Voter model dynamics in complex networks: Role of dimensionality, disorder, and degree distribution

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We analyze the ordering dynamics of the voter model in different classes of complex networks. We observe that whether the voter dynamics orders the system depends on the effective dimensionality of the interaction networks. We also find that when there is no ordering in the system, the average survival time of metastable states in finite networks decreases with network disorder and degree heterogeneity. The existence of hubs, i.e., highly connected nodes, in the network modifies the linear system size scaling law of the survival time. The size of an ordered domain is sensitive to the network disorder and the average degree, decreasing with both; however, it seems not to depend on network size and on the heterogeneity of the degree distribution.

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I. INTRODUCTION

Equilibrium order-disorder phase transitions, as well as nonequilibrium transitions and the kinetics of these transitions [1], have been widely studied by spin Ising-type models in different lattices [2]. Given the recent widespread interest in complex networks [3–6] the effect of the network topology on the ordering processes described by these models has also been considered [7–11]. In particular, models of opinion formation, or with similar social motivations, have been discussed when interactions are defined through a complex network [12–18].

A paradigmatic and simple model where a systematic study of network topology effects can be addressed is the voter model [19], for which analytical and well established results exist in regular lattices [20,21]. The dynamics of ordering processes for the voter model in regular lattices [22] is known to depend on dimensionality, with metastable disordered states prevailing for $d>2$ [23]. In this paper we address the general question of the role of network topology in determining if a system orders or not, and on the dynamics of the ordering process. Quenched disorder in regular networks is known to be able to modify equilibrium critical properties [24]. A different question is the role of disorder that changes the effective dimensionality of the network. For example, disorder in a small-world network is measured by a rewiring parameter p. The degree of disorder affects the critical properties of equilibrium phase transitions in these networks with a crossover temperature to mean-field behavior that depends on p [25]. A general related question that we address here is how disorder, of the type considered in a small-world network, modifies qualitative features of nonequilbrium dynamics on these networks. We do that in the voter model which does not incorporate thermal fluctuations. Specifically, analyzing the voter model in several different networks, we consider the role of the effective dimensionality of the network, of the degree distribution, and of the level of disorder present in the network.

The paper is organized as follows. In Sec. II we briefly review the basics as well as recent results on the voter model. Section III considers the voter model in scale-free (SF) networks [3] of different effective dimensionality, showing that voter dynamics can order the system in spite of a SF degree distribution. In Sec. IV we consider the role of network disorder by introducing a disorder parameter that leads from a structured (effectively one-dimensional) SF (SSF) network [26] to a random SF (RSF) network through a small-world [27] SF (SWSF) network. The role of the degree distribution is discussed, comparing the results on the SSF, RSF, and SWSF networks with networks with an equivalent disorder but without a power law degree distribution. Some general conclusions are given in Sec. V.

II. VOTER MODEL

The voter model [19] is defined by a set of "voters" with two opinions or spins $\sigma_i=\pm 1$ located at the nodes of a network. The elementary dynamical step consists in randomly choosing one node (asynchronous update) and assigning to it the opinion, or spin value, of one of its nearest neighbors, also chosen at random. In a general network two spins are nearest neighbors if they are connected by a direct link. Therefore, the probability that a spin changes is given by

$$P(\sigma_i \to -\sigma_i) = \frac{1}{2}\left(1 - \frac{\sigma_i}{k_i}\sum_{j\in\mathcal{V}_i}\sigma_j\right), \quad (1)$$

where k_i is the degree of node i, that is, the number of its nearest neighbors, and $\mathcal{V}_i$ is the neighborhood of node i, that is, the set of nearest neighboring nodes of node i. In the asynchronous update used here, one time step corresponds to updating a number of nodes equal to the system size, so that each node is, on the average, updated once. In our work we

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choose initial random configurations with the same proportion of spins +1 and −1.

The dynamical rule implemented here corresponds to a *node-update*. An alternative dynamics is given by a *link-update* rule in which the elementary dynamical step consists in randomly choosing a pair of nearest neighbor spins, i.e., a link, and randomly assigning to both nearest neighbor spins the same value if they have different values, and leaving them unchanged otherwise. These two updating rules are equivalent in a regular lattice, but they are different in a complex network in which different nodes have different number of nearest neighbors [28]. In particular, both rules conserve the ensemble average magnetization in a regular lattice, while in a complex network this is only a conserved quantity for link-update dynamics. Node-update dynamics conserves an average magnetization weighted by the degree of the node [28,29]. We restrict ourselves in this paper to the standard node-update for better comparison with the growing literature on the voter model in complex networks [30–33].

The voter model dynamics has two absorbing states, corresponding to situations in which all the spins have converged to the $\sigma_i=1$ or to the $\sigma_i=-1$ states. The ordering dynamics towards one of these attractors in a one-dimensional lattice is equivalent to the one of the zero temperature kinetic Ising model with Glauber dynamics. In more general situations, as in regular lattice of higher dimension or in a complex network, the ordering dynamics is still a zero-temperature dynamics driven by interfacial noise, with no role played by surface tension. A comparison of the voter model and the zero temperature Ising Glauber dynamics in complex networks [11] has been recently reported [33]. A standard order parameter to measure the ordering process in the voter model dynamics [22,30] is the average interface density ρ, defined as the density of links connecting sites with different spin values:

$$\rho = \left(\sum_{i=1}^{N} \sum_{j \in \mathcal{V}_i} \frac{1-\sigma_i\sigma_j}{2} \right) \Bigg/ \sum_{i=1}^{N} k_i . \tag{2}$$

In a disordered configuration with randomly distributed spins $\rho \simeq 1/2$, while when ρ takes a small value it indicates the presence of large spatial domains in which each spin is surrounded by nearest neighbor spins with the same value. For a completely ordered system, that is, for any of the two absorbing states, $\rho=0$. Starting from a random initial condition, the time evolution of ρ describes the kinetics of the ordering process. In regular lattices of dimensionality $d<2$ the system orders. This means that, in the limit of large systems, there is a coarsening process with unbounded growth of spatial domains of one of the absorbing states. The asymptotic regime of approach to the ordered state is characterized in $d=1$ by a power law $\langle\rho\rangle \sim t^{-1/2}$, while for the critical dimension $d=2$ a logarithmic decay is found $\langle\rho\rangle \sim (\ln t)^{-1}$ [22]. Here the average $\langle\cdot\rangle$ is an ensemble average.

In regular lattices with $d>2$ [20], as well as in small-world networks [30], it is known that the voter dynamics does not order the system in the thermodynamic limit of large systems. After an initial transient, the system falls in these cases in a metastable partially ordered state where coarsening processes have stopped: spatial domains of a given attractor, on the average, do not grow. In the initial transient of a given realization of the process, ρ initially decreases, indicating a partial ordering of the system. After this initial transient ρ fluctuates randomly around an average plateau value ξ. This quantity gives a measure of the partial order of the metastable state since $l=\xi^{-1}$ gives an estimate of the average linear size of an ordered domain in that state. In a finite system the metastable state has a finite lifetime: a finite size fluctuation takes the system from the metastable state to one of the two ordered absorbing states. In this process the fluctuation orders the system and ρ changes from its metastable plateau value to $\rho=0$. Considering an ensemble of realizations, the ordering of each of them typically happens randomly with a constant rate. This is reflected in the late stage exponential decay of the ensemble average interface density from its plateau value

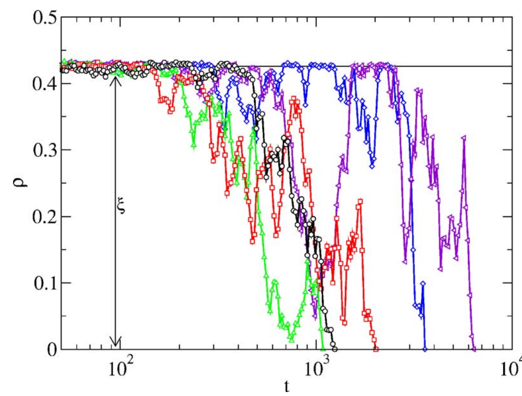

FIG. 1. (Color online) Evolution of the interface density ρ, Eq. (2), for ten realizations of Barabási-Albert networks of system size $N=10\,000$ and average degree $\langle k\rangle=8$. Note the plateau value ξ indicated by the solid line.

$$\langle\rho\rangle \propto e^{-t/\tau}, \tag{3}$$

where τ is the survival time of the partially ordered metastable state. Note then that the average plateau value ξ (see Fig. 1) has to be calculated at each time t, averaging only over the realizations of the ensemble that have not yet decayed to $\rho=0$.

The survival time τ, for a regular lattice in $d=3$ [20] and also for a small-world network [30], is known to scale linearly with the system size N, $\tau \sim N$, so that the system does not order in the thermodynamic limit. More recently the same scaling has been found for random graphs [32,33] while a scaling $\tau \sim N^{0.88}$ has been numerically found [28,33] for the voter model in the scale-free Barabási-Albert network [34]. This scaling is compatible with the analytical result $\tau \sim N/\ln N$ reported in Ref. [32]. Other analytical results for random networks with arbitrary power law degree distribution are also reported in Ref. [32]. We note that a conceptually different, but related quantity, is the time τ_1 that a finite system takes to reach an absorbing state when coarsening processes are at work, that is in situations in which the system would order in the thermodynamic limit. The time scale

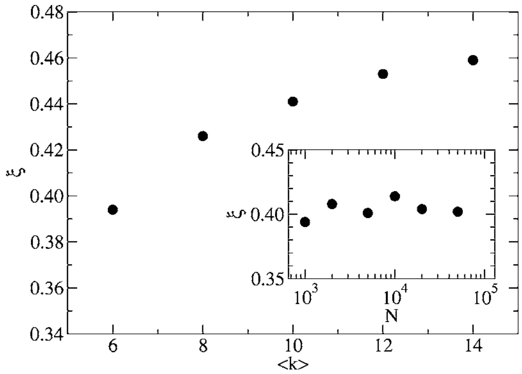

FIG. 2. Average plateau height ξ for BA networks of system size $N=10\,000$ for different average degree. Inset: Plateau height dependence on network size with average degree $\langle k\rangle=6$. Data are averaged over 1000 realizations.

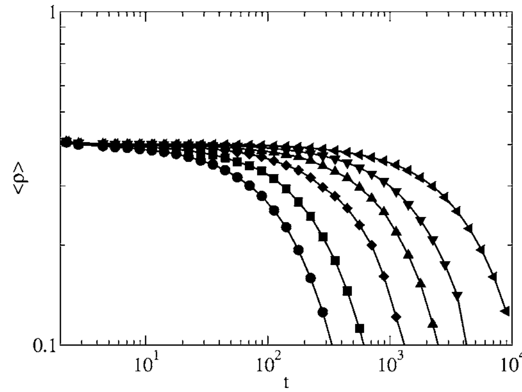

FIG. 3. Evolution of the average interface density $\langle\rho\rangle$, Eq. (2), in BA networks of different sizes (increasing from left to right: $N=1000$, 2000, 5000, 10 000, 20 000, and 50 000). Data are averaged over 1000 realizations for $\langle k\rangle=6$.

is here determined by the presence of boundaries, while τ is determined by the likelihood of a finite size fluctuation. The time τ_1 is known to scale as $\tau_1\sim N^2$ for a regular $d=1$ lattice and $\tau_1\sim N\ln N$ for a regular $d=2$ lattice. We will be here mostly concerned with situations in complex networks of large dimensionality for which there is no coarsening, so that the relevant characteristic time is the survival time τ.

In the next sections we discuss the time evolution of ρ and the characteristic properties of the plateau value ξ and survival time τ for asynchronous node-update voter dynamics in a variety of different complex networks.

III. DIMENSIONALITY AND ORDERING: VOTER MODEL IN SCALE-FREE NETWORKS

One of simplest models that displays a scale-free degree distributions is the well known Barabási-Albert network [34]. In this model, the degree distribution follows a power law with an exponent $\gamma=3$, the path length grows logarithmically with the system size [3] while the clustering coefficient decreases with system size [35]. It has been shown that critical phenomena on this class of networks are well reproduced by mean-field calculations valid for random networks [36]. Thus we will consider in the remainder the Barabási-Albert (BA) networks as a representative example of a random scale-free (RSF) network [37]. Results for the voter model in the BA network are shown in Figs. 1–3. The qualitative behavior that we observe is the same as the one-described above for regular lattices of $d>2$ or also observed in a small-world network [30]: The system does not order but reaches a metastable partially ordered state. The interface density ρ for different individual realizations of the dynamics is shown in Fig. 1. In this figure we see examples of how finite size fluctuations take the system from the metastable state with a finite plateau value of ρ to the absorbing state with $\rho=0$. The level of ordering in this finite lifetime metastable state can be quantified by the plateau level ξ shown in Fig. 2. We obtain the plateau level ξ from the prefactor in the exponential decay [Eq. (3)] of the average interface density. We find that the level of ordering decreases significantly with the average degree of the network, a result consistent with the idea that total ordering is more easily achieved for effective lower dimensionality. On the other hand the level of ordering is not seen to be sensitive to the system size, for large enough sizes.

The survival time τ can be calculated from the ensemble average interface density $\langle\rho\rangle$ as indicated in Eq. (3). The time dependence of $\langle\rho\rangle$ for systems of different size (Fig. 3) shows an exponential decrease for which the result mentioned above $\tau\sim N^{0.88}$ can be obtained [28]. We note that the value τ is found to be independent of the average degree of the network and that a linear scaling $\tau\sim N$ is obtained if a link-update dynamics is used [28].

The fact that the presence of hubs in the BA network is not an efficient mechanism to order the system might be counterintuitive, in the same way as the presence of long range links in a small-world network is also not efficient to lead to an ordered state. However, in both cases the effective dimensionality of the network is infinity and the result is in agreement with what is known for regular lattices with $d>2$. A natural question is then the relevance of the degree distribution versus the effective dimensionality in the ordering dynamics. To address this question we have chosen to study the voter model dynamics in the structured scale-free network introduced in Ref. [26]. The SSF networks are nonrandom networks with a power law degree distribution with exponent $\gamma=3$ but with an effective dimension $d=1$ [38,39].

Our results for the time dependence of the average interface density in the SSF network are shown in Fig. 4. For comparison the results for a regular $d=1$ network are also included. For both networks we observe that the system orders with the average interface density decreasing with a power law with characteristic exponent 1/2,

$$\langle\rho\rangle \sim t^{-1/2}. \tag{4}$$

The only noticeable difference is that the SSF network has a larger number of interfaces at any moment, but the ordering process follows the same power law. Additionally we find that for a finite system the time τ_1 to reach the absorbing state scales as $\tau_1\sim N^2$, as also happens for the regular $d=1$ network.

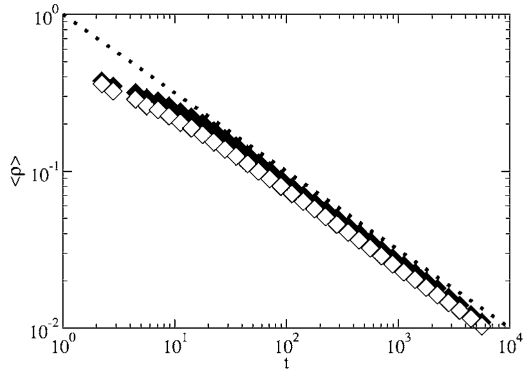

FIG. 4. Evolution of the average interface density $\langle\rho\rangle$, Eq. (2), in one-dimensional systems with system size $N=10\,000$ and average degree $\langle k\rangle=8$: (solid symbols) with scale-free topology and (empty symbols) regular lattice. For reference, the dotted line is a power law with exponent $-1/2$. Average over 1000 realizations.

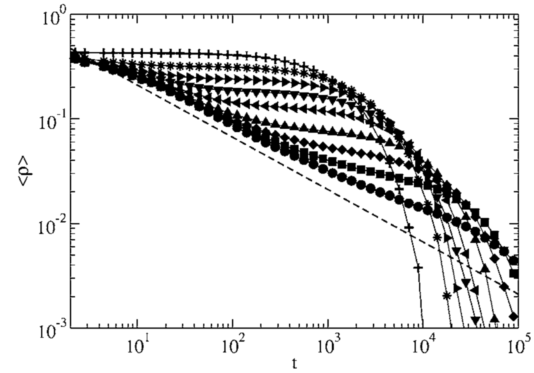

FIG. 5. Evolution of the mean interface density $\langle\rho\rangle$, Eq. (2), for SWSF networks of system size $N=10\,000$, $\langle k\rangle=8$, with different disorder parameter p (increasing from circles to stars: $p=0.0004$, 0.001, 0.002, 0.004, 0.01, 0.02, 0.04, and 0.1), and comparison with the BA network (+). For reference, the dotted line is a power law with exponent $-1/2$. Data averaged over 100 realization for $p<0.01$ and over 1000 realizations for $p\geqslant 0.01$.

The network is completely ordered when the last interface disappears. At this point, the density is simply $(N\langle k\rangle)^{-1}$, where $N\langle k\rangle$ is the total number of links in the network. Since the interface density decreases as $\langle\rho\rangle\sim t^{-1/2}$, then the time to order τ_1 is given by

$$(N\langle k\rangle)^{-1}=\tau_1^{-1/2}, \tag{5}$$

leading to $\tau_1\sim N^2$.

Therefore we conclude that the effective dimensionality of the network is the important ingredient in determining the ordering process that results from a voter model dynamics, while the fact that the system orders or falls in a metastable state is not sensitive to the degree distribution.

IV. ROLE OF NETWORK DISORDER AND HETEROGENEITY OF THE DEGREE DISTRIBUTION

Once we have identified in the previous section the crucial role of dimensionality we now address the role of network disorder and heterogeneity of the degree distribution in quantitative aspects of the voter model dynamics. We do that by considering a collection of complex networks in which the system falls into partially disordered metastable states, except for the regular one-dimensional lattice and SSF networks of Ref. [26] in which the system shows genuine ordering dynamics.

(1) Structured scale-free (SSF) network as defined in the previous section.

(2) Small-world scale-free (SWSF) network. This is defined by rewiring with probability p the links of a SSF network. In order to conserve the degree distribution of the unperturbed ($p=0$) networks, a randomly chosen link connecting nodes i, j is permuted with that connecting nodes k, l [40].

(3) Random scale-free (RSF) network. This is defined as the limit $p=1$ of the SWSF network. The RSF network shares most important characteristics with the BA network.

By changing the parameter p from $p=0$ (SSF) to $p=1$ (RSF) we can analyze how increasing levels of disorder affect the voter model dynamics while keeping a scale-free degree distribution. On the other hand, the consequences of the heterogeneity of the degree distribution characteristic of SF networks can be analyzed by comparing the voter model dynamics on these networks with networks with the same level of disorder and a non-SF degree distribution. These other networks are constructed introducing the same disorder parameter p, but starting from a regular $d=1$ network; namely we consider the following.

(4) Regular $d=1$ network that can be compared with a SSF network.

(5) Small-world (SW) network defined by introducing the rewiring parameter p in the regular network as in the prescription by Watts and Strogatz [27]. The SW network can be compared with the SWSF network.

(6) Random (RN) network corresponding to the limit $p=1$ of the SW network.

Likewise, one can consider a random network with an exponential degree distribution. The exponential (EN) network is constructed as in the BA prescription but with random instead of preferential attachment of the new nodes. These two random networks, RN and EN, can be compared with the RSF network.

A. Role of disorder

Figure 5 shows the evolution of the mean interface density for SWSF networks with different values of the disorder parameter p. It shows how by varying p one smoothly interpolates between the results for the SSF network and those for a RSF network. In general, increasing network randomness by increasing p the system approaches the behavior in a BA network, causing it to fall in a metastable state of higher disorder, but with finite size fluctuations causing faster ordering. This trend is quantitatively shown in Figs. 6 and 7 where the survival time τ and plateau level ξ for SWSF networks are plotted as a function of the disorder parameter p. We observe that τ and the size of the ordered domains $l=\xi^{-1}$ decrease with p but without following any clear power law.

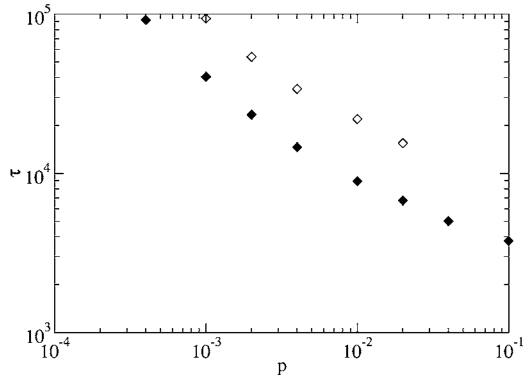

FIG. 6. Survival times τ for a SWSF networks of system size $N=10\,000$ and $\langle k\rangle=8$, with different disorder parameter p (solid symbols). For comparison results for SW networks are also included (empty symbols). Average over 1000 realizations.

As a general conclusion, when extrapolating to $p=1$, we find that $\tau_{\mathrm{SWSF}}>\tau_{\mathrm{RSF}}$ and $l_{\mathrm{SWSF}}>l_{\mathrm{RSF}}$.

The role of increasing disorder in the network can also be analyzed in networks without a scale-free degree distribution by considering SW networks with different values of the rewiring parameter p. The survival time τ and plateau level ξ for SW networks are also plotted in Figs. 6 and 7. We observe that the effect of disorder is qualitatively the same for SW as for SWSF networks [41]. Extrapolating the results in Figs. 6 and 7 to $p=1$ where the SW network becomes a RN network we find that $\tau_{\mathrm{SW}}>\tau_{\mathrm{RN}}$ and $l_{\mathrm{SW}}>l_{\mathrm{RN}}$.

B. Role of degree distribution

To address the question of the role of the degree distribution of the network in the voter model dynamics we compare the evolution in networks with a scale-free degree distribution with the evolution in equivalent networks but with a degree distribution involving a single scale. A first comparison was already made between the dynamics in a regular $d=1$ network and the SSF network of Ref. [26] (Fig. 4). This is included for reference in Fig. 8 where we compare the evolution of the mean interface density in a SWSF network with the evolution in a SW network with the same level of disorder. We observe for the SWSF network a similar plateau value (similar but slightly more disordered state) at any time before the exponential decay of $\langle\rho\rangle$ which is faster for the SWSF than for the SW networks. Finite size fluctuations that order the system seem to be more efficient when hubs are present, causing complete ordering more often, and therefore a faster exponential decay of $\langle\rho\rangle$. These claims are made quantitative in Figs. 6 and 7 where it is shown that $\tau_{\mathrm{SW}}>\tau_{\mathrm{SWSF}}$ and $l_{\mathrm{SW}}\simeq l_{\mathrm{SWSF}}$. In addition, extrapolating to the limit $p=1$ we have that $\tau_{\mathrm{RN}}>\tau_{\mathrm{RSF}}$ and $l_{\mathrm{RN}}\simeq l_{\mathrm{RSF}}$

It is also interesting to compare the dependence with system size of the voter model dynamics in SW [30] and SFSW networks: The time dependence of the mean interface density for a SWSF network with an intermediate fixed value of p is shown in (Fig. 9). The qualitative behavior is the same as the one found for SW networks. However, the survival times shown in Fig. 10 deviates consistently from the linear power

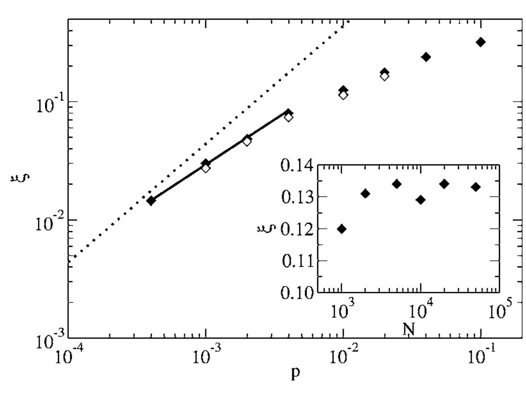

FIG. 7. Average plateau heights ξ for SWSF networks (solid symbols) of system size $N=10\,000$ and $\langle k\rangle=8$, with different disorder parameter p. For reference, dotted line is a power law $\xi\sim p$ while the solid line $\xi\sim p^{0.76}$. For comparison results for SW networks are also included (empty symbols). Average over 100 realization for $p<0.01$ and over 1000 realizations for $p\geqslant0.01$. Inset: Plateau heights for SWSF networks of different system size N. Average over 1000 realizations, with $p=0.01$ and $\langle k\rangle=8$.

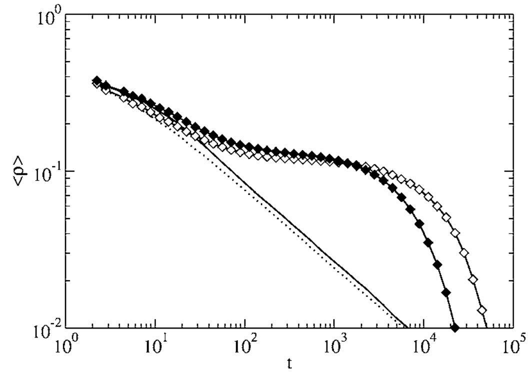

FIG. 8. Evolution of the average interface density $\langle\rho\rangle$, Eq. (2), for SFSW (solid symbols) and SW networks (empty symbols) of system size $N=10\,000$ and $\langle k\rangle=8$, with the same level of disorder $p=0.01$. Average over 1000 realizations. For reference, the evolution of the average interface density $\langle\rho\rangle$ for the one-dimensional lattice (dotted line) and the SSF (solid line) is also plotted.

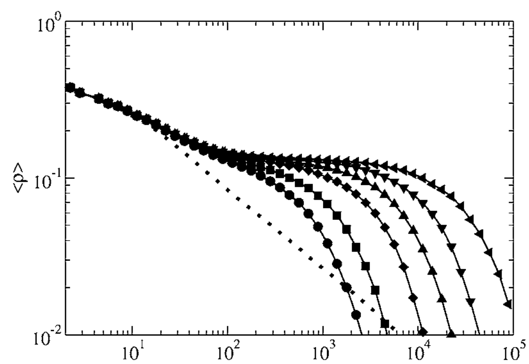

FIG. 9. Evolution of the average interface density $\langle\rho\rangle$, Eq. (2), for SWSF networks of different system size N (increasing from left to right: $N=1000$, 2000, 5000, 10 000, 20 000, 50 000). Average over 1000 realizations, with $p=0.01$ and $\langle k\rangle=8$.

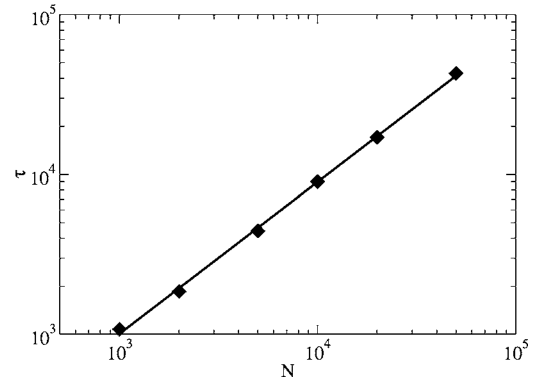

FIG. 10. Survival times τ for SWSF networks of different system size N. Average over 1000 realizations, with $p=0.01$ and $\langle k\rangle=8$. The solid line is a power law fit $\tau \sim N^{0.95}$.

law $\tau \sim N$ found for SW networks [30]. This deviation might possibly have the same origin as the deviation from the linear power law observed for BA networks, that is the lack of conservation of magnetization in the node-update dynamics of the voter model in a complex network [28]. This nonconservation becomes much more important in a SWSF network than in a SW network because of the high heterogeneity of the degree distribution. On the other hand we note that the analytical results for survival times in Ref. [32] apply only to uncorrelated networks and therefore do not help us in understanding our numerical result for SWSF networks. We also mention that the plateau level ξ for SWSF networks does not show important dependence with system size (see inset of Fig. 7).

The role of the heterogeneity of the degree distribution can be further clarified considering the limit of random networks $p=1$ where the SW network becomes a RN network and the SWSF network becomes a RSF network essentially equivalent to the BA network. The evolution for the mean interface density for different random networks is shown in Fig. 11. We find again that when there are hubs (large heterogeneity of the degree distribution) there is a faster exponential decay of $\langle\rho\rangle$, so that ordering is faster in BA networks than in RN or EN networks, while the plateau level or level

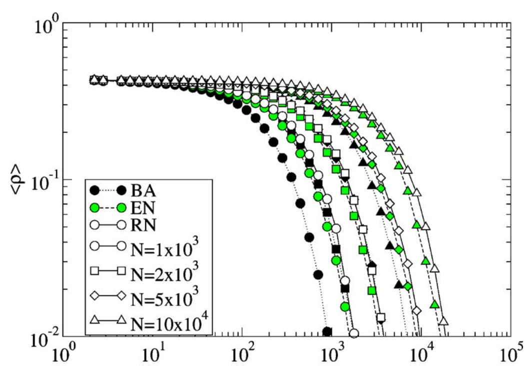

FIG. 11. (Color online) Evolution of the average interface density $\langle\rho\rangle$, Eq. (2), for RN and EN networks of different sizes N. BA network of size 10 000 is also shown for comparison. Average over 1000 realizations, and $\langle k\rangle=8$.

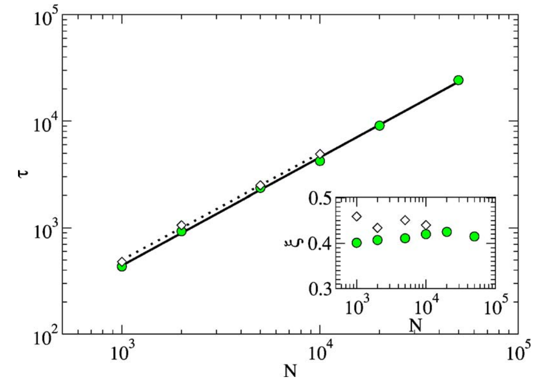

FIG. 12. (Color online) Survival times τ for RN (diamonds) and EN (circles) networks of different sizes N. Average over 1000 realizations, with $\langle k\rangle=8$. For reference, the solid and the dotted lines correspond to a linear dependence of the survival time with system size $\tau \sim N$. Inset: Plateau heights for the same networks.

of order in that state does not seem to be sensitive to the degree distribution. This coincides with the extrapolation to $p=1$ of the data in Figs. 6 and 7 which indicate that $\tau_{\mathrm{RN}} > \tau_{\mathrm{RSF}}$ and $l_{\mathrm{RN}} \simeq l_{\mathrm{RSF}}$. Our results for the system size dependence of the survival times and plateau levels for RN and EN networks are shown in Fig. 12. The size of the ordered domains $l=\xi^{-1}$ is again found not to be sensitive to system size. The survival times for RN and EN networks follow a linear scaling $\tau \sim N$ in agreement with the prediction in Ref. [32]. We recall that, as discussed earlier, in random networks with scale-free distribution such as the BA network a different scaling is found ($\tau \sim N^{0.88}$) [28,33] compatible with the prediction $\tau \sim N/\ln N$ [32].

V. CONCLUSIONS

We have analyzed how the ordering dynamics of the voter model is affected by the topology of the network that defines the interaction among the nodes. First we have shown that the voter model dynamics orders the system in a SSF network [26], which is a scale-free network with an effective dimension $d=1$. This result, together with the known result that in regular lattices the voter model orders in $d \leqslant 2$, suggests that the effective dimension of the underlying network is a relevant parameter to determine whether the voter model orders. In fact we find the same scaling law for the ordering process in a regular $d=1$ network than in a SSF network with the density of interfaces decreasing as $\langle\rho\rangle \sim t^{-1/2}$ in both cases. This seems to indicate that such laws are not sensitive to the degree distribution which is a delta function in the $d=1$ regular network while it has power law behavior for the SSF network. The relevance of the effective dimensionality of different scale-free networks has also been observed in other dynamical processes [14,38,42,43].

Second, we have introduced standard rewiring algorithms to study the effect of network disorder. Disorder is characterized by a parameter p that changes continuously from a one-dimensional network to a random network. We have studied this variation in the case of networks with a degree distribution characterized by a single scale (from a regular

$d=1$ network to a RN network through SW networks) or by a scale-free distribution (from a SSF to a RSF network through SWSF networks). In general we find that network disorder decreases the lifetime of metastable disordered states so that the survival time to reach an ordered state in finite networks is smaller,

$$\tau_{\text{SWSF}} > \tau_{\text{RSF}}, \quad \tau_{\text{SW}} > \tau_{\text{RN}}.$$

Likewise, the average size of ordered domains in these metastable states decreases with increasing disorder,

$$l_{\text{SWSF}} > l_{\text{RSF}}, \quad l_{\text{SW}} > l_{\text{RN}}.$$

Third, the heterogeneity of the degree, that is the presence of nodes with rather different number of links, also facilitates reaching an absorbing ordered configuration in finite networks by decreasing the survival time. We have analyzed this question comparing networks with same level of disorder (same value of the parameter p) but which have or do not have a scale-free distribution of degree. We conclude that finite size fluctuations ordering the system are more efficient when there are nodes with a very large number of links ("hubs") in the network, so that

$$\tau_{\text{SW}} > \tau_{\text{SWSF}}, \quad \tau_{\text{RN}} > \tau_{\text{RSF}}.$$

The presence of hubs also invalidates the scaling law for the survival time $\tau \sim N$ found in SW and RN networks. However we did not find differences in the average size of ordered domains depending on the heterogeneity of the degree distribution,

$$l_{\text{SW}} \simeq l_{\text{SWSF}}, \quad l_{\text{RN}} \simeq l_{\text{RSF}}.$$

In summary, we find for the different classes of networks considered in this work that

$$\tau_{\text{SW}} > \tau_{\text{RN}} > \tau_{\text{RSF}},$$

$$l_{\text{SW}} \simeq l_{\text{SWSF}} > l_{\text{RN}} \simeq l_{\text{RSF}}.$$

In general our results illustrate how different features (dimensionality, order, heterogeneity of the degree) of complex networks modify key aspects of a simple stochastic dynamics.

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