Cooperation and the Emergence of Role Differentiation in the Dynamics of Social Networks[1]

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By means of extensive computer simulations, the authors consider the entangled coevolution of actions and social structure in a new version of a spatial Prisoner's Dilemma model that naturally gives way to a process of social differentiation. Diverse social roles emerge from the dynamics of the system: *leaders* are individuals getting a large payoff who are imitated by a considerable fraction of the population, *conformists* are unsatisfied cooperative agents that keep cooperating, and *exploiters* are defectors with a payoff larger than the average one obtained by cooperators. The dynamics generate a social network that can have the topology of a small world network. The network has a strong hierarchical structure in which the leaders play an essential role in sustaining a highly cooperative stable regime. But disruptions affecting leaders produce social crises described as dynamical cascades that propagate through the network.

INTRODUCTION

Social traps (Platt 1973) are situations in which rational individual choices result in an undesirable collective outcome for the social group. A well-known example is the problem of establishing cooperation in a social

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group (Axelrod 1984) or, more generally, problems of public goods (Olson 1965; Hardin 1968). Understanding collective social behavior, once individual attitudes are known, requires taking into account the interactions among the individuals of the group and acknowledging that these interactions are mediated by a network of social relations (Granovetter 1973, 1978). Such a network constitutes the social structure of the group. The embeddedness of the interactions in the social structure (Granovetter 1985) has been identified as an important factor in explaining the evolution of cooperation (Macy and Skvoretz 1998). In the same way that the actions of individuals are affected by the social network, it has been documented that the network is not an exogenous structure but is created by individual choices (Lazer 2001). However, there are not many specific models of social dynamics that explicitly incorporate the concept of coevolution of individual and network (Lazer 2001). In fact, in the long-term research agenda posed by Macy (1991), a central point is that the structure of the network should not be considered as given, but should be seen as variable. Macy poses the question of how social structure might evolve in tandem with the collective action it enables. This question goes beyond models in which some network evolution is decoupled from the evolution of the actions of the individuals in the group. In the context of reciprocal altruism and the building of cooperation, this general question was implicitly considered within a game theory simulation model called the social evolution model (SEM; Zeggelink, de Vos, and Elsas 2000; de Vos, Smaniotto, and Elsas 2001). In this article, we address the problem of coevolution in a version of the Prisoner's Dilemma (PD; Rapoport and Chammah 1965) in which players interact through a network that adapts to the results of the game and therefore to the actions of the players. We focus on the resulting type of social structure (cohesive group vs. social hierarchy), as well as on the dynamical mechanisms needed to produce the topological properties of the network of interactions that stabilize a collective cooperative behavior.

An important finding from our analysis—extensively based on computer simulations (Zimmermann, Eguíluz, and San Miguel 2001)—is the *emergence* of a process of social differentiation together with the building-up of a network with hierarchical relations. Starting from random partnership among equivalent individuals, a social structure emerges. In this emerging structure, the topology of the network of social relations identifies individuals with different social roles: leaders, conformists, and exploiters. These roles have been spontaneously selected during the complex

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adaptive evolution of the social group that entails a learning process. This social structure sustains global cooperation, with exploiters surviving in hierarchical chains in the network. This result is at variance with other simulation models, such as SEM, in which partner selection leads to the formation of cohesive egalitarian clusters (Zeggelink et al. 2000; de Vos et al. 2001): the emergence of these groups in a socially segmented population, with the exclusion of free riders, seems to be the basis of the survival of cooperation in these other studies. In considering the topology of the coevolved network that sustains cooperation, we find a hierarchical network with an exponential connectivity distribution. Our results show that clustering is not needed to sustain cooperation. However, the additional inclusion of local neighboring partner selection in the model generates the celebrated small world connectivity (Watts 1999*a*) of the network.

From a philosophical point of view, the concept of *emergence* is used in contemporary sociology in contradictory ways, its proper meaning being debatable (Sawyer 2001). We use it here as it appears in multiagent models of social systems (Gilbert and Conte 1965; Schelling 1978; Epstein and Axtell 1996; Axelrod 1997). In this context, it refers, as is often the case in the natural sciences, to complex dynamical behavior or properties that cannot be reduced to or predicted from a detailed description of the units that compose a system, so that the reductionist hypothesis does not imply a constructionist one (Anderson 1972). These ideas put forward a hierarchical structure of science in which different concepts and descriptions are needed at different levels. The key idea is that sociology cannot be reduced to psychology, as molecular biology is not just applied chemistry (Anderson 1972). From this perspective, and recognizing the limitations of game theory in describing human interactions, we learn much from a metaphor like the PD when considering emergent properties that are not simply linked to special features of individual human behavior.

A canonical example of emergence is the V shape of bird flocks (Sawyer 2001). The shape of the flock and the fact that a particular bird plays the role of a leader, with other birds lining up behind it, is a nontrivial result of simple interaction rules, but the leader is neither genetically determined nor externally appointed. Our results parallel this example, showing how equivalent agents confronted with the choice of an action (cooperating or defecting) and with the possibility of choosing partners, differentiate and acquire different roles while building up a social structure. This process is the result of interactions among neighbors that determine an entangled coevolution of the choice of actions and of the social network. A particularly enlightening example of the general process that we address here is the cooperative relations among researchers in a given scientific field. A given researcher might choose to work with or not work with another scientist and, depending on the degree of success of this collaboration, he might search in the community to find other scientists with whom a profitable cooperative relation can be alternatively established. This coevolution process results in a network of collaborations in which different scientists play different roles.

Our results might be compared with other studies of group formation beyond those associated with the SEM. In Carley's theory of group stability (1991), as also happens in our simple model of equivalent agents, individuals assume multiple roles. In addition, these diverse roles produce, at the group level, group stability or change depending on the social structure. However, while Carley's theory is based on the sharing of knowledge, our model is not. Stability is in our case strictly dependent on the adapting structure of the network of interactions.

In the remainder of this introduction, we give a brief discussion of the main components of our analysis. These components are models of cooperation, models of social networks, and the implications of the will of the agents that manifests itself in the ability to make choices. The second section of the article presents our model and some general conclusions from its analysis. The main body of our computer simulation results are summarized in a third section. We conclude with a discussion of the results and limitations of our study.

Routes to Cooperation

Why do people cooperate? Why is cooperation empirically observed when there is a conflict between self-interest and the common good? Classical answers to these questions are formulated in terms of kin (Hamilton 1964), group selection (Wilson and Sober 1994), or cooperation based upon reciprocity (Trivers 1971; Axelrod and Hamilton 1981; Axelrod 1984). The first two answers are biologically based—the former on the increased biological fitness of kinship and the latter on the adaptive success of groups of cooperators. However, the idea of social kin selection has also been put forward (Riolo, Cohen, and Axelrod 2001): cooperation can arise from similarity, and tags that identify such similarity can be based on cultural attributes instead of being genetically determined. Indeed, cultural transmission has been invoked as a potential reason for the prevalence of cooperation in human populations (Mark 2002). Another route to cooperation, based on a reputation score for each individual and information sharing (Nowak and Sigmund 1998), also emphasizes the cultural forces present in society. Cooperation grounded on reputation can be seen, however, as a form of indirect reciprocity.

Cooperation based upon reciprocity is often formalized through the iterated PD (Rapoport and Chammah 1965). The game theoretical formulation of the PD shows that two perfectly rational agents interacting once and confronted with the choice of cooperating or defecting would both choose to defect, which corresponds to the Nash equilibrium of the game. When the game is repeated or iterated, the Folk theorem classifies the many possible outcomes that can be sustained, particularly full cooperation. It is the *shadow of the future* in the repeated interaction that makes cooperation sustainable (Axelrod 1984). Although basic concepts and results on the PD were well known in game theory (Binmore 1998), the evolutionary ideas for the selection of an equilibrium pioneered by Axelrod (1984, 1997) have been extremely influential. They have lead to the consideration of the virtue of different strategies and to the concept of evolutionary stable strategies. It is now well accepted that dynamical models of cultural evolution and social learning hold a greater chance for success than models merely based on rational choice. This body of knowledge has been reviewed by Hoffmann (2000), while some recent advances in the theory have been reviewed by Axelrod (2000).

Generally speaking, locality is not taken into account in the different studies of models of cooperation mentioned so far. Individuals or players interact globally through continuous changes of random pairings among them. However, local spatial interactions introduce another possibility of reciprocity that is not based on history-dependent or backward-looking strategies. In this line of thinking, spatial PD games have been introduced (Axelrod 1984; Nowak and May 1992) and further generalized by Lindgren and Nordahl (1994). Spatial games and local interactions were also introduced in economic contexts by Blume (1993) and Ellison (1993). Nowak and May (1992, 1993) and Nowak, Bonhoeffer, and May (1994*a*, 1994*b*) started by considering a group of individuals placed at the nodes of a regular two-dimensional square lattice. What they showed is that cooperation might arise from spatially distributed interactions: if agents play only with a local neighborhood in the lattice, clusters of cooperators may survive the interaction with defectors, and cooperation may be sustained. This can happen by bypassing any consideration of memory and strategy, in situations when noncooperative behavior would prevail in the global game. Admittedly, the game theory formulation of cooperation neglects interpersonal relations driven by emotional processes (Lawler and Yoon 1998). In spatial PD games, the fundamental relationship is that of exchange conditioned by structural position. Still, the basic mechanism of imitating successful neighbors is introduced. This mechanism is certainly prevalent in many human interactions. It must be pointed out that the analysis of spatial games, especially when searching for collective emergent behavior in societies with a large number of individuals, requires the use of intensive computer simulations. Some analytical results, however, have been obtained (Schweitzer, Behera, and Muhlenbein 2002), especially in simplified one-dimensional models (Eshel, Samuelson, and Shaked 1998).

There are other routes to cooperation, among which we can mention optional participation (Hauert et al. 2002) and stochastic collusion (Macy 1991). Optional or volunteer participation is a mechanism that incorporates, in addition to cooperators and defectors, players (*loners)* that refuse to participate in the game. This has been shown to be an effective mechanism to escape from the social dilemma without invoking any form of reciprocity. Incorporating locality and spatial interactions, cooperators also tend to fare better (Szabo and Hauert 2002). Stochastic collusion stems from a forward-looking route to learning and adaptive behavior (Macy and Flache 2002): stochastic search by adaptive individuals in response to immediate outcomes allows escape from the noncooperative social trap. The beneficial implications of locality (*bounded social space*) on sustaining cooperation is another component of this mechanism (Macy 1991).

Social Networks

A new field of opportunities for modeling is opened when considering the network of social interactions—links between individuals are not randomly established, but are dependant on neighborhood or interest relationships. As a consequence, clusters and stable linkages are developed, giving way to new aspects of cooperation in which interactions among individuals have an important effect, as they are crucial to the performance of the system as a whole (Holland et al. 1986).

The consideration of the spatial version of the PD discussed above is a step in the direction of considering a social network. But a social network is different from a regular two-dimensional lattice, just as social interactions are different from a continuous random pairing of individuals. The popular phenomenon of *six degrees of separation* (Guare 1990) that follows from the early experiments of Milgram (1967) demonstrates that if the average separation between two individuals is given by only six intermediate acquaintances (six links), social networks radically differ from the regular ones often considered in spatial games. Such a small world effect has been repopularized by Watts and Strogatz (1998). Beyond the physical and topological properties of the network, it is also important to recall that, in economic terms, a social network has been associated with a social capital (Coleman 1988), in the sense that it gives the basis for stable, repeated social interactions.

The spatial version of the PD had been revisited by Axelrod and coworkers (Axelrod 2000; Cohen, Riolo, and Axelrod 2001) in trying to understand the role of social structure and geographically based networks in the maintenance of cooperation. They have shown that geographically dispersed social networks are efficient in maintaining cooperation, provided the links are stable. They conclude, therefore, that the important aspect is not clustering (i.e., locality or correlation of linkage patterns), but what they label as context preservation, that is, the continuity of interactions. Another interesting contribution in this context is the study of Buskens and Weesie (2000) on the role of social structure in supporting cooperation via reputation. By including a social structure, this study goes beyond those of Nowak and May (1992) and Riolo et al. (2001), and it shows that these two aspects—social structure and reputation—reinforce each other in building up a cooperative collective behavior (Axelrod 2000). Reputation established via information sharing leads to sustained cooperation in social structures that are less rigid than the ones fixed by geographic positions, therefore adding a higher degree of freedom.

From the early mathematical analysis of random networks (Erdos and Renyi 1959), there is a fast-growing literature devoted to uncovering the topological properties of technological, biological, and social networks (Wasserman and Faust 1984; Watts and Strogatz 1998; Watts 1999*a*, 1999*b*; Barabasi and Albert 1999; Amaral et al. 2000; Newman 2001; Newman, Strogatz, and Watts 2001), partially reviewed by Albert and Barabási (2002), Barabási (2002), and Watts (2003). Strogatz (2001) emphasized that although many networks present similar characteristics, it is important to note that differences do exist. The properties of a network that are usually analyzed are average distance between nodes, clustering, and degree distribution. A short average distance, that is, the fact that the number of steps needed to connect any pair of individuals in a social network is few, is what originally characterized a small world effect. But it was long ago recognized (Wasserman and Faust 1984) that social networks also show a high tendency to form cliques, that is, that friends of friends are also friends. Such a high clustering property and a small path length characterize the small world networks formalized by Watts (1999*a*, 1999*b*). These small world networks are halfway between random and regular networks. The degree distribution is the probability distribution of the number of links or connections of a node of the network. Many networks have a power-law distribution, which implies a scale-free distribution of degree (Barabasi and Albert 1999). However, it has been reported (Amaral et al. 2000) that social networks do not generally display such heterogeneity in their connectivity, but instead show a more homogeneous degree distribution, with the number of connections being characterized by a narrow distribution around a single scale.

Our concern in this article is not the characterization of the topological properties obtained in examining a snapshot of a social network. Rather, the question addressed is how the network is dynamically formed or how a given network structure is reached after social agents interact for a long time (Zimmermann, Eguíluz, and San Miguel 2004). As Watts indicates (Watts 1999*a*), networks affect the dynamics of the system in a passive and an active way. Examples of the passive way are the spatial version of the PD game and the variants thereof previously mentioned, or the study of a PD game in small world network (Abramson and Kuperman 2001). We are here interested in the active manner in which the network of connections evolves by the will of the agents. We seek to unveil possible dynamic mechanisms to achieve a small world connectivity.

Making Choices and Beyond: Spontaneous Social Differentiation

Incorporating the will of the agents seems to be a crucial component of any model of human behavior. In the setting of a spatial version of the PD game in a social network, the agents should have some way of choosing their actions (cooperate or defect) and choosing their partners. Making choices is an instrument for learning in an adaptive evolution.

Partner and action selection has been taken into account in the iterated PD as reviewed by de Vos et al. (2001), but without considering local spatial interactions and also without reference to a social network. The volunteering mechanism of Hauert et al. (2002) does not give a choice of action, since agents refusing to participate are fixed from the outset, but it is an indirect mechanism to determine who interacts with whom. More closely related to changes in the network structure, or to the choice of partners, is the mechanism of allowing players to exit from an unsatisfactory relationship with partners, as discussed by Axelrod (2000) on the grounds of the Edk-Group's analysis of a set of 15 strategies related by the possibility of opting out (Edk-Group 2000). The possibility of exiting generally reinforces cooperation, but again, this study does not include local interactions and social networks.

The formation of a social network based on individual decisions has been studied by Bala and Goyal (2000); however, the individuals forming the network do not have dynamics on top of the network of interactions: there is no game mediated by the interactions defined by the network. Skyrms and Pemantle (2000) do consider the simultaneous evolution of actions and the structure of the network (fluid network), emphasizing that these types of models are largely unexplored. A generic result from their study is that important changes in global behavior occur when moving from frozen to fluid social networks. Their approach is similar to the one also considered by Macy (1991): starting from a random network, the network is regenerated at each time step of the game by random pairings among the agents, but the probability of a pairing depends on the previous results of the game. This is a basic idea of coevolution, but it does not take into account aspects of locality, or a social network, that albeit evolving, has well-established, stable links among individuals.

Our starting point is the observation that the influence of social structure in human behavior, with feedback between actions and social structure, might be seen as an entangled coevolution of the choice of actions and the formation of a social structure. Specifically, we address here the issue of the evolution of social networks in the context of cooperation, allowing individuals both to choose actions and to choose exiting from unsatisfactory relationships. We propose a simple model where the actions of individuals that form the social network are driven by their level of satisfaction, as they choose their actions by imitation of their best neighbor. Moreover, we introduce *social plasticity*—other definitions are introduced by Lazer (2001)—as the capacity of the individuals to choose partners, being able to change their neighborhood as time goes on. Evolving interactions thus appear, allowing individuals to choose different partners on the grounds of the obtained performance.

The significant step forward in the results of our approach is that it naturally leads to a process of social differentiation. Starting from random partnership among equivalent individuals, a social structure emerges. In this emerging structure, the topology of the network of social relations identifies individuals with different social roles. These roles have been spontaneously selected during the complex adaptive dynamics of the social group. They are not the direct consequence of initial differences in strategy or location. Rather, they emerge in a probabilistic dynamic in which the social network is constructed.

PRISONER'S DILEMMA IN AN ADAPTIVE NETWORK

The simplest form of the PD game consists of two agents that may choose from either of two actions: to cooperate (C) or to defect (D). If both agents choose C, each agent gets a payoff (reward) R; if one defects while the other cooperates, the former gets a payoff T (with $T > R$), while the latter gets the "sucker's" payoff S (with $S < R$); if both defect, both get a payoff P. Under the standard restrictions $T > R > P > S$, $T + P < 2R$, defection is the best choice in a single-shot game (Nash equilibrium). Thus, the social dilemma of how cooperation may be sustained arises, due to the fact that rational agents would defect. When no social structure is assumed, agents are drawn randomly in pairs to play the game, and the dynamics may be described by a replicator-type equation (Hofbauer and Sigmund 1998). This equation can be understood from the biologically motivated fact that strategies with fitness greater than the average in the whole population replicate with a positive rate, while those that underperform compared to the average die away. In our context, if Π_C is the average payoff obtained using strategy C, and $\langle\Pi\rangle$ the average payoff for all the population, then the time evolution of the fraction of the agents using strategy C, labeled f_C, obeys the equation

$$\frac{df_C}{dt} = f_C(\Pi_C - \langle\Pi\rangle). \tag{1}$$

Given the payoffs of the PD game we obtain

$$\Pi_C = \mathrm{R}f_C + (1 - f_C)S, \tag{2}$$

$$\langle\Pi\rangle = Rf_C^2 + (S + T)f_C(1 - f_C) + P(1 - f_C)^2, \tag{3}$$

and thus

$$\frac{df_C}{dt} = f_C(1 - f_C)[(R - T)f_C + (S - P)(1 - f_C)]. \tag{4}$$

The analysis of this equation indicates that it has two equilibrium solutions in $f_C \in [0,1]$: $f_C = 0$ is a stable fixed point, while $f_C = 1$ is an unstable fixed point. Thus, it also follows from this dynamical analysis that in the absence of an interaction network describing a social structure, the only stable solution is a pure defective state.

In the spatial version of the PD (Axelrod 1984; Nowak and May 1992, 1993; Nowak et al. 1994*a*, 1994*b*), agents sit on the nodes of the network and they play the game with their neighbors. Two agents are said to be "neighbors" if they are directly connected by a link of the network. A detailed analysis of the different spatiotemporal patterns that can arise in the dynamics shows a phase space where cooperators and defectors can coexist depending on the parameter values (Schweitzer et al. 2002; Lindgren and Nordahl 1994). There are threshold values in parameter space beyond which defection dominates the whole network.

We will consider a spatial version of the PD, generalized in order to consider adapting evolving networks of interaction instead of fixed, regular lattices.

Social Plasticity

We want to study how the results obtained with fixed interaction networks change if a certain *social plasticity* is allowed. This term addresses the common observation that social interactions in most societies adapt in time by a learning process. We will implement a rule for social plasticity that depends on both the strategies and payoffs of the individual players. The motivation of this rule comes from the following analysis of the mutual benefit obtained from each pair of allowed PD interactions: two symmetric (C–C and D–D) and one asymmetric (C–D). Clearly, the symmetric interaction C–C should be reinforced because both agents get the maximum payoff from their selected strategies. However, the opposite occurs in a D–D interaction, where both agents have aligned incentives to change neighbors and to possibly find a C neighbor. The asymmetric interaction C–D forms an intermediate class, where the C agent will not support the interaction, but the D agent will try to reinforce it. As a first approximation to the problem, we assume that this type of interaction does not change, because the overall effect of both agents is balanced. From this analysis, we conclude that the simplest nontrivial social plasticity rule should allow interactions among two D agents to adapt, providing a self-interest mechanism of social adaptation where the agents can increase their payoffs by changing their partners.

The Model

We consider a fixed population of agents placed in the nodes of a network and connected by links to their neighbors. The dynamics evolve in discrete time steps divided in three stages, starting from a random choice of strategies and a random network:

1. *Interacting*.—Each agent i plays the PD game with its neighbors and collects an aggregate payoff, Π.
2. *Strategy update*.—Each agent updates its current strategy, imitating the strategy of the neighbor with the largest payoff including itself (whenever more than one equivalent neighbor with a larger payoff exists, one of them is randomly selected).
3. *Neighborhood update*.—If agent i imitates a defector, then the agent i replaces with probability p this link with the imitated D neighbor with a new one pointing to a randomly chosen partner from the whole network. This process updates the network.

In order to clarify the rules of the model, we would like to make the following points. First, we specify that in the interaction step, we only consider bidirectional (undirected) links, so if agent i plays with j, then we also assume that j plays with i. Second, we do not consider complex strategies which involve the history of past encounters with neighbors (as in Lindgren and Nordahl 1994), but we consider instead only zero-memory strategies, and assume that each agent plays the same strategy (action) C or D with all its partners. Third, the game is played synchronously; that is, at each discrete time step, agents decide their strategy in advance and they all play at the same time.

We consider in figure 1 an example that illustrates the rules of the

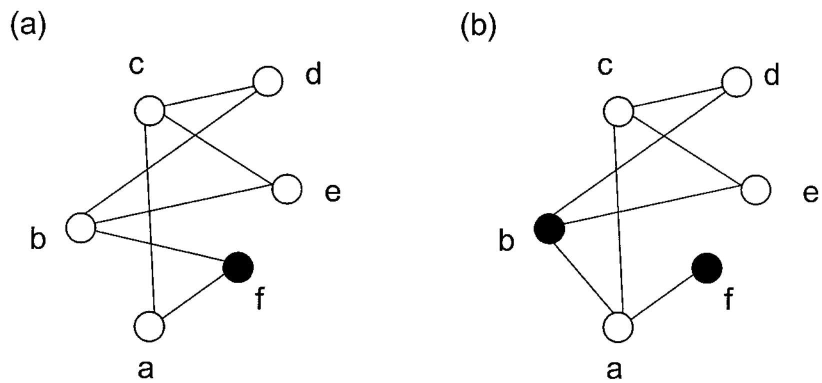

FIG. 1.—(*a*) At time step t, the system is composed of cooperators (white circles $a - e$) and defectors (black circle f) whose interactions are given by the links. Since the payoff matrix is given by $P = S = 0$, $R = 1$, and $T = 1.3$, the payoff for each agent is $\{\Pi_a,\Pi_b,\Pi_c,\Pi_d,\Pi_e,\Pi_f\} = \{1,2,3,2,2,2.6\}$. Therefore, the cooperator c is *leader*, because it has the maximum payoff. Agents a and b get a lower payoff than their common neighbor, the defector f. However, agent a has another neighbor c that gets an even larger payoff than f. (*b*) In the next time step $t + 1$, a imitates cooperation from c, which has the largest payoff among its neighbors, and only agent b changes strategy and imitates the D strategy from f. The figure also shows that b broke the link with agent f and started a new interaction with the randomly selected agent a.

model. The system is initially formed by five cooperators (agents a to e) and one defector (agent f). According to the payoff they obtain, agents a, d, and e are unsatisfied, and they will imitate in the next time step the cooperator c. Agents c and f are satisfied because they do not have a neighbor with a larger payoff. Finally, agent b is also unsatisfied, but the agent with the largest payoff is the defector f. Thus in the next time step, it changes its strategy to become a defector and (with probability p) breaks the link with agent f and selects a new neighbor as partner (in this case it selects agent a). Note that in the neighborhood update rule, it is not necessary for an agent to change its strategy from cooperator to defector in order to sever a link. This example shows that the neighborhood update rule facilitates the survival of the defectors by increasing their payoff when a cooperator is selected at random.

The strategy and neighborhood update rules implemented in this model represent a useful device to unveil basic mechanisms behind global cooperative behavior. Other devices are discussed later in section 4. On the one hand, the extreme learning (imitation) rule of the strategy update can be justified by the psychological bias to focus on confirmation and neglect disconfirmation of beliefs (Strang and Macy 2001). Bounded rational agents seek to learn from limited and biased information: they respond to perceived failure by imitating their most successful peer. In our example of scientific collaborations, each scientist takes as a model the most successful among his collaborators rather than, for example, alternating between imitating both successful and not-so-successful collaborators.

Regarding the neighborhood update rule, we assume first that only D–D links are broken, and second that any targeted agent always accepts a new partner. This rule can be justified as a conservative assumption when considering minimal conditions for the emergence of cooperation. As discussed in section 4, alternative rules allowing cooperators to sever their links would further enhance cooperation. The second assumption can be justified by invoking the absence of cost to sustain a link. Within this assumption, the new link may only increase the payoff, never decrease it, and thus it will always be accepted. In the context of scientific collaboration, if the cost to sustain a research collaboration was zero, then scientists would accept offers from any collaborator with the expectation of getting benefit. Beyond sustaining a link, there is a cost to establish a new one. In our model, the plasticity parameter p measures how easy it is both to sever a collaboration and to find a new partner. While for $p = 0$ this cost is extremely large (so large that it prevents the formation of new links), for $p = 1$ the cost is small.

Finally, although in real social dynamics one experiences both spontaneous creation and suppression of interactions, for simplicity, our model assumes that unsuccessful relations are replaced by randomly selected new ones, always leaving the total number of links constant.

Our strategy update introduces the first classification of the agents in the network: *satisfied* and *unsatisfied* agents. An agent is satisfied if it does not imitate any other agent, thus having the largest payoff in its neighborhood; otherwise it is unsatisfied. Thus, strategy *imitation* and *social plasticity* (adaptation) is restricted only to unsatisfied agents.

The probability p measures the social plasticity of the agents, controlling the rate at which the network structure evolves, as compared to the timescale of evolution of the strategies. For values of $p \ll 1$, strategies change much faster than network evolution (a situation similar to the frozen network of $p = 0$), while for $p = 1$, strategies and network evolve at the same rate (fluid social network).

Stationary States

The proposed model naturally leads to a time evolution of the local connectivity of the network, featuring agents with heterogeneous neighborhoods. Starting from an initial condition of random partnership with links randomly placed among initially equivalent agents, the feedback between choice of strategies and choice of partners results in a dynamical evolution from which a well-defined social structure is expected to emerge in the form of *stationary states.*

In our context, a stationary state is reached when both the strategy of each agent and its neighbors remain fixed in time. Given the discrete nature of the dynamics and the finite number of possible states, these states may be reached in a finite number of time steps. The simplest stationary state consists of all agents being cooperators. Note however, that a state composed exclusively of defectors is a stationary state only when the network is fixed ($p = 0$); for $p > 0$, the agents' strategies remain fixed, while the network is continuously evolving. According to the dynamical rules of the model, each agent's being a defector is not a stationary configuration, even in the case of a social network where each agent has the same number of neighbors (remember that in the case of several neighbors sharing the largest payoff in the neighborhood, one of them is picked at random). In an all-D network, only interaction links change, but not strategies. For the sake of clarity, we will refer to this state as the all-D network state, even if the interactions are not stationary. Our system has a multiplicity of different stationary states, and the system is expected to reach one of these states. This assumption is confirmed by numerical simulations. The specific stationary state reached by the system depends on the stochastic process that shapes the network evolution. A proper characterization of the general properties of these stationary states relies then on statistical tools.

Two requirements are needed for a stationary state to exist. The first is that there be no links between two defectors, so that the neighborhood update rule ($p > 0$) does not produce any change. The second condition relates the respective payoffs in each neighborhood of any cooperator i interacting with a D agent, say d. Their respective payoffs must satisfy

$$\Pi_j > \Pi_d > \Pi_i, \tag{5}$$

where agent j is the cooperator imitated by i. Thus, a stable situation occurs when the payoff of defectors is accommodated "in between" the payoff of two cooperators, and they only *exploit* cooperators. Whenever the payoff of d becomes larger than the best neighbor of i, the configuration is no longer stationary, because the cooperator i will switch to imitate the strategy of agent d. Thus, in the stationary state, defectors do not interact with other D agents, and no cooperator imitates their strategy. An interesting consequence of this result is that the agent with the largest payoff is a cooperator if the system settles to a stationary state. However, this does not prevent a defector from holding the largest payoff in a transient state.

This simple analysis highlights the role that the "imitation of the best neighbor," coupled to the social plasticity rule, has in shaping the social structure of the system. A related consequence is that the social structure developed becomes hierarchical in terms of payoff and dynamics, as we will show below. One way to visualize the hierarchal structure of the network is by constructing a subnetwork referred to as an *imitation network*, where each *directed* link indicates who is imitating whom. As in stationary states, no other agent imitates defectors; in this representation, D agents are isolated.

Figure 2 shows part of the imitation network in a stationary configuration. The agents in the outer layers imitate the ones in the inner layers: by our previous definition they are unsatisfied agents. In a stationary state, cooperators form *trees* of agents hierarchically ordered according to their payoff. This description highlights some special agents that do not imitate any other agent: the agent at the top of the chain has the highest payoff of the tree to which it belongs, and it is the only agent that is satisfied in the tree. They are easily recognized at the top of the trees. All other agents in the tree are unsatisfied, but they imitate the same strategy they were playing in the previous time step. An implication of this representation is that the size of the *group* formed by following directed links to the leader gives an indication of the influence of the leader with respect to the rest of the system. In a stationary state, a leader has no links to D agents. Among the leaders, there is an absolute leader defined as the one with the largest payoff in the system. As a consequence, the absolute leader is also the agent with the largest number of links in the imitation network. All the links of the absolute leader are imitation links shown in the imitation network; any other leader in the system has a smaller number of cooperating partners. The whole system is dominated by the few leaders that control a large fraction of the population. The structure of the social network is then expected to be very sensitive to perturbations acting on those leaders.

When the system is close to a stationary state (i.e., if there are a few cooperators not satisfying eq. 5, as happens when, for example, a cooperator imitates a defector), then the imitation network is useful in understanding how this "perturbation" will propagate along the rest of the social structure. Note that at each time step, the D strategy replicates on all those agents connected to the agent where the perturbation started, causing an "avalanche" of replication events. This chain of events may end in an all-D network, or, because of the existence of cooperator leaders, cooperation may recover, as we will show below.

Social Differentiation

Our previous description of the resulting stationary states also offers a description of how our social model with network adaptation naturally

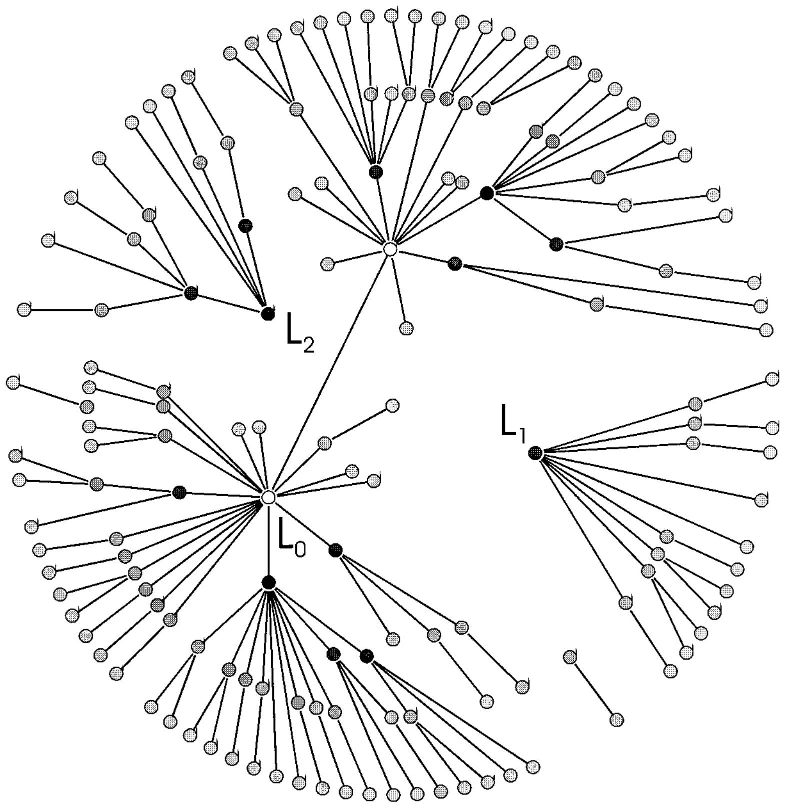

FIG. 2.—Imitation network obtained from a numerical simulation with $T = 1.75$, $R = 1$, $P = S = 0$, and $p = 0.10$. The simulation starts from a random network with $N = 10{,}000$ agents and a number $K = 8$ of average links per node of the network. Agents are organized in imitation layers coded with the same gray level. Starting from the outer layer of a tree, new nodes are introduced for each new link in the path to reach the leader at the top of the tree. Each agent imitates the agent of the inner layer to which it is linked in the tree. The network contains several trees. Three leaders (L_0, L_1, and L_2) are identified. L_0 is the absolute leader with the largest payoff in the system. Its tree contains approximately half the number of agents of the system. All nodes are cooperators, because defectors are isolated in the imitation network. Most of the network's nodes, which belong to the lowest layer of the hierachical structure, are not shown for clarity.

gives rise to a process of social differentiation with the spontaneous emergence of different social roles. Even if all the agents are driven by the same dynamical rules and they are initially statistically equivalent, their role in the network diversifies. Our previous analysis of the imitation network allows us to identify three types of agents in a stationary state:

1. *Leaders*.—Satisfied cooperators that have the maximum payoff in their corresponding neighborhood. The absolute leader is the agent with the largest payoff in the whole network, and its corresponding group of influenced agents is in general the largest. Typically, the other leaders have also a large number of links and a payoff above the average. Admittedly, the sociological concept of leader has several different characteristics, not all of them included in our definition. Still, we use this term to emphasize that these are agents strongly influencing other agents to adopt their strategy. In this context, leaders do not have the will to seek their leadership, nor do they try to preserve it.
2. *Conformists*.—Unsatisfied cooperators, that is, cooperators that do not have maximum payoff in their neighborhood, but imitate other agents by playing their same strategy. They constitute the large majority of the nodes of the imitation network except for the leaders.
3. *Exploiters*.—Defectors that take advantage of others' actions. Defectors have a larger payoff then their C neighbors, showing they succeed in exploitation and are satisfied. In the imitation network, they do not have any directed link, because in the stationary state, no other agent imitates their strategy.

SIMULATING SOCIAL DYNAMICS

To facilitate comparison with previous results of the PD game in fixed networks (e.g., Nowak and May 1992), we have performed numerical simulations of the above model using as main parameters the incentive to defect $b \equiv T$ in the range $1 < b < 2$ and the social plasticity $p \in [0,1]$. The rest of the PD payoff matrix parameters have been kept fixed, $R = 1$, $P = 0$, and $S = 0$ as in Nowak and May (1992). It has been previously found (Lindgren and Nordahl 1994) that turning $P = S = 0$ does not change the main results, although the single PD game is not in the strict PD conditions. We have made sure that changing the parameter P in the region (0, 0.1) does not significantly affect our results.

All simulations were performed with $N = 10{,}000$ agents. The strategy initial condition was always set to 60% of randomly distributed cooperators. The network initial condition was set by distributing $KN/2$ undirected links between random pairs of nodes. The number K corresponds to the average connectivity per agent, and we considered $K = 8$. The initial degree (number of links of each agent) distribution constructed in this manner is a Poisson distribution with parameter K. Simulations with other values parameters have been performed, and no qualitative differences were found. Likewise, we have ensured that using an asynchronous updating in our simulations does not change any meaningful qualitative result. For the above parameters, a stationary state is reached in a time scale that depends very much on p, requiring about 1,000 time steps for $p = 0.01$ and about 10 time steps for $p = 1$. For smaller N the typical time to reach a stationary state decreases, while for larger K it increases. A proper characterization of the general properties of these stationary states relies then on averaging over many different numerical experiments. Our results are given as averages over 100 different experiments.

Simulations in our adaptive dynamic network game show how cooperation is in general enhanced, and no threshold to an all-D network is observed. In fact, our results indicate that for a given set of parameters, there is a *coexistence* of a multitude of cooperative stationary states and the all-D network state. This means that for a fraction of the initial conditions, the stochastic dynamics of the network may lead either to the all-D network or to a stationary state. Once in the all-D network, the system gets trapped, and no recovery is possible (remember that an in an all-D state the interaction links change, but not the strategies). The probability of reaching the trapping all-D network decreases as the number of agents increases (it also decreases as the incentive to defect b decreases and the initial distribution of cooperators increases), and thus the trap appears to be a finite size effect. For instance, for an incentive to defect $b = 1.6$ and plasticity $p = 0.01$, only 6% of the realizations get trapped for a large population $N = 10{,}000$, while for $N = 1{,}000$, 80% get trapped. Here we concentrate on stationary states where cooperators coexist with defectors.

Fraction of Cooperators

Figure 3 shows a first global characterization of the stationary states for different values of the parameter p and the incentive to defect b. The fraction of cooperators, f_C, is measured for a fixed network ($p = 0$) and is shown to decrease with b, approaching zero at a threshold value of $b \simeq 1.85$. Thus, context preservation (Cohen et al. 2001) without social plasticity provides partial cooperation. In clear contrast, for positive p, social plasticity facilitates the establishment of a highly cooperative state with a fraction of cooperators essentially independent of the incentive to defect b. Even for small plasticity p, the fraction of cooperators is above 90% in the range of parameter b considered. A similar result is obtained for other values of the average connectivity $K > 2$. In this sense, context preservation is not a necessary condition to build up cooperation, but rather the social structure is a consequence of an adaptive dynamics in which cooperation is greatly enhanced.

It is worth mentioning that the result of enhanced cooperation works against the direct effects of the neighborhood adaptation rule, introduced

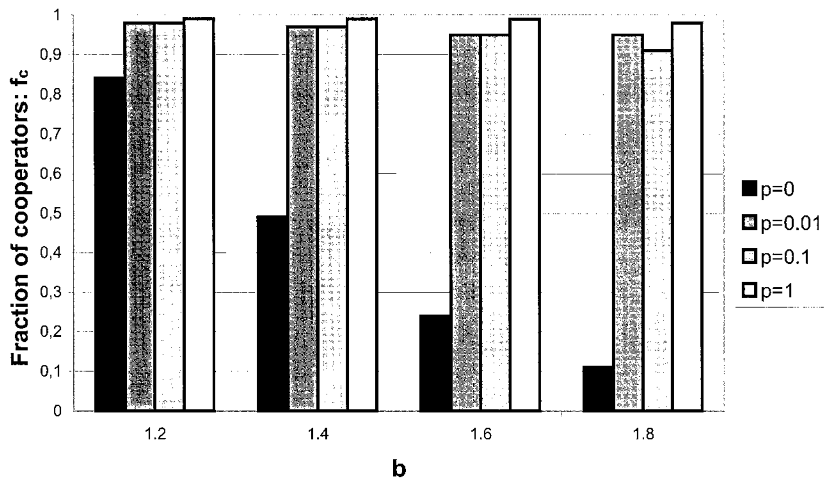

FIG. 3.—Average fraction of cooperative agents, f_C, for different values of the plasticity p and incentive to defect, b. For a fixed network ($p = 0$), f_C decreases with b. However, in the presence of social plasticity ($p \neq 0$), f_C is kept above 90%.

to facilitate the survival of defectors. Defectors are allowed to find new cooperators to exploit, exiting from unsatisfactory interactions with other defectors. However, the final outcome is that the number of defectors decreases. It seems that successful defectors become isolated because they are role models: their victims "run away." The new defectors—those that imitated successful defectors—establish links with cooperators that have a high concentration of other cooperators in their neighborhoods. Thus, the formerly exploited cooperators, now defectors, turn again into cooperators. To achieve cooperation, it seems that cooperators need enough cooperator partners to have a payoff higher than any defector. This hypothesis is analyzed in the following sections.

Distribution of Cooperators' and Defectors' Payoffs

The total average payoff is larger in the dynamic network ($p \neq 0$) than for a frozen network ($p = 0$). However, we have measured systematic differences when considering the average payoffs Π_D and Π_C for each of the respective subpopulations D and C. While the number of cooperators is larger, their average payoff is smaller than that of the defectors, reversing the situation of a frozen network (see fig. 4). The payoff distribution in a stationary state for each subpopulation is shown in the inset of figure 4 for a particular value of b. This graph shows that the most probable payoff is larger for defectors, also explaining the larger average payoff of the defectors. However, close inspection reveals that the distribution for cooperators always has a larger tail, indicating that a number of cooperators have a payoff larger than any other defector in the network. Most of these agents are leaders and are necessary for a cooperative stationary state to exist.

Thus, the main characteristic of our adaptive social game is that an altruistic social network, that is, a network composed of a large number of cooperators, may develop with a few defectors that have a large average payoff. The neighborhoods of the agents adapt to conform to the requirements of the stationary states (D agents only interact with cooperators), and the cooperator with the largest payoff in each branch of the imitation network corresponds to a "leader" with a payoff much larger than the average, whose strategy all other agents imitate.

Transient Dynamics, Leaders, and Social Crisis

The evolution toward a stationary state is typically not a smooth, monotonous buildup of a globally cooperating state. Starting from a random initial condition, the transient dynamics are characterized by small fluc-

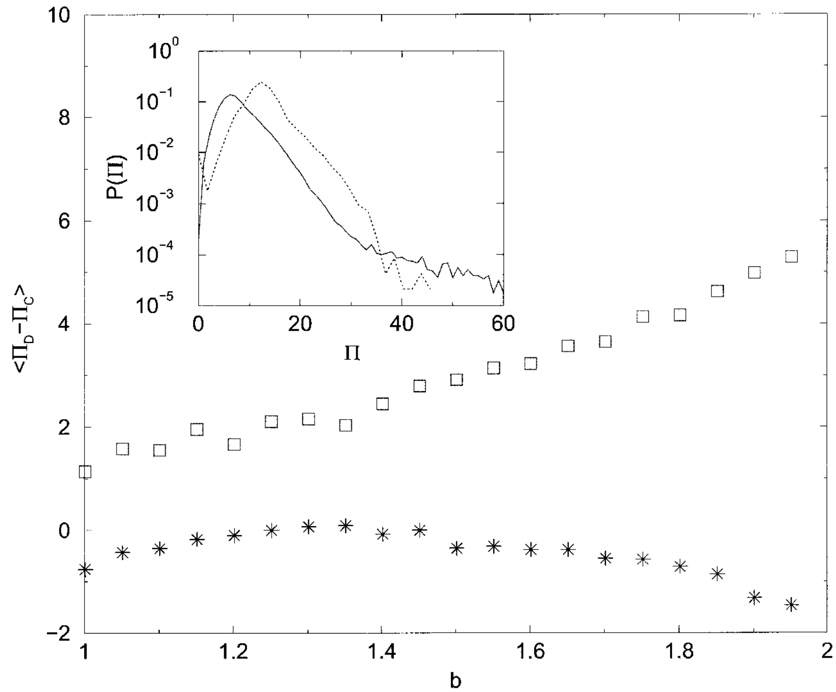

FIG. 4.—Difference between the average payoff of defectors, $\langle \Pi_D \rangle$, and cooperators, $\langle \Pi_C \rangle$, as a function of b for $p = 0$ (stars) and $p = 0.10$ (squares). The effect of the social plasticity is that the defectors get a larger payoff on average: $\langle \Pi_D - \Pi_C \rangle$ is positive for $p \neq 0$. Inset: probability distribution (normalized to each subpopulation) of individual payoff, for cooperators (solid line) and defectors (dotted line). The most probable payoff, which corresponds to the maximum of the distribution, is larger for defectors than for cooperators. However, there are a few cooperators getting the largest payoff in the population, as is seen in the tails of the distribution for large values of Π. Parameter values $b = 1.75$, $p = 0.1$.

tuations and distinctive, large oscillations in the proportion of cooperators as a function of time that can be understood in terms of *social crisis*.

An example of a frustrated attempt to build cooperation is the evolution of the proportion of cooperators shown in figure 5. Starting from a nonstationary state of low cooperation in the initial fixed random network for $t < 200$, network dynamics then lead to a social network with a high degree of cooperation after several large oscillations in the time interval $220 < t < 300$. The large oscillations in f_C are frustrated attempts to build cooperation. They indicate that the defecting behavior is so rewarding that the cooperation has to find a specific network configuration in order to be robust against eventual changes of strategy. In such a configuration, the most connected agent (the one that has the largest number of links) in the imitation network is also the one with the largest payoff. In the

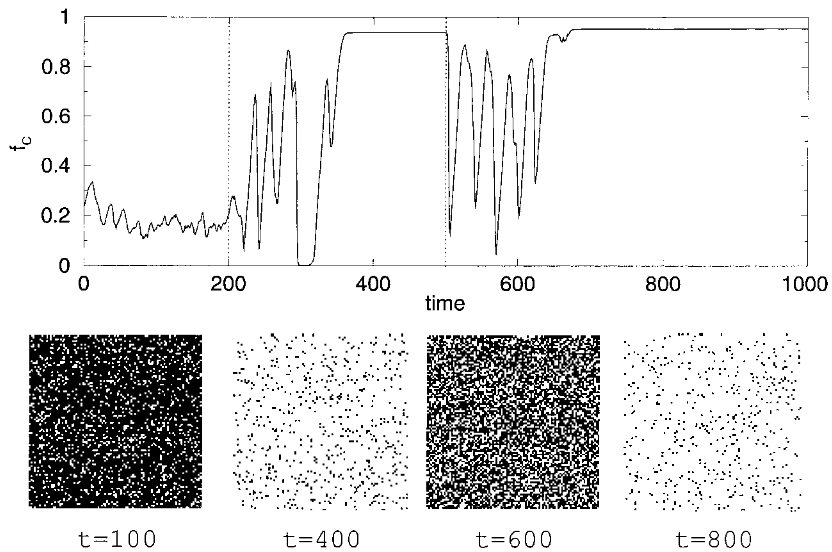

FIG. 5.—Time series for the fraction of cooperators f_C for $b = 1.7$. The evolution is in a fixed random network ($p = 0$) up to time $t = 200$ when network dynamics are switched on, so that $p = 1$ for $t > 200$. At time $t = 500$ the network leader is forced to change action from C to D, to show how cooperation may recover after a social crisis. After an oscillatory transient, a new stationary state is reached. Below, snapshots of the instantaneous configurations of actions are shown at the indicated times. A white (black) point is a C (D) agent in a node of the network (links are not shown). The spatial location of the agents does not have any meaning because of the building of a network of interactions. It is only used for display purposes.

frustrated attempts to reach a global stationary cooperative state, the proportion of cooperators becomes large. A few time steps before f_C reaches each maxima, a cooperator receives a new link from a defector with a larger payoff and switches to D strategy, starting a social crisis. Given that at those time steps there is a large proportion of cooperators in the network, an avalanche of D-strategy imitation starts. This avalanche will certainly affect the neighbors of a cooperator leader, thus decreasing the payoff of most leaders. Precisely when the avalanche begins, there is a defector with a payoff *larger* than the *absolute cooperative leader.* The social crisis propogates through most of the system, producing a change in the connectivity of the social network (Zimmermann et al. 2001). The initial distribution of links for the C and D populations are Poisson distributions around the average connectivity, typical of random networks. At a local maximum of f_C, the two distributions have exponential tails for large values of links. Then, very rapidly, the network switches to the almost defective solution with a large number of D–D links and Poissonian degree distributions. However, the existence of a small number of cooperators with a large payoff permits, thanks to the plasticity of the network, the gradual buildup of cooperation by creating C–C links. This requires two steps. First, D–C links are created (initiated by the defectors), and then the defectors imitate the more successful cooperator in a later stage. Finally, in the stationary state, the distribution of links for the D population becomes very narrow, while the distribution for the C population displays a tail approaching an exponential decay. The stationary network configuration is thus dominated by a few cooperators—the leaders—with a large number of links (the tails of the distribution of links for cooperators). These highly connected agents dominate the collective behavior of the network.

These characteristic dynamics reveal the *functional property* of C leaders; on the one hand, through the imitation process, they sustain cooperation, while on the other, they enhance cooperation whenever the C leader has the largest payoff of the whole network. In fact, there is a sort of competition between the leader and the defector with the largest payoff. We remark that the latter process is due to social plasticity: whenever a defector selects a leader for partnership, there is a great probability that the leader has a larger payoff and the defector will imitate cooperation, *enhancing* the number of cooperators.

Figure 5 also illustrates the sensitivity of the stationary network structure to exogenous perturbations acting on the leaders, which reflects their key role in sustaining cooperation. At time $t = 500$, the system already has reached a stationary state. However, at this time step, we have forced a strategy switch of the cooperative absolute leader, leading once again to a social crisis similar to the one previously described in the endogenous dynamics. In summary, the system self-organizes into one of several possible cooperative states where avalanches and social crises are likely to occur following spontaneous, focused local perturbations.

Structure of the Social Network

A first characterization of the topology of the stationary network reached by the dynamics is given by the degree distribution, that is, the probability distribution of the number K of links of a node. This distribution has a long tail that distinguishes it from the Poisson distribution of the random network. This is reflected in the values of the normalized standard deviation $\sigma_n = \sigma/K$ shown in figure 6 for different values of p and b. We recall that for a Poisson distribution, $\sigma_n = K^{-1/2}$, while for an exponential distribution $\sigma_n = 1$. We find that for small values of b, the distribution departs significantly from the Poisson distribution only for large values of the plasticity parameter p, while for increasing b, the tail of the distribution expands and approaches an exponential form. In other words, the hierarchical structure of the network is accentuated as b increases, with fewer leaders that have a larger payoff.

The distribution of payoffs (inset of fig. 4) is similar to the degree distribution because that payoff of the cooperators is given by the number of links with other cooperators, and there is a very small number of defectors in the stationary state. Therefore, the variations of σ_n also provide a measure of social inequality in the network. In fact, σ_n is closely related to the Gini coefficient (Kakwani 1980) used in the characterization of economic inequalities in a social group. We have measured the Gini coefficient of the resulting network configuration in a stationary state and found that it can be twice as large as one of a random (Poissonian distribution) network. Social plasticity generates a flux of payoff toward richer individuals.

Finally, we address the question of whether the social structure generated in our dynamical model has the characteristics of a small world network. Two requirements have to be fulfilled: the clustering (or cliquishness) has to be much larger than in a random network, and the average path length between two nodes should be similar to that of a corresponding random network. The clustering coefficient, c, measures the fraction of neighbors of a node that are connected among them, averaged over all the nodes in the network. Most very complex networks show a clustering larger than in random networks given by $c_{rand} = K/N$ (Amaral et al. 2000). In our original formulation, numerical simulations show (fig. 7) that for increasing b the clustering coefficient increases very slightly with respect to the clustering of a fixed random network (i.e., c/c_{rand} may change up to 1.06). We have tested a slight enhancement of the network adap-

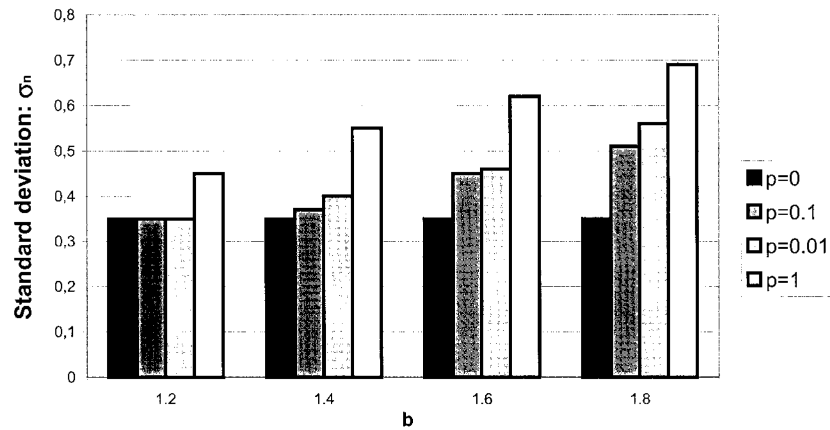

FIG. 6.—Normalized standard deviation σ_n of the degree distribution of the social networks obtained for different values of the plasticity parameter p and the incentive to defect b. For a Poisson distribution and $K = 8$, $\sigma_n = K^{-0.5} = 0.35$, while for an exponential distribution, $\sigma_n = 1$. For $p = 0$, σ_n takes the value of a Poisson distribution. As p and b are increased, the interaction network departs from a Poisson distribution and approaches a distribution with an exponential tail.

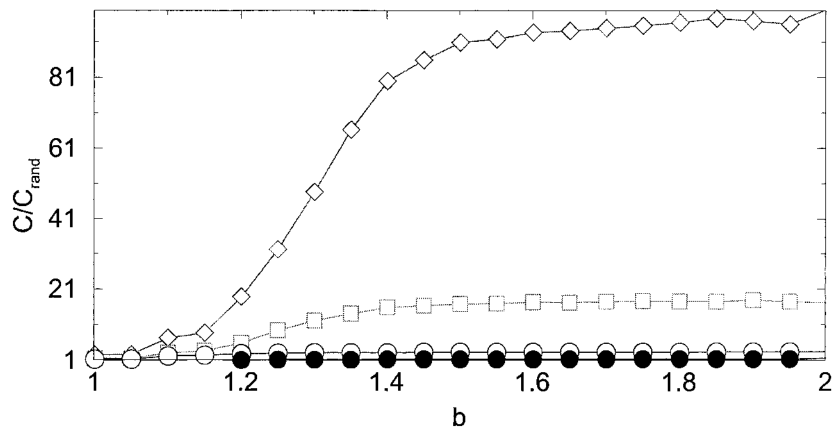

FIG. 7.—Normalized clustering coefficient c/c_{rand}, with $c_{rand} = K/N$. Filled circles correspond to $p = 1$ and $q = 0$. Other curves are for $q = 0.01$ and $p = 0.01$ (diamonds), $p = 0.1$ (squares), and $p = 1$ (empty circles). Social plasticity ($p \neq 0$) and a local selection of new partners ($q \neq 0$) are needed in order to reproduce the clustering observed in social networks.

tation which easily accounts for high clustering. Very often, new acquaintances are made based on the relationships of current neighbors. To implement this idea, the social neighborhood adaptation step of our original formulation was augmented: if a neighbor is replaced by a new one with probability q, a *local* selection of a new partner happens among the neighbors of the neighbors, while with probability $1 - q$, the previous random selection is performed. The limiting case $q = 0$ corresponds to our original formulation, while $q = 1$ corresponds to the case that all the new partners are chosen from the neighbor's neighbors. It is natural that this new mechanism will increase the clustering. Numerical simulations show that while most of our previously discussed results are qualitatively independent of the value of q, with a very small value of q, the clustering coefficient becomes very large. For instance, 1% of local partner selection is enough to increase c 100 times, being the clustering largest for a slow evolution of the network ($p \ll 1$). As for the second requirement for small world topology, we find that the average path length remains in all cases very close to that of a random network. Together, our results indicate that, allowing for local partner selection, the social network generated in our adaptive dynamics has the structure of a small world network.

DISCUSSION

We have presented a minimal model that incorporates simultaneous and coupled evolution (*coevolution*) of the strategies of the agents and of the social network, providing a first step in the investigation of the processes of social differentiation in a globally cooperating social group. Different agents play different social roles that are acquired through social interaction and are not externally imposed or determined by genetic mechanisms. Rather, the roles *emerge* from the self-organizing dynamics of the complex system. It is important to note that the initial differences in number of links among the agents do not determine the final roles of each agent, since each agent temporarily changes its role in the stochastic dynamics until a final stationary state is reached. Moreover, we have made sure—provided the initial condition of a random network in which each agent has the same number of links—that the system dynamics lead to the same emergent role differentiation.

Our results indicate that cooperation is stabilized by a hierarchical self-organized structure, so that the formation of cohesive clusters of cooperators with the exclusion of free riders is unnecessary for persistent global cooperation. Allowing for local partner selection, this self-organization leads from a random network to a final social network with the topology of a small world network, demonstrating how small world connectivity, in which clustering is larger than in a random network, can be dynamically achieved.

Our study reveals that if defectors have the ability to choose partners—breaking interactions with other defectors—the number of cooperators increases, but the average payoff of cooperators is less than that of greedy agents. The dynamics naturally generate *leaders*—individuals getting a large payoff that are imitated by a considerable fraction of the population, *conformists*—unsatisfied cooperative agents that keep cooperating, and *exploiters*—defectors with a payoff larger than the average obtained by cooperators. The most prominent role is the one of leader, a cooperator that not only sustains cooperation, but also drives the whole system toward more cooperation. Defectors are found to remain stable whenever they *exploit* cooperators. The formation of a social hierarchy in the population is the source of possible unstable behavior. It promotes the occurrence of social crisis that can affect a large fraction of the population. These crises take the form of global cascades that might be easily triggered by the spontaneous change in action of a highly connected agent. This result highlights the importance of highly connected agents that, as illustrated by the imitation network, play a leadership role in the collective dynamics of the system. Such sensitivity of the network stability to local special perturbations reveals an interesting feature of globalization: the group of agents organizes itself into a state where an exogenous or stochastic perturbation may produce drastic changes, at distance, within a limited time period.

Even though our model has many interesting features, the strategy and neighborhood update rules represent an extreme and conservative choice of rules. Thus, it would be important to investigate how the modification of the model rules affects the results reported here. Some of the points that may merit further research in connection with our strategy update and network update rules are outlined in the remainder of this article.

Starting with our strategy update rule, we note that an important assumption made in our model is that the satisfaction of an agent is determined by comparison of absolute payoffs. Taking into account the evolution of the social interaction, this assumption implies that two neighbors might have different payoffs because they have a different number of neighbors. On the one hand, this assumption highlights the importance of being highly connected. On the other hand, basing a comparison on the relative payoff per neighbor implies that each agent knows how many neighbors each of its neighbors has. Additionally, our strategy update rule is an extreme "copy best" rule, while alternatives like probabilistic imitation of better role models (Schlag and Pollock 1999) or probabilistic selection of neighbors' strategy (Nowak and May 1993) could also be considered.

For the network update rule, we note that, in our model, only links between two defectors can be broken. In a stationary state, most cooperators are unsatisfied because they imitate an agent with a larger payoff. In our model, this unsatisfactory situation does not induce action to improve payoff beyond the imitation of a neighbor in the social network. One way to test the implications of this hypothesis would be to modify the network update rule, letting the C–C link also be broken. For example, we could let cooperators have some probability of changing a D neighbor with a random agent. In this setting, cooperation would be further enhanced, and the occurrence of large social crises could be limited. Thus, our rule for neighborhood update assures in particular that the highly successful defectors will not be abandoned by the cooperators whose exploitation has made them so successful. Hence, we make defection an attractive option in that exploited agents cannot simply leave their exploiter because of, for example, changed costs of relationship or some (semi-)rational calculation of the expected benefit of creating new ties. A related point is that one could question why D–D links cannot be broken except for those that involve an unsatisfied agent. We have in fact assumed that only agents having at least one neighbor with higher payoff are "forced" to do something to improve their payoff. Thus, we assume some kind of "aversion to change": one will do something only when one is not satisfied, otherwise one will do nothing. This rule can be justified by the cost implicit in the change of a social relation, and it allows, in principle, the maintenance of interaction between two defectors as long as they are both satisfied.

There is another point that deserves further investigation and that relates both to the strategy and network update rules: our choice of strategy update is based on the aggregate payoff, while our justification of the network adaptation rule is based on comparison of individual actions. This a priori assumption involves two different mechanisms, and it is clear that our choice is one among several possibilities. We have already mentioned a strategy update (Nowak and May 1993) based on probabilistic selection among the neighbors' strategies weighted by the aggregate payoff. Cohen et al. (2001) compares different such mechanisms on fixed networks. On the other hand, fewer results are known about network update mechanisms. In the economics literature, a common rule is that a link between two economic agents is accepted if *both* agents improve their payoff (Jackson and Watts 2002; Bala and Goyal 2000). It would be interesting to test new strategy and network adaptations involving a more sophisticated evaluation of the strategy of the opponent and its aggregate payoff. One possible way would be to contemplate a more complex strategy space, involving strategies dependent on past encounters. Work in this direction with a fixed social network was initiated by Lindgren and Nordahl (1994). Another interesting variable to consider is nonequivalent agents that may differ in their attractiveness as exchange partners. Flache (2001) has incorporated this situation in a model that combines the agents' decisions about cooperation with the decisions about selection of new partners. This also resulted in a process of social differentiation sustaining cooperation. It is unclear how much of the process of social differentiation originates in the unequally attractive agents or in the simpler mechanisms contained in our model.

Beyond our update rules, the addition of random perturbations in strategy and network is also a very relevant feature to be explored in the future. These perturbations may originate from errors in imitation or payoff determination, for example. Based on our results, we can see that strategy perturbations may cause large social crises, especially if a well-connected agent makes an error.

We finally remark that many situations in human societies follow the dynamic scheme considered here—new collaborations are frequently formed, while other long-lasting partnerships die out. From scientific collaboration to sports teams—not to mention political parties—our study offers an example of a simple mechanism by which leadership and other social roles might appear and consolidate.

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