Transients in Multivariable Dynamical Systems Depend on Which Parameter is Switched as Illustrated in Lasers

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Noise driven transients in single mode semiconductor, solid state, and molecular gas lasers after they are "switched on" have strong (nearly linear) correlations between the time at which the laser intensity becomes significantly different from zero and the peak intensity of the first large pulse emitted by the laser. We present evidence of a clear difference in the signs of the slopes of the linear correlations for these lasers for transients resulting from gain switching and loss switching and demonstrate the physical origin for this difference.

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For a broad class of lasers there is a distinct difference between switching the laser on by abruptly increasing the excitation process which leads to optical amplification and switching the laser on by abruptly decreasing the losses, commonly referred to as "gain switching" (GS) and "Q switching" (QS) (loss switching), respectively. Focusing as one often does on a single variable of a dynamical system, such as the intensity of the laser, it is hard to imagine how these two processes, which both seem to cause a change in the ratio of the linear gain to the linear loss in the system, might lead to different phenomena. However, by studying these differences, we learn about the subtleties of the nonlinear dynamics in the multidimensional variable space (phase space) of the system that are masked by the simplicity of the terminology.

Many physical systems can be forced to switch from one state to another by changing a parameter. As examples, a laser which is initially off may be switched on by increasing the amplification of the laser medium; for a magnetic material above its Curie temperature T_C, a nonvanishing magnetization can be achieved by lowering the temperature to less than T_C; or a structural phase transition can be caused by a change in the temperature of a material or the pressure applied to it. Of special interest are those cases in which the system has only (except for broken symmetries) two steady states, one stable and the other unstable, for which the change of the parameter only modifies the stability of these solutions. If the initial state is stable before the parameter is switched and becomes unstable after the parameter is switched, noise or perturbations are needed to trigger the departure from it. As a consequence, the variables describing the evolving state of the system have large statistical spreads during the transient dynamics [1,2].

Generally the characteristics of transients depend on the number of variables needed to describe the dynamical evolution. If only a single variable is needed, usually one finds exponential departure from the unstable state and exponential approach to the stable state [1] regardless of which parameter is switched. However, if several variables are involved in the dynamics, the behavior of a particular variable during a switching process depends on which parameter is varied.

We illustrate these general principles with the specific example of single-mode lasers described by two variables. If the variable describing energy storage in the medium is more lethargic than the variable for the electric field ("class B") [3], there are notable differences in the responses to the two switching mechanisms (GS and QS). The cavity losses affect the laser electric field and its evolution and they can be directly controlled (and abruptly varied) in an experiment. In contrast, the gain is not experimentally adjustable directly, rather one can only adjust the rate of excitation of the active medium. An increase in this pumping rate does not immediately yield an increased gain because of the finite time required to build up the corresponding degree of excitation of the medium. However, if the buildup time for the population inversion is much shorter than that for the electric field, the gain reaches a new value almost instantaneously (this essentially defines the class A lasers for which GS and QS give the same transients).

We have measured the laser intensity during switch-on transients from two different class B lasers, both a CO_2 laser (Fig. 1) and a Nd:YAG laser (Fig. 2), after QS. The insets in Figs. 1 and 2 plot the peak intensity during the first pulse vs the time τ^* at which the intensity first reaches a small prescribed value. This time can be taken as the random escape time from the unstable state [elsewhere referred to as the first passage time (FPT)] [4]. The peak intensity is strongly correlated with

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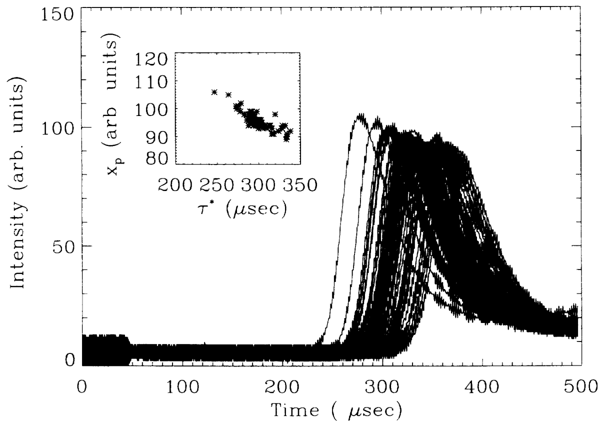

FIG. 1. Multiple traces of intensity vs time for a Q-switched CO_2 laser. Switching duty cycle was 650 μsec on out of 97 msec. For cycling below 20 Hz the laser transient shows no memory of the previous cycle. Inset: peak intensity x_p vs passage time τ^*. τ^* is taken when the intensity reaches $x_r = 20$.

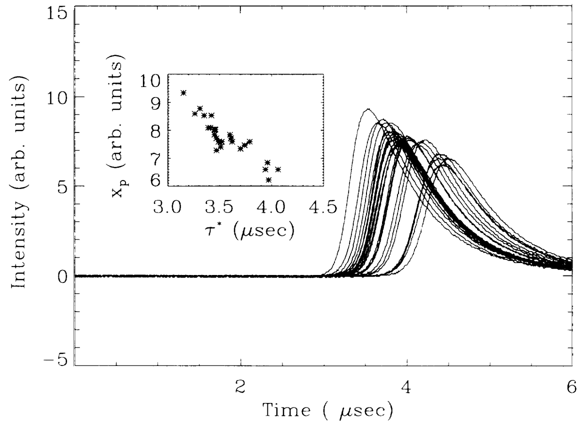

FIG. 2. Multiple traces of intensity vs time and associated inset of peak intensity vs passage time τ^* for a Q-switched Nd:YAG laser. τ^* is taken when the intensity reaches $x_r = 1$

the FPT, with decreasing pulse height corresponding to longer FPT. Similar results were found for a Q-switched NMR laser [5], though other measurements on a CO_2 laser [6] did not show this correlation.

In contrast with these QS results, the correlation between pulse height and FPT found in measurements of gain-switched semiconductor lasers [7] had the opposite sign—larger pulse heights appeared at later times. This correlation for GS has been analyzed previously [8–10].

To explain the differences in the almost linear correlation of peak intensities and FPT's, for QS and GS we consider a model which contains two variables, the complex electric field and the energy stored in the medium. A general dimensionless form of the evolution equations is

$$\partial_\tau z = F_z(z, D) + a_1(D)\xi_z(\tau) \,, \qquad (1)$$

$$\partial_\tau D = F_D(|z|^2, D) - a_2(D)[z^*\xi_z(\tau) + z\xi_z^*(\tau)] \,, \qquad (2)$$

where z is the complex slowly varying amplitude of the electric field, D is the population inversion, and F_z, F_D, $a_1(D)$, and $a_2(D)$ are system-dependent functions. $\xi_z(\tau)$ is a complex white noise of zero mean and delta autocorrelation, $\langle \xi_z(\tau)\xi_z^*(\tau')\rangle = 2\delta(\tau - \tau')$ modeling spontaneous emission. In (1) and (2) we neglect other sources of noise describing, for example, nonradiative population decay and pump noise. The main effect of these noise sources is to set the distribution of initial conditions, but they typically can be neglected during the transient evolution [8,10].

The deterministic version of Eqs. (1) and (2) [obtained by setting $a_1(D) = a_2(D) = 0$] has two different steady state solutions: the "off" ($|z|^2 = 0$) and the "on" ($|z|^2 = x_{ss} \neq 0$) states, which are stable below and above threshold, respectively. Numerical solutions of the stochastic equations (1) and (2) for a variety of functions F_z and F_D indicate that three different time domains can be identified during the decay process from the "off" to the "on" state [8,10]. After the control parameter is switched at $\tau = 0$ there is a time interval when the laser intensity $|z|^2$ is very small and its dynamics is strongly influenced by noise. (In the GS case there is also deterministic drift in the phase space by the inversion variable from its initial condition toward the unstable fixed point.) Once the intensity has reached a macroscopic value $|z|^2 = x_r \ll x_{ss}$, which happens at the random FPT τ^*, it drifts deterministically towards the on steady state value through the so-called relaxation oscillations. Finally, when the vicinity of this stable steady state has been reached, the intensity has small fluctuations due to the various noise sources.

Experimental [11,12] and theoretical [8,10] results are available for the FPT statistics of GS for class B lasers. FPTs are calculated by taking advantage of the smallness of the laser intensity during the first interval, $0 < \tau \leq \tau^*$, which allows for approximately solving (1) and (2) by neglecting in these equations the terms of order $|z|^2$ [8,10]. τ^* is then a function of the sequence of spontaneous emission events and the initial conditions, D_0 and z_0.

The initial conditions for the nonlinear deterministic evolution during the second interval are x_r and $D^* = D(\tau^*(D_0, z_0), D_0)$. The dynamical evolution is then described by (1) and (2) neglecting noise terms. In general [13], $F_z(z, D) = zG_z(|z|^2, D)$ so that defining $z = \sqrt{x}\exp(i\varphi)$, the field intensity x and the population inversion satisfy a closed set of equations. Since there is no explicit time dependence, these equations can be integrated to give

$$x = \mathbf{X}(D, x_r, D^*) \,, \qquad (3)$$

$$D = \mathbf{D}(\tau, \tau^*, x_r, D^*) \,, \qquad (4)$$

which define a nonlinear mapping of the FPT statistics onto the statistics of the other variables during the second stage of evolution.

The condition for the peak intensity, x_p, is $\mathrm{Re}G_z = 0$ which, together with (3) leads to

$$x_p = \mathbf{X_p}(x_r, D^*) . \quad (5)$$

In order to obtain the dependence of x_p on τ^* from (5) we must first calculate D^*, but if the FPT variance is small enough, we can expand x_p in a power series around the mean FPT, $\langle\tau^*\rangle$, obtaining

$$x_p = \bar{\mathbf{X}}_\mathbf{p} + \frac{\partial \mathbf{X_p}}{\partial D^*} \frac{dD^*}{d\tau^*}\bigg|_{\langle\tau^*\rangle} (\tau^* - \langle\tau^*\rangle) + \cdots , \quad (6)$$

where $\bar{\mathbf{X}}_\mathbf{p}$ denotes the value of the peak intenisity corresponding to $\langle\tau^*\rangle$. $\langle\tau^*\rangle$ is calculated by averaging over different switch-on events, which implies the average over the distribution of spontaneous emission noise and over the distribution of initial conditions. The sign of the slope depends on two different factors, $\partial_{D^*}\mathbf{X_p}$ and $d_{\tau^*}D^*$, evaluated at the mean FPT. The first one contains the nonlinear dynamics describing the deterministic evolution from the reference intensity onwards; it is always positive because the higher the population inversion at the beginning of the optical pulse, the larger the intensity overshoot. The second factor describes change of the population inversion at τ^* with τ^*, so it can be determined in the linear regime; its explicit expression reads

$$\frac{dD^*}{d\tau^*}\bigg|_{\langle\tau^*\rangle} = \left[\frac{\partial D^*}{\partial \tau^*} + \frac{\partial D^*}{\partial D_0}\left(\frac{d\tau^*}{dD_0}\right)^{-1}\right]_{\langle\tau^*\rangle} . \quad (7)$$

It is worth noting that this factor determines whether the peak intensity grows or decreases with increases in τ^*.

The physical differences between the QS and GS mechanisms give different weights to the two terms in (7), which give different signs for $d_{\tau^*}D^*|_{\langle\tau^*\rangle}$. More generally both terms are present, and these contributed to a negative slope for GS with the same Nd:YAG laser, though it was approximately 1/70 of the QS value. In our CO_2 laser QS experiments, the fluctuating initial conditions of the effective population inversion variable depended on both noise in the pump excitation mechanism and fluctuations in the laser cavity detuning. *A priori* selection of transients which have the same final output power (which selects instances of the same detuning) reveals the expected slight positive slope due to residual population inversion fluctuations arising from either pumping rate noise or spontaneous emission as predicted by our analysis [14]. The clear positive slope of GS experiments for semiconductor lasers [7] results, in part, from the absence of any significant dependence of the gain on detuning fluctuations. Effects such as thermal lensing (well known to occur in Nd:YAG lasers) are not included in our plane-wave model, but deterministic transformations of the transverse intensity pattern after the FPT should not change the slope of the correlation between the peak intensity and the FPT.

An explicit calculation of the slope of x_p vs τ^* requires a specific form for F_z, F_D, $a_1(D)$, and $a_2(D)$ in (1) and (2), and solving these equations in the nonlinear regime. This procedure cannot be carried out exactly, although some approximations to calculate this slope lead to good agreement with numerical and experimental results [8,9]. However, the interesting physical problem here is to explain the *different sign* of the slopes, which only requires solving (1) and (2) for $\tau \leq \tau^*$. The qualitative result does not depend on nonlinear gain saturation used in most semiconductor laser models and some CO_2 laser models [8,10,11,15], hence we take

$$F_z(z, D) = \frac{1 + i\theta}{2\epsilon} z\,[D - 1 - L(\tau)] , \quad (8)$$

$$F_D(|z|^2, D) = \epsilon\,[1 + \lambda(\tau) - D - D|z|^2] , \quad (9)$$

and $a_1(D) = a_2(D)/\epsilon^2 = \sqrt{2(\eta_1 + \eta_2 D)}$, where η_1 and η_2 describe the strength of spontaneous emission noise, which is taken proportional to the population of the upper level [16]. ϵ ($\ll 1$) defines the time scale for the evolution of the system. θ is the detuning parameter in two-level lasers (for which there is an additional factor of $1 + \theta^2$ in the denominator of the stimulated emission terms) and the linewidth enhancement factor in a semiconductor laser. The functions $L(\tau)$ and $\lambda(\tau)$ define the kind of switching of the laser. For QS, $\lambda(\tau) = 0$ and $L(\tau) = L_{\mathrm{off}}\Theta(-\tau) - L_{\mathrm{on}}\Theta(\tau)$, where $\Theta(x)$ is the Heaviside function and L_{off} and L_{on} are the values of the loss parameter before and after QS, respectively. For GS, $L(\tau) = 0$ and $\lambda(\tau) = -\lambda_{\mathrm{off}}\Theta(-\tau) + \lambda_{\mathrm{on}}\Theta(\tau)$, where λ_{off} and λ_{on} are the values of the pump parameter before and after GS, respectively.

For GS, one obtains [8] $D(\tau \leq \tau^*) = 1 + \lambda_{\mathrm{on}}\{1 - \exp[-\epsilon(\tau - \bar{\tau})]\}$. Unless λ_{on} is very small, $\epsilon(\tau^* - \bar{\tau}) \ll 1$, hence $D(\tau \leq \tau^*) \approx 1 + \epsilon\lambda_{\mathrm{on}}(\tau - \bar{\tau})$, which yields

$$\tau^* - \bar{\tau} = \sqrt{(2/\lambda_{\mathrm{on}})\ln(x_r/|h|^2)} , \quad (10)$$

where we have defined

$$h = x(\bar{\tau}) + \int_{\bar{\tau}}^{\infty} dt \sqrt{2[\eta_1 + \eta_2 D(\tau)]}\,\xi_z(t)\,e^{-\frac{1+i\theta}{2\epsilon}A(t)} ,$$

$$\bar{\tau} = \epsilon^{-1}\ln\frac{1 + \lambda_{\mathrm{on}} - D_0}{\lambda_{\mathrm{on}}} , \quad A(t) = \int_{\bar{\tau}}^{t} dt'[D(t') - 1] ,$$

where $\bar{\tau}$ denotes the time at which the laser threshold is first reached. Equation (10) determines the statistics of τ^* as a transformation of the statistics of D_0 and the Gaussian random variable h. It is seen from (10) that the weak dependence of D^* on D_0 is through $\bar{\tau}$. Neglecting this dependence, we obtain from (7) and (10) that $d_{\tau^*}D|_{\langle\tau^*\rangle} \approx \epsilon\lambda_{\mathrm{on}} > 0$.

For QS, it is found that $D(\tau \leq \tau^*) \approx D_0$, so

$$\tau^* = \epsilon(D_0 - 1 + L_{\mathrm{on}})^{-1}\ln(x_r/|h|^2) , \quad (11)$$

with

$$h = z_0 + \int_0^\infty dt \sqrt{2(\eta_1 + \eta_2 D_0)}\, \xi_z(t)\, e^{-\frac{1+i\theta}{2\epsilon}(D_0 - 1 + L_{\rm on})t}\,.$$

Then, $d_{\tau^*} D|_{\langle\tau^*\rangle} \approx -L_{\rm on}/(\langle\tau^*\rangle - \epsilon/L_{\rm on}) < 0$, because now D^* only depends on D_0 and not on the sequence of spontaneous emission events.

In summary, fluctuations in the initial conditions of the inversion and in spontaneous emission contributions to the field have different importance to the switch-on statistics in class B lasers depending on which parameter is switched. During the linear regime after GS, D grows until τ^*, when the optical pulse is emitted. The later the FPT, the larger D^*, and $\partial_{\tau^*} D^*|_{\langle\tau^*\rangle}$ is positive. However, amplification of spontaneous emission contributions to the field is not possible before the threshold is reached at $\bar{\tau}$. $\bar{\tau}$ depends only on the initial conditions and the operating point of the laser. If the operating point of the laser is stable before the switch on, the spread in passage times for different switch-on events is mostly due to spontaneous emission noise triggering the growth of the light intensity, so in practice, $\partial D^*/\partial D_0 \simeq 0$ and the second term in (7) can be neglected.

If the operating point of the laser fluctuates significantly (a phenomenon enhanced for parameters near threshold) then both terms in (7) come into play.

In contrast, Q switching is far more sensitive to fluctuations in the initial conditions, since amplification is immediately possible because the population inversion begins greater than its final equilibrium value. In the linear regime D remains nearly constant at its initial value D_0. Hence, the first term in (7) vanishes and the only remaining dependence of D^* on τ^* is through the initial condition D_0. Since the amplification of spontaneous emission is more efficient for higher values of D_0, $d_{\tau^*} D_0|_{\langle\tau^*\rangle} < 0$ and x_p decreases linearly with τ^* with a small slope. For a laser with stabilized excitation detuning, the fluctuations of D_0 are limited to those arising from spontaneous emission and these may be difficult to measure (as their percentage of the total decreases) if the laser is not operated close to threshold or if care is not taken to avoid detector saturation [6]. As shown here, when pump or loss noise is important or the initial state is only slightly below threshold, the spread of D_0 allows observation of the negative slopes. In QS, fluctuations in the initial conditions of the inversion may cause a double-peaked passage-time distribution when the proximity to the threshold value is less than the dispersion in the initial population inversion fluctuations [17].

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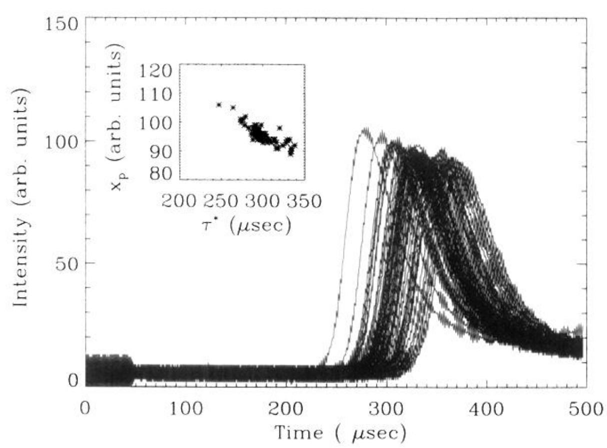

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