too close to threshold it is a good approximation to take $h(0) \approx 0$ in order to calculate first-passage-time statistics. Then, it is expected than in the same conditions, it is here a good approximation to take $h(0) + M(z_0) \approx 0$. Following the comparison, we see that the expression for the linear dynamics in the presence of feedback is equivalent to that in the absence of feedback but with $\int_{t_{th}}^{t} q(t')dt'$ replaced by $z_0 t + \int_{t_{th}}^{t} q(t')dt'$ and the noise intensity $\sqrt{\beta}$ replaced by $\sqrt{\beta}/(1 + \tau z_0)$. By noting that only the intensity, that is, the modulus squared of $E(t)$ plays a role in the calculation of passage times, and using the explicit expression for $q(t)$ (9), we find that we can calculate passage times from the formulae developed in absence of feedback but with γ and β replaced by γ_{eff} and β_{eff} in (10) and (11). The replacement (10) takes into account that the exponential factor in (32) starts to amplify spontaneous emission noise for different parameter values than in the absence of feedback. This can be understood as a change in threshold current (see Section II-C) or alternatively as a change in the time after which noise amplification is possible. On the other hand, the replacement (11) amounts to a change in the noise intensity caused by the field reinjection. In our calculations, we will take in (10) and (11) $z_0 \approx \hat{K}(\tau)$. The inclusion of higher order terms from (30) is only important when $\tau\hat{K}(\tau)$ is such that the SPA looses its validity. When this happens the feedback is no longer weak and the contribution of the other poles cannot be neglected.

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E. Hernández-García, photograph and biography not available at the time of publication.

C. R. Mirasso, for a photograph and biography, see the June 1993 issue of volume 29 of this JOURNAL, p. 1630. (The photograph of C. R. Mirasso is mistakenly placed next to the biography of Pere Colet.)

K. A. Shore, photograph and biography not available at the time of publication.

M. San Miguel, for a photograph and biography, see the June 1993 issue of volume 29 of this JOURNAL, p. 1630.