Dependence of Timing Jitter on Bias Level for Single-Mode Semiconductor Lasers under High Speed Operation

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Abstract—**The dependence on the bias level of some quantities characterizing optical pulse statistics, such as the turn-on time, pulsewidth, maximum output photon number, and average output power, of single-mode semiconductor lasers are numerically analyzed at frequencies in the GHz range. Periodic modulation and pseudorandom word modulation are considered. In the former regime timing jitter is shown to be rather independent of the bias current. In the latter regime timing jitter becomes larger when biasing above threshold than when biasing below threshold. This large jitter is found to be associated with a bimodal probability distribution of the turn-on time, which yields undesirable pattern effects. A privileged bias, slightly below threshold, suppresses these pattern effects making the laser response almost independent of previous input bits. For such bias value the probability distribution functions of the turn-on time in the case of the periodic and pseudorandom word modulation coincide.**

I. Introduction

THE effect of semiconductor laser noise in optical communication systems was recently reviewed [1]. When the laser is directly modulated by varying the injection current, a limiting factor of its performance, in addition of intensity and phase fluctuations, is the timing jitter in the emission of the optical pulse, as reviewed by Spano *et al.* [2]. Timing jitter causes a degradation of temporal resolution, also important in optical sampling. Such fluctuations in the delay between the electrical and optical pulses (turn-on delay) can result in a system timing error limiting the performance of communication systems working at GHz rate. An additional complication is mode-partition noise leading to the occasional turn-on of a side mode in nearly single-mode semiconductor lasers [3]–[6]. The randomness of the turn-on time of the optical pulse is originated by intrinsic spontaneous emission noise. Nonlinear dynamics amplify the indeterminacy of the turn-on time so that pulses of different height and width occur [7]. The difference in pulse shapes manifests itself in statistical, transient intensity [8] and phase fluctuations. In fact, characteristics of pulse statistics such as the distribution of pulse height or chirp range are easily related [9] to the statistics of the turn-on time. In intensity modulation / direct detection optical communications, one wishes to minimize the turn-on delay and timing jitter, but it is also important to maximize the on–off ratio and to avoid pattern effects. A compromise among these, sometimes contradictory, requirements has to be established to determine the best possible operating conditions of the laser. A small timing jitter is also important in some other applications in which narrow, high-power pulses are required. In general, a central question concerning the operating condition is the dependence of timing jitter on the bias level, and a possible different dependence on the bias when varying the modulation rate. Also such dependence might be different in a situation of signal transmission (pseudorandom word modulation) and in a strictly periodic regime (periodic sequence of pulses).

Fluctuations in the turn-on delay have been studied by a number of authors [2]–[5], [9]–[20] either experimentally or by theoretical calculations including numerical simulations of stochastic rate equations. Recently, attention has also been focused on transient mode-partition noise effects [3]–[6], [17], [18] as well as in transient fluctuations [8], [9], [19]. Most of these studies are based on ensemble averages, that is the statistics is based on a large number of essentially independent gain-switching events in which the laser reaches a steady state fixed by the bias level before each turn-on event. For this situation, and for single-mode lasers, the turn-on delay and timing jitter are reduced biasing above threshold, while the on–off ratio is improved biasing below threshold. A detailed study of the dependence of timing jitter on bias level was reported in [16]. However, there are clear indications [11], [15] that timing jitter has an important dependence on the modulation frequency which cannot be discussed in terms of such ensemble averages. In addition, it has been shown [20] that for pseudorandom word modulation, the turn-on time probability distribution might become double peaked when biasing below threshold, being single peaked and essentially Gaussian when the bias is above threshold, and being also single peaked for periodic modulation and bias either below or above threshold.

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In this paper we report a detailed theoretical study for single-mode lasers of the combined dependence of timing jitter and other statistical properties of optical pulses on the bias level and on modulation frequency in the GHz range. We also consider the different behavior under strictly periodic modulation and the situation of signal transmission, where pattern effects are more important due to the nonperiodic sequence of pulses. We provide evidence that, in general, timing jitter is not strongly reduced by biasing above threshold when modulating in the GHz range [21]. We also find that, by biasing slightly below threshold, pulse statistics are grossly independent on the modulation frequency, and they are the same in the periodic and pseudorandom word modulation (signal transmission) regimes. An explanation of the mechanism that allows to identify such privileged bias level is given. We also show that for pseudorandom word modulation the probability distribution of the turn-on time is broad and with two main peaks when biasing above threshold, so that a large timing jitter is found due to pattern effects; but it becomes single peaked with a small associated timing jitter for bias slightly below threshold. This apparently contradicts the results of Shen [20] mentioned above, which suggested the need of biasing above threshold for high speed data transmission to avoid large timing jitter. We show that, in agreement with his results, at a modulation frequency of 1 GHz the turn-on time probability distribution is double-peaked when biasing 10% below threshold and becomes single peaked for bias 10% above threshold. However, the situation changes at larger modulation speeds: The distribution is always single peaked for bias around 2–3% below threshold, while for a given bias above threshold the distribution becomes double peaked for large enough modulation speed (except for large bias, greater than 20–25% the threshold value). The consequence is that to avoid timing jitter and pattern effects the bias should be changed from above to slightly below threshold when increasing the modulation rate. Our results are largely based on extensive numerical simulations of stochastic rate equations. The advantage of this method for this problem is that we can monitor the stochastic evolution of the carrier number which allows to identify the dominant mechanisms at play.

The outline of the paper is as follows. In Section II we describe the stochastic rate equations and the statistical quantities to be calculated. In Section III we study the situation of a periodic modulation. In Section IV we analyze the response of the system to a pseudorandom word modulation. In Section V a privileged bias level is discussed in detail. A summary and general conclusions are given in Section VI.

II. Stochastic Rate Equations

The description given here is based on noise driven rate equations for a single-mode semiconductor laser. These equations, for the number of photons I and the carrier number N inside the active layer, are [22]:

$$\frac{dI}{dt} = (G - \gamma)I + 4\beta N + \sqrt{4\beta N I}\,\xi(t) \tag{1}$$

$$\frac{dN}{dt} = C(t) - \gamma_e N - GI \tag{2}$$

where $G = g(N - N_0)/\sqrt{(1 + sI)}$. A gain-saturation factor of the form $(1 + sI)^{-1/2}$ is included [23]. The meaning of the symbols and typical values [19] of the different parameters involved in these equations are listed in Table I. The random spontaneous emission process is modeled by a Gaussian white noise term $\xi(t)$ of zero mean and correlation $\langle \xi(t)\xi(t')\rangle = 2\delta(t - t')$. The term $4\beta N$ yields the mean photon number emitted by spontaneous emission, and the term $\sqrt{4\beta N I}\,\xi(t)$ describes the fluctuations of this mean photon number. We neglect the effect of the radiative and nonradiative carrier generation and recombination noise in the rate equation for $N(t)$ since it is negligible as compared with the fluctuations induced by $\xi(t)$ in the regime of pulse emission relevant here [1], [4]. The stochastic differential equations (1) and (2) are defined in the Ito sense [24], [25].

Equations (1) and (2) predict that the laser threshold occurs for

$$C_{th} = \left(\frac{\gamma}{g} + N_0\right)\gamma_e. \tag{3}$$

For our parameter values, $C_{th} \simeq 3.76 \times 10^{16}\ \text{s}^{-1}$ and $N_{th} = C_{th}/\gamma_e \simeq 7.51 \times 10^7$. For $C > C_{th}$ the stationary off solution $I = 0$ and $N = C/\gamma_e$ becomes unstable and the stable lasing solution is given by

$$N_{st} = \frac{\gamma\gamma_e s}{2g^2}\left[\sqrt{1 + \frac{4g^2}{\gamma\gamma_e s}\left[\frac{C}{\gamma_e} + \frac{\gamma}{\gamma_e s} - N_0\right]} - 1\right] + N_0 \tag{4}$$

$$I_{st} = \frac{1}{s}\left[\frac{g^2}{\gamma^2}\left(N_{st} - N_0\right)^2 - 1\right]. \tag{5}$$

For our parameter values, and $C = C_{\text{on}} = 1.4 \times 10^{17}\ \text{s}^{-1}$, $N_{st} \simeq 7.62 \times 10^7 \approx 1.015 N_{th}$ and $I_{st} \simeq 2.55 \times 10^5$. The difference between N_{st} and N_{th} is due to gain saturation effects.

We consider three different operating regimes according with the time-dependence of the injection current $C(t)$:

Gain switching: At time $t = 0$ the injection current is changed from a value $C = C_b$ to a constant value above threshold $C(t) = C_{\text{on}}$ during a time t_{on}. For $t > t_{\text{on}}$, $C = C_b$ and the laser relaxes to steady-state conditions.

Periodic modulation: The injection current $C(t)$ follows a square-wave modulation of period $T = t_{\text{on}} + t_{\text{off}}$ taking values C_{on} during t_{on} and C_b during t_{off} in each period. We allow for a finite rise and fall times of 12 ps included in t_{on} and t_{off}, respectively. For very large t_{off} the gain-switching regime is recovered.

Pseudorandom word modulation: The response of the system, when it is used for transmission of a signal in the return to zero (RTZ) modulation scheme, can be modeled considering the response to a random modulated injection current composed of a stochastic sequence of "0" and "1" bits. A bit "1" has an injected current of C_{on} during t_{on} and C_b during t_{off}. On the other hand, a bit "0" has an injected current C_b during the full period T.

We are interested in the statistical properties of the light pulses emitted in any of these three situations. Relevant quantities are the mean turn-on time $\langle t \rangle$, the mean pulsewidth $\langle w \rangle$, the average maximum photon number $\langle I_{\max} \rangle$, and their respective mean square deviations σ_t, σ_w, and σ_i, for modulation frequencies in the GHz range and different bias levels. The turn-on time is defined as the time, after C rises to C_{on}, at which the intensity I reaches 50% of its steady-state value for $C = C_{\text{on}}$. The width of the pulse is also defined at this reference level. $I_{\max}$ is defined as the instantaneous maximum value of the photon number in each optical pulse. We will also consider the average output power per pulse (per facet) $\overline{P}$, calculated as the temporal average, during a period of modulation, $\overline{P} = 1/T \int_0^T P(s)ds$, of the output power $P(s)$. For the value of the facet loss α_m and the wavelength λ given in Table I, $P(s) = [hc^2\alpha_m/(2\mu_g\lambda)]I(s) \approx 2.19 \times 10^{-5}\, I(s)$ mW [22], where c is the speed of light in vacuum and h is the Planck constant.

A typical time trace obtained from (1) and (2), under periodic modulation, can be described as follows. For bias below threshold [Fig. 1(a), (b)] and very large t_{off}, at the end of a period of modulation, the number of photons I is very small and N is below its threshold value $N_{\text{th}} = C_{\text{th}}/\gamma_e$. As $C(t)$ raises to C_{on}, N grows linearly beyond N_{th}, and it reaches a maximum as the laser turns on. The randomness of this time is due to the spontaneous emission noise and causes timing jitter. After the pulse is originated, N diminishes and the pulse in the photon number I develops. The later the pulse is emitted, the larger the maximum value of N, and consequently the higher the emitted pulse [9]. A typical time trace when biasing above threshold is shown in Fig. 1(c), (d). The main difference with the former case is that now the laser switches-on during the off-semiperiod. Typical time traces for shorter t_{off} are shown in Fig. 2. The shape of the time traces has a strong dependence on t_{off} since it determines the initial conditions for the following pulse. It is clear that the pulses are rather similar in Fig. 2 independently of C_b, while this is not the case for large t_{off} (Fig. 1). In fact, the value of t_{off} has to be compared with two different times:

1) The relaxation time needed for N to reach its steady state when $C(t) = C_b$. For bias close to threshold this time scale is given by $\gamma_e^{-1} = 2000$ ps. For $t_{\text{off}} < \gamma_e^{-1}$ differences in pulse statistics with respect to

TABLE I
MEANINGS AND VALUES OF THE PARAMETERS IN (1) AND (2)

Parameter	Meaning	Value	Units
g	Gain parameter	5.6×10^4	s^{-1}
γ	Inverse photon lifetime	4×10^{11}	s^{-1}
γ_e	Inverse carrier lifetime	5×10^8	s^{-1}
β	Spontaneous emission rate	1.1×10^4	s^{-1}
s	Gain-saturation factor	1.3×10^{-6}	adimensional
N_0	Carrier number at transparency	$6.8\ 10^7$	adimensional
C_{th}	Threshold current	3.76×10^{16}	s^{-1}
α_m	Facet loss	45	cm^{-1}
λ	Operating wavelength	1.5	μm
μ_g	Group refractive index	4	adimensional

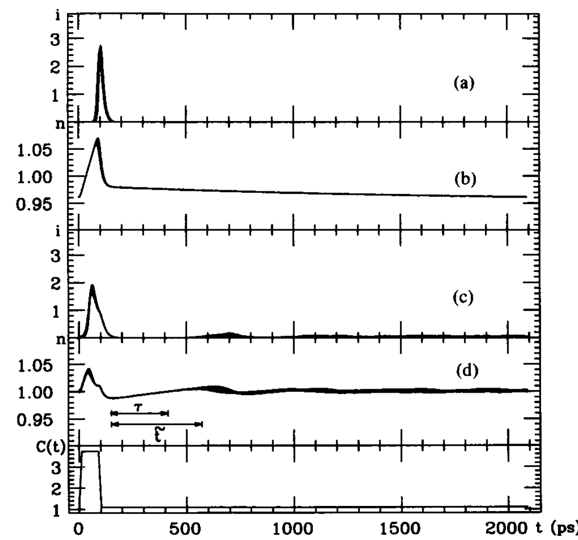

Fig. 1. Fifteen time traces showing the evolution of the photon number (a) and (c), and carrier number (b) and (d) for $C_b = 0.95\,C_{th}$ and $C_b = 1.1\,C_{th}$, respectively, with $t_{\text{on}} = 90$ ps and large t_{off} (= 2000 ps). I has been normalized to its steady-state value for C_{on} [(4) and (5)], while N has been normalized to the threshold value: $i = I/I_{st}$, $n = N/N_{th}$. (d) Electrical pulse, normalized to the threshold value $c = C/C_{th}$, for $C_b = 1.1\,C_{th}$.

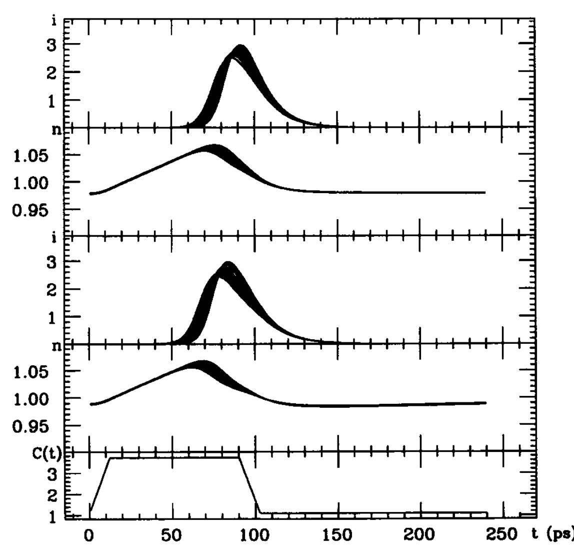

Fig. 2. Same as in Fig. 1 but with $t_{\text{off}} = 150$ ps.

the situation of gain switching should be generally observed.

2) The turn-on time $\tilde{t}$ during the off-semiperiod for $C_b > C_{th}$. If t_{off} is chosen to be smaller than $\tilde{t}$, the fact that C_b is larger than C_{th} has no important effect in the following pulse. For the values used here for C_{on} and C_b, we numerically find $\tilde{t} \approx 500$ ps. An analytical estimation of $\tilde{t}$ is given below.

III. Periodic Modulation

From the previous discussion of typical time traces, it is clear that the value of t_{off} is very important in determining the statistical properties of the pulses, since it determines the conditions for the beginning of the next pulse. We first study such statistical properties as a function of t_{off}, in the regime of periodic modulation, for a fixed value of t_{on}. A convenient pulsed operation of the laser is obtained taking t_{on} such that $C(t)$ changes from C_{on} to C_b at a time intermediate between the pulse emission time and the time at which N goes through its minimum value (see Fig. 2). For a value of t_{on} such that $C(t)$ reaches C_b when N has started to grow from its minimum value the pulse becomes wider and a subsidiary pulse associated with relaxation oscillations could also occur. We have chosen $t_{\text{on}} \simeq 90$ ps between these limits. In Fig. 3 we show, in the regime of $t_{\text{off}} < \tilde{t}$ and for different C_b, the dependence on t_{off} of $\langle t \rangle$, σ_t, $\langle w \rangle$, σ_w, $\langle i_{\max} \rangle$, and σ_i, as obtained from numerical simulations of (1) and (2). The averages have been taken over 10^4 optical pulses, after a periodic regime is reached. We allow the system for 10^2 periods for reaching this periodic regime. We have checked that for large enough t_{off}, gain-switching results are reproduced. These results are indicated with arrows on the right side of Fig. 3. For such large t_{off} there exists a strong dependence on the bias current [14]–[19] with a significant reduction in the jitter σ_t when changing the bias from $C_b = 0.95\ C_{th}$ to $C_b = 1.1\ C_{th}$. However, in the regime considered here ($t_{\text{off}} < \tilde{t}$) the value of N at the end of the period is quite independent of the value of C_b, as can be seen from Fig. 2. For this reason, the strong dependence on the bias current obtained for large t_{off} is drastically reduced, due to the fact that neither I nor N has time enough to reach its stationary value associated with $C(t) = C_b$. As a consequence, a plateau appears in most curves plotted in Fig. 3 for bias close to threshold and t_{off} in between 100 and 200 ps (which correspond to modulation frequencies of 5.21 and 3.42 GHz, respectively). Our results for $t_{\text{off}} \leq 100$ ps merit a separate comment. The turn-on time and associated jitter are reduced because the number of photons remains at a relatively high value at the end of the period of modulation. A faster turn-on reflects the fact that the maximum value reached by the carrier number is smaller. As a consequence, pulses are wider and with a lower maximum, causing a degradation of the on–off ratio. For smaller t_{off} the possibility of pulse overlap is greatly enhanced.

In order to check that $t_{\text{on}} \simeq 90$ ps is a reasonably good value, we have also studied the dependence of $\langle t \rangle$, σ_t, $\langle w \rangle$, σ_w, $\langle i_{\max} \rangle$ and σ_i with t_{on}, for a fixed value of $t_{\text{off}} = 150$ ps. Our results are shown in Fig. 4. It can be seen that for $t_{\text{on}} < 90$ ps the jitter increases while the width and the maximum output photon number are strongly reduced giving a very small output power. This reduction is due to the fact that the injected current is turned-off to its bias value before N has arrived to it natural maximum value, causing an earlier turn-on of the laser, and giving also a small on-off ratio. With that small output power it might happen that the minimum value needed for the detection of the signal is not reached causing an error in the transmission. This situation of an early turn-on of the laser

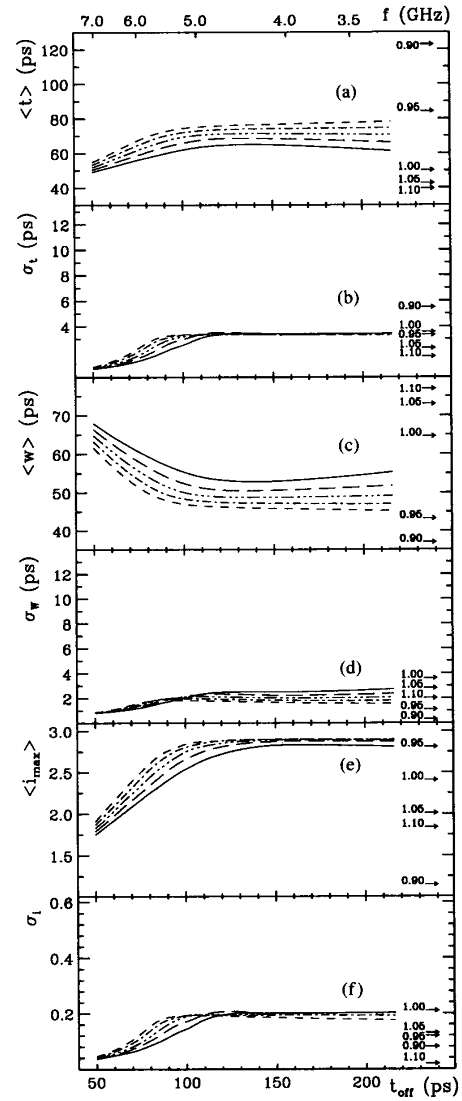

Fig. 3. (a) Mean turn-on time. (b) Time jitter. (c) Mean and (d) standard deviation of the pulsewidth. (e) Mean and (f) standard deviation of the maximum output photon number are plotted as a function of t_{off}, for different C_b. The modulation frequency f associated with different t_{off} is indicated at the top. Short dash corresponds to $C_b = 0.9\,C_{th}$; dot-dash to $C_b = 0.95\,C_{th}$; three-dot-dash to $C_b = 1.0\,C_{th}$; long dash to $C_b = 1.05\,C_{th}$ and the solid line to $C_b = 1.1\,C_{th}$.

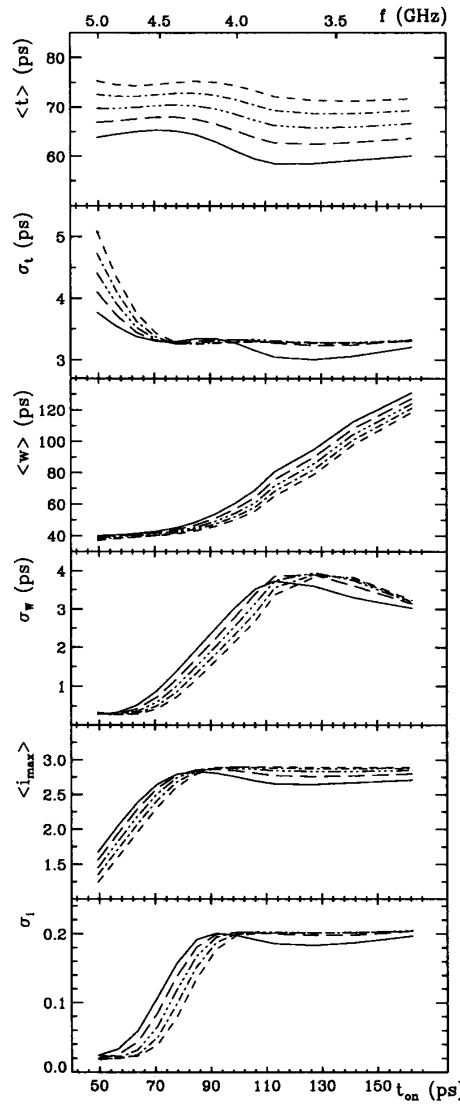

Fig. 4. The same statistical quantities as in Fig. 3 but plotted as a function of t_{on}.

causes the anomalous behavior for $C_b = 0.9\ C_{th}$, $t_{on} \simeq$ 90 ps and large t_{off} shown on the right side of Fig. 3, where timing jitter is seen to be considerably larger than for $C_b = C_{th}$. We have checked that for larger values of t_{on} (90 ps $< t_{on} <$ 150 ps) and a large t_{off}, corresponding to a gain-switching condition, σ_t for $C_b = 0.90 C_{th}$ is smaller than for $C_b = C_{th}$; if t_{on} becomes even larger ($\sim$ 150 ps), w becomes also larger and a secondary pulse associated with relaxation oscillations appears in the time trace of I and N, so that undesired pattern effects become important. It can also be seen from Fig. 4 that $\langle \sigma_t \rangle$, $\langle i_{max} \rangle$ and $\langle \sigma_i \rangle$ become almost constant and quite independent on C_b for 90 ps $< t_{on} <$ 180 ps. For these reasons we fix t_{on} = 90 ps during the remaining of our analysis.

Possible source of errors in the transmission of a pulse are a very large turn-on time or a very small output power

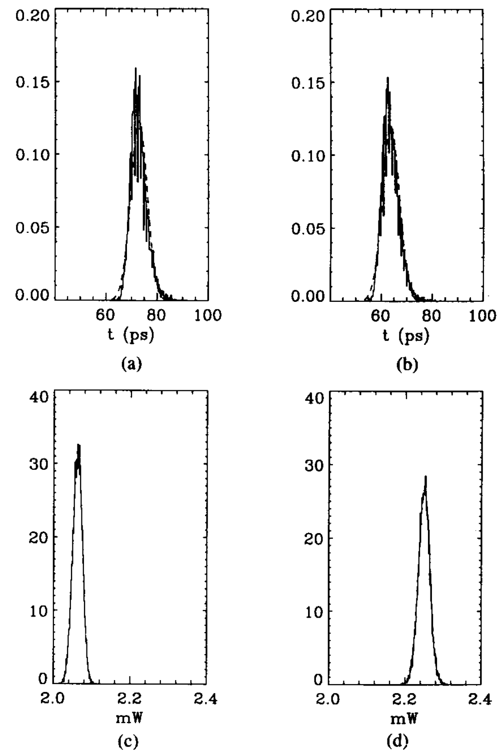

Fig. 5. Probability distribution function for the turn-on time [(a) and (c)], for $C_b = 0.95 C_{th}$ and $C_b = 1.1 C_{th}$, respectively, and average output power per pulse [(b) and (d)], for $C_b = 0.95 C_{th}$ and $C_b = 1.1 C_{th}$, respectively.

of the pulse. These events are not determined by the mean values and variances of these quantities if their probability distribution is not Gaussian. In fact, one is interested, when looking for errors, in the tails of the probability distribution. We show in Fig. 5 the probability distributions for the turn-on delay time and the average output power for t_{off} = 150 ps. The distribution functions are here compared with Gaussian distributions of the same mean value and variance. The same comparison is given throughout the paper with Gaussian distributions indicated in dashed lines in the figures. The distributions look rather Gaussian, except that, specially the distribution functions of the turn-on time are rather asymmetric. The deviation from a Gaussian distribution can be quantitatively characterized through their skewness and kurtosis, which are 0 and 3, respectively, for Gaussian distributions. For the turn-on time (average power) probability distribution the skewness is 0.88 (−0.01) and 0.82 (0.005) and the kurtosis 3.45 (2.9) and 3.3 (3.27) for $C_b = 0.95 C_{th}$ and $C_b = 1.1 C_{th}$, respectively. This indicates a better Gaussian fit for the average output power than for the turn-on time.

An important result of our analysis of pulse statistics that can be extracted from Fig. 3 is that statistical quantities for $C_b \simeq 0.95 C_{th}$, in particular σ_t, $\langle i_{max} \rangle$ and σ_i, are almost independent of t_{off}, taking here the same value as for the gain-switching conditions. There seems to exist a bias value for $C(t)$, which is below the threshold, for which all the studied statistical quantities have a value almost independent of t_{off}. Such a bias value should correspond to a situation in which every pulse is grossly independent of the previous one in a way in which the independence is maintained when changing t_{off}. From Figs. 1 and 2 one can think that this particular bias value should be associated with the minimum value N_m reached by $N(t)$, which is found to depend upon C_{on} but to be rather independent of C_b. When C_b is fixed to a value $\overline{C}_b = \gamma_e N_m$, $N(t)$ remains essentially constant at $N(t) = N_m$ during the off-semiperiod independently of the value of t_{off}. In this way, the initial conditions that modifies the statistics of turn-on time and pulse-shape become essentially the same for any t_{off}, including the very large t_{off} corresponding to a gain-switching condition. This particular value $\overline{C}_b$ can be self-consistently estimated as follows. Given an initial condition at the beginning of one period $N(t = 0) = N_m = \overline{C}_b/\gamma_e$ and $I = 0$, one imposes that this value of $N(t) = N_m$ is again obtained at the first minimum of $N(t)$. From the initial conditions, the maximum value of N, N_M, is obtained solving (2) with $I = 0$ until the mean turn-on delay time $\langle t \rangle$

$$N_M = \delta + (N_m - \delta)e^{-\gamma_e \langle t \rangle} \tag{6}$$

where $\delta = C_{on}/\gamma_e$. This maximum value is then used as the initial value for half a period of a relaxation oscillation with $C(t) = C_{on}$:

$$N_m - N_{th} = (N_M - N_{th})e^{\alpha_R T_R/2} \tag{7}$$

so that,

$$N_m = \frac{N_{th}(1 + e^{\alpha_R T_R/2}) + \delta e^{\alpha_R T_R/2}(e^{-\gamma_e \langle t \rangle} - 1)}{1 + e^{-\gamma_e \langle t \rangle + \alpha_R T_R/2}}. \tag{8}$$

The damping constant α_R is

$$\alpha_R = -\frac{1}{2}\left[\frac{4\beta N_{st}}{I_{st}} + \frac{g s I_{st}}{2}\frac{N_{st} - N_0}{(1 + sI_{st})^{3/2}} + \frac{g I_{st}}{\sqrt{1 + sI_{st}}} + \gamma_e\right]$$

and the period of the relaxation oscillation

$$T_R = 2\pi\left[\frac{4\beta N_{st}}{I_{st}}\gamma_e + g\frac{N_{st} - N_0}{\sqrt{1 + sI_{st}}}\left(4\beta + \frac{sI_{st}(\gamma_e - 4\beta)}{2(1 + sI_{st})}\right) + \frac{\gamma g I_{st}}{\sqrt{1 + sI_{st}}} - \alpha_R^2\right]^{-1/2}$$

where N_{st} is the steady-state value for the carrier number for $C = C_{on}$. For our parameters values, $\alpha_R = -3.12 \times 10^{10}$ s^{-1}, $T_R = 99.34$ ps and $\langle t \rangle \simeq 65$ ps. Then from (8) $N_m = 0.983 N_{th}$, so that we find $\overline{C}_b \simeq 0.98 C_{th}$. This explains the fact that for $C_b = 0.95 C_{th}$ we have observed that σ_t, $\langle i_{max} \rangle$ and σ_i are almost independent of t_{off}.

Having estimated N_m we can also estimate the mean turn-on time $\langle \tilde{t} \rangle$ for $C_b > C_{th}$ during the off-semiperiod, as follows. We assume that at the beginning of the off-semiperiod $N(t) = N_m$ and that N_m is independent of C_b. The regime of exponential amplification of $N(t)$, starting from the initial condition $N_m = 0.983 N_{st}$, can be calculated integrating (2) with $I = 0$ and $C(t) = C_b$, until the time τ at which the carrier number reaches the threshold value. This time is:

$$\tau = -\frac{1}{\gamma_e}\ln\frac{1 - C_b/C_{th}}{0.983 - C_b/C_{th}}. \tag{9}$$

For $C_b = 1.1\, C_{th}$, $\tau \simeq 300$ ps. This agrees with the typical time trace shown in Fig. 1. The turn-on time starting with an initial condition $I = 0$, $N = N_{th}$ can be calculated using the same procedure than in [9]. The turn-on time is then given by:

$$\langle \tilde{t} \rangle = \tau + \frac{1}{\sqrt{g(C - \gamma_e N_{th})}}\left[\sqrt{2\theta} - \frac{\psi(1)}{\sqrt{2\theta}}\right] \tag{10}$$

where $\theta = \ln\,[I_{ref}^{off}/\langle\, |h(\infty)|^2 \rangle]$, with $I_{ref}^{off} = I_{st}^{off}/2$ being the reference value used to define the turn-on time during the off semiperiod, $\psi(1)$ is the digamma function [26] and $\langle\, |h(\infty)|^2 \rangle$ is given by:

$$\langle\, |h(\infty)|^2 \rangle = \frac{4\beta}{g} + 4\beta\left(\frac{\gamma}{g} + N_0\right)\sqrt{\frac{\pi}{2g(C - \gamma_e N_{th})}}.$$

The mean turn-on time during the off-semiperiod for a bias above threshold, $C = C_b = 1.1\, C_{th}$, calculated from (10) gives $\langle \tilde{t} \rangle \simeq 500$ ps which is consistent with the time trace shown in Fig. 1(d). The overestimation of $\langle \tilde{t} \rangle$ is due to the calculations in (10) which becomes more accurate for C_b far from C_{th}. From the above discussion it should be clear that $\tilde{t}$ has a different physical meaning than the relaxation oscillation period T_R. The time T_R has different values for either $C = C_{on}$ or $C = C_b$ and it gives information on damped small oscillations around steady state, while $\tilde{t}$ gives a time scale of a regime of exponential amplification with $C = C_b$ induced by the fact that the laser has gone through the threshold. It is after this regime in which N reaches its maximum value that relaxation oscillations of N toward the steady state, fixed by C_b, occur. We note that even if the initial condition for N is rather close to its final value, a relaxation oscillation does not take place when threshold is crossed. The relevant characteristic time when biasing above threshold is $\tilde{t}$ rather than T_R.

In summary, we have shown that for $t_{off} < \tilde{t}$ pulse statistics are rather independent of C_b being above or below threshold and for $C_b = \overline{C}_b < C_{th}$ we have shown that pulse statistics are rather independent of t_{off}.

IV. Pseudorandom Word Modulation

We next consider the situation of pseudorandom word modulation, described in Section II, with $t_{on} \approx 90$ ps and $t_{off} > 50$ ps. It is interesting to note that a varying value of t_{off} in the situation of periodic modulation can be understood as a periodic signal consisting of a "1" bit followed by a varying number of "0" bits. As a consequence one might already expect that for $C_b = \overline{C}_b$ pattern effects will be greatly diminished since pulse statistics were shown to be independent of t_{off} for such bias. In our numerical calculations for pseudorandom word modulation averages are taken over 2×10^4 periods of modulation, which on the average correspond to 10^4 pulses associated with "1" bits.

A. Statistical Averages

In Fig. 6 we show the dependence on t_{off} of $\langle t \rangle$, σ_t, $\langle w \rangle$, σ_w, $\langle i_{\max} \rangle$, and σ_i, for different C_b. Strong differences with the periodic modulated regime can be seen by comparing with Fig. 3. The "plateau" observed for the periodic regime has disappeared in the pseudorandom word modulation regime, except for $C_b = 0.95\,C_{th}$. This confirms the existence of a special value for the bias current for which the statistical quantities should be independent of t_{off}. On the other hand, timing jitter is now considerably larger when biasing above threshold than when biasing below threshold. An important increase of σ_t, σ_w, and σ_i is observed, specially for $t_{\text{off}} > 80$ ps and $C_b = 1.1\,C_{th}$. This is a reason against biasing above threshold in this frequency range of operation of the laser.

For very long t_{off} the statistics of a pulse should be independent on whether the preceding bit was "0" or a "1", so the results pointed by arrows on the right side of Fig. 3 should be also obtained in this case. This implies that, for some t_{off}, the curves of Fig. 6 must cross. In particular, and as opposed to what is seen in Fig. 6, the curves of σ_t and σ_i for $C_b = 1.1\,C_{th}$ must be below those for $C_b = 0.95\,C_{th}$ for large t_{off}. This is shown in Fig. 7. It can be clearly seen that, for our parameters, at $t_{\text{off}} \sim$ 400-500 ps (~ 2 GHz) the two curves for σ_t and σ_i cross each other. This value of t_{off} should be identified with the time $\langle \tilde{t} \rangle$ discussed in Section III. For larger values of t_{off} the laser turns-on in the off-semiperiod, when biasing above threshold, and as a consequence timing jitter becomes smaller. The result in Fig. 7 makes a preference for biasing slightly below threshold for frequencies greater than 2 GHz and above threshold for smaller frequencies. That questions the suggestion in [20] of biasing above threshold for frequencies larger than 1 GHz to reduce timing jitter and to avoid pattern effects. A discussion of this seemingly contradiction in terms of turn-on probability distributions is given below.

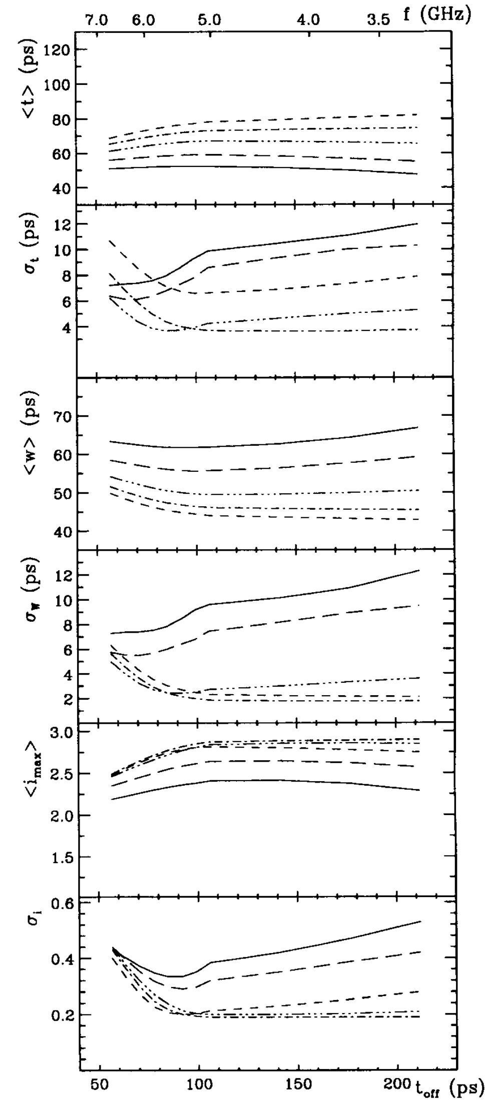

Fig. 6. The same statistical quantities as for Fig. 3 but for a pseudorandom word modulation.

B. Probabilities Distributions Functions for the Turn-on Time and Average Output Power

In order to better understand the behavior of the system for bias above and below threshold, we analyze the distribution functions of the turn-on time and the average output power for $C_b = 0.95$ and $1.1\ C_{th}$. We will show that the observed large timing jitter for $C_b = 1.1\,C_{th}$ originates from bimodal probability distribution associated with pattern-dependent effects. In Fig. 8 the probability distribution functions for the turn-on delay time for $C_b = 0.95\,C_{th}$ and $1.1\,C_{th}$ are shown. The distribution function for $C_b = 0.95\,C_{th}$ is very similar to the one shown in Fig. 5 for a periodic modulation. This fact confirms that for a bias around this value the response of the system is almost independent of t_{off}. The probability distribution function for $C_b = 1.1\,C_{th}$ must be also compared with the one in Fig. 5. It can be seen that strong differences, between periodic and pseudorandom word modulation, appear when biasing above threshold. Under pseudorandom word modulation we find a bimodal distribution which is very different from the corresponding Gaussian. The two peaks of the distribution yield the large jitter observed in Fig. 6 for $t_{\text{off}} < 500$ ps. There are clear indications that, at least in a first approximation, there are two characteristic turn-on times, which could be associated with a "1 1" sequence and a "0 1" sequence. Moreover, each part of the bimodal distribution has a width which is smaller than that of the single peaked distribution for $C_b = 0.95\,C_{th}$. The probability distribution functions of the average output power are also shown in Fig. 8. Again for $C_b = 1.1\,C_{th}$, a bimodal distribution appears, which could be also associated with the sequences "1 1" and "0 1" already mentioned.

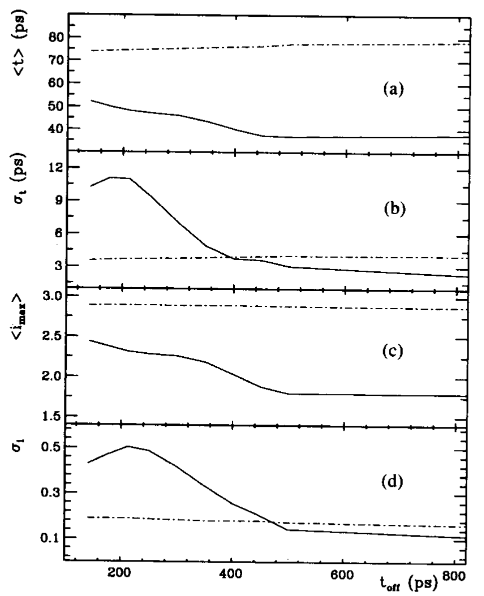

Fig. 7. (a) Mean turn-on time. (b) Time jitter. (c) Mean and (d) standard deviation of the maximum output photon number are plotted as a function of t_{off}, for 150 ps $\leq t_{off} \leq$ 800 ps for different C_b: $C_b = 0.95\, C_{th}$ (dot-dash line) and $C_b = 1.1\, C_{th}$ (solid line).

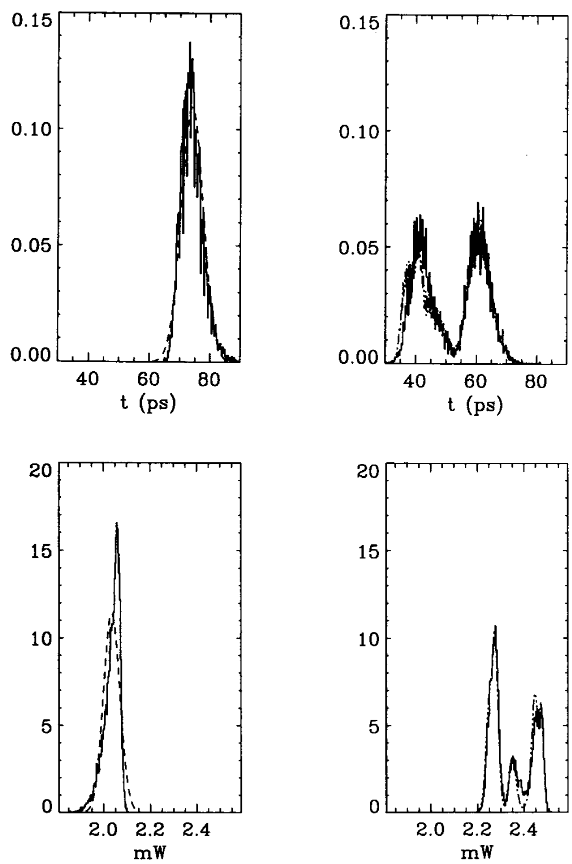

Fig. 8. The same probability distribution functions as in Fig. 5 but for a pseudorandom word modulation.

The origin of the bimodal distributions for $C_b = 1.1\, C_{th}$ can be understood as follows. As a first approximation the two peaks can be associated with the two sequences "1 1 1 1 · · ·" and "0 1 0 1 · · ·," as has been already suggested. However, these two sequences are not enough for a proper quantitative description of Fig. 8. We have calculated the probability distribution associated with the last bit "1" when the laser is periodically modulated with the following eight sequences of four digits: "1 1 1 1," "0 1 1 1," "1 0 1 1," "0 0 1 1," "0 1 0 1," "1 1 0 1," "1 0 0 1," and "0 0 0 1." The normalized probability distribution functions associated with the eight sequences are shown in Fig. 9. The normalized distribution function given by the superposition of the ones in Fig. 9, each one with the same weight, is shown (dashed-dotted line) in Fig. 8. A very good agreement with the distribution function for the pseudorandom word modulation can be observed. This means that the superposition of the 8 independent sequences of 4 pulses can describe, in a very good way, the response of the laser to a pseudorandom word modulation. This can be understood as a memory of the system, which only remembers a finite sequence of bits before a "1" bit. For our laser parameters, we can say that the system has a "four bit memory," for $C_b = 1.1\, C_{th}$. For $C_b = 0.95\, C_{th}$ the system has almost "no memory." There exists a precise value of C_b for which the system loses its memory, so that pattern effects are avoided. We consider this special bias value in the next section.

The problem of bimodal distributions has been also discussed previously by Shen [20]. For his laser parameters, bias values about 10% far from the threshold value, and frequencies of 1 GHz, a double-peaked probability distribution function of the turn-on delay time appeared when biasing below threshold, and a single peaked probability distribution function when biasing above threshold. This result indicates a change from single peaked to bimodal distributions in an opposite direction than the one discussed above. The reason for this difference lies in the change of qualitative behavior discussed in connection with Fig. 7. If we consider larger values of t_{off} reducing the frequency to 1 GHz the results by Shen are reproduced. This is explicitly shown in Fig. 10 for $t_{off} = 850$ ps and $t_{on} = 150$ ps ($f = 1$ GHz), where a bimodal distribution occurs for $C_b = 0.9\, C_{th}$ and a single peaked distribution for $C_b = 1.1\, C_{th}$. However, if C_b approaches $\overline{C}_b < C_{th}$ the probability distribution function becomes single peaked for $f = 1$ GHz and also for any other frequency. For $C_b = 1.1\, C_{th}$, the probability distribution function becomes bimodal when going to $f \gtrsim 2$ GHz. To avoid bimodal distributions at $f \gtrsim 2$ GHz, when biasing above threshold, the bias has to be increased further and further beyond the 10% value above threshold used by Shen.

V. Pattern-Effects Suppression Bias

The response of the laser to an input bit (a "1" bit or "0" bit chosen randomly) depends on previous bits for almost all values of the bias current. Moreover, for a bias

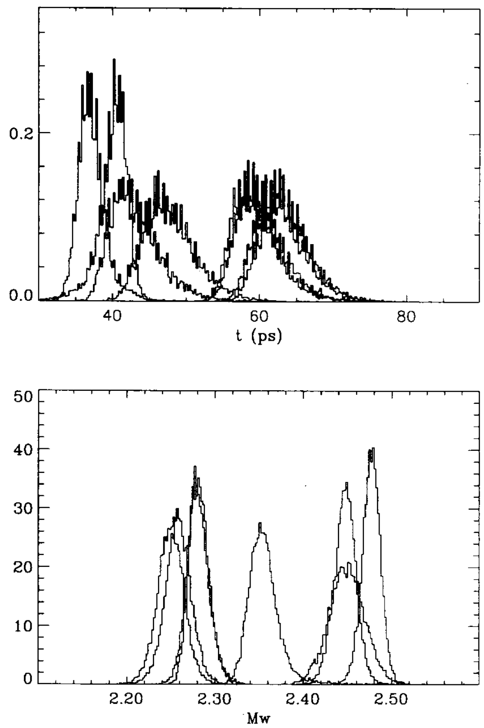

Fig. 9. Probability distribution functions for the 8 different sequences of periodic modulation: "1 1 1 1," "0 1 1 1," "1 0 1 1," "0 0 1 1," "0 1 0 1," "1 1 0 1," "1 0 0 1," and "0 0 0 1" for the turn-on time (upper panel) and average output power per pulse (lower panel).

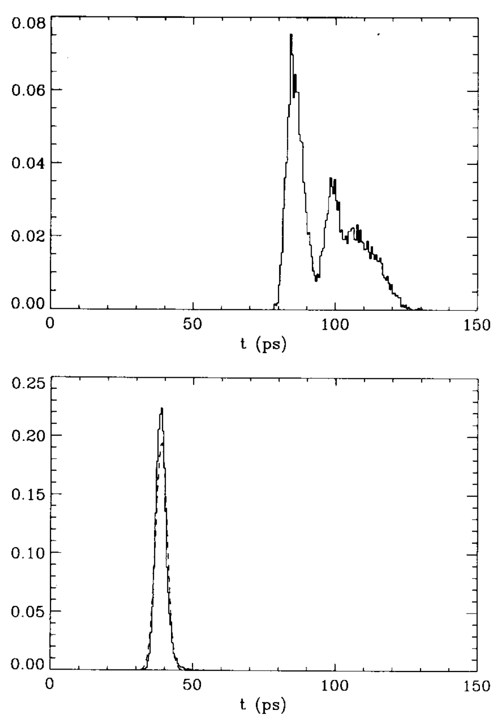

Fig. 10. Probability distribution function of the turn-on time for $C_b = 0.9\,C_{th}$ (upper panel) and $C_b = 1.1\,C_{th}$ (lower panel) for a modulation frequency of 1 GHz (t_{on} = 150 ps and t_{off} = 850 ps).

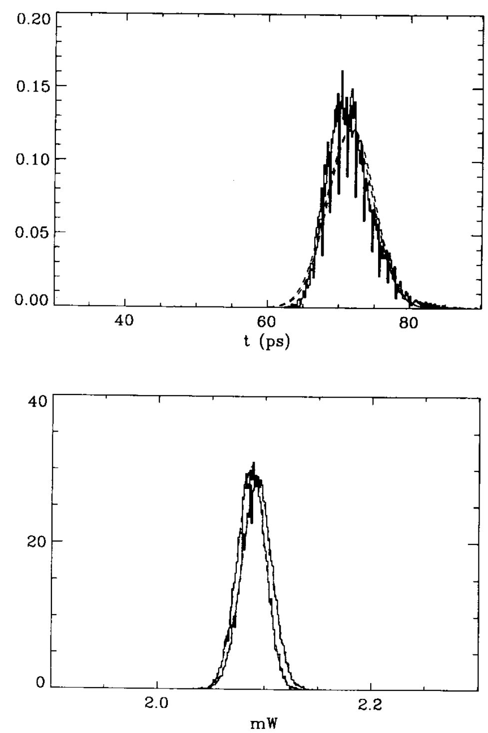

Fig. 11. Probability distribution functions of the turn-on time (upper panel) and average output power per pulse (lower panel) for the privileged bias $C_b = 0.98\,C_{th}$ (t_{off} = 150 ps).

current above threshold we find, in a convenient range of parameters, a bimodal distribution for the turn-on time and for the average output power per pulse (and also for w and i_{max}). Such pattern dependent effect makes very difficult to predict the behavior of the output signal. However, a special value of the bias current, slightly below threshold, seems to make the response of the laser to an input bit, randomly chosen, independent of previous bits. This special value is associated with the minimum value N_m reached by the carrier number N during the first relaxation oscillation, from $C = C_{on}$, which we already discussed in Section III. When the bias current is set at the value $\overline{C}_b = N_m \gamma_e$, after a "1" bit, the system evolves with $I = 0$ and $N = N_m$ constant, independently of t_{off}. Therefore, the initial conditions for the following input pulse are exactly known. If the following input bit is a "0", the system remains with $I = 0$ and $N = N_m$ during the full period, giving the same initial conditions for the following input pulse, as if the input was a "1." For our parameter values this minimum is $N_m \approx 0.98\ N_{th}$. Clear evi dence to substantiate this idea is given in Fig. 11, where the distribution functions for $C_b = \overline{C}_b = 0.98\ C_{th}$ are shown. The probability distribution functions obtained under periodic or pseudorandom word modulation are superimposed. As can be seen both distributions coincide to a very good accuracy, which clearly shows that the laser response is almost independent of previous input bits. In this situation pattern effects are greatly suppressed.

VI. Summary and Conclusion

We have studied the dynamical behavior of a single-mode semiconductor laser under high speed modulation of the injected current. We have analyzed pulse statistics under two different type of modulations: periodic modulation and pseudorandom word modulation. In the former case the statistical quantities are quite independent of the bias current, for a bias close to threshold, and frequencies greater than 2 GHz. For smaller frequencies, the results approach the ones obtained for gain switching, where the statistical quantities (specially the jitter) strongly depend on the bias current. In the latter case the system behaves in a very different manner. For high frequencies, timing jitter and the dispersion in the maximum output photon number become larger when biasing above threshold than when biasing below threshold. This large jitter appears as a consequence of a bimodal distribution of the turn-on time associated with pattern dependent effects. This fact makes difficult to predict the response of the system to the following input pulses. We have shown that there exists a special value for the bias current, slightly below threshold, for which the response of the system is almost independent of previous bits. This special value is associated with the minimum value reached by N in the first relaxation oscillation. It has been analytically estimated and its value is, for our parameter values, $\overline{C}_b \simeq 0.983\ C_{th}$. For such a bias value pulse statistics are independent of t_{off} and the probability distribution functions of the turn-on time and average output power are single peaked both for the periodic and pseudorandom word modulation regimes. The strong suppression of pattern effects for this bias value is made clear by showing the coincidence of the probability distribution functions of turn-on time and average output power in the case of both periodic and pseudorandom word modulation.

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References

[1] G. P. Agrawal, "Noise in semiconductor lasers and its impact in optical communication systems," in *Proc. SPIE*, vol. 1376, R. Roy, Ed., pp. 224-235, 1990.

[2] P. Spano, A. Mecozzi, A. Sapia, and A. D'Ottavi, "Noise and transient dynamics in semiconductor lasers," Third Int. Workshop on Nonlin. Dynamics Quant. Phenom. in Opt. Systems, R. Corbalan and R. Vilaseca, Eds. New York: Springer-Verlag, 1991.

[3] A. Mecozzi, A. Sapia, P. Spano, and G. P. Agrawal, "Transient multimode dynamics in nearly single-mode lasers," *IEEE J. Quantum Electron.*, vol. 27, pp. 332-343, 1991.

[4] P. Spano, A. D'Ottavi, B. Daino, and S. Piazzolla, "Experimental measurements and theory of first passage time in pulse-modulated semiconductor lasers," *IEEE J. Quantum Electron.*, vol. 25, pp. 1440-1449, 1989.

[5] S. E. Miller, "On the prediction of the mode-partitioning floor in injection lasers with multiple side modes at 2 and 10 Gb/s," *IEEE J. Quantum Electron.*, vol. 26, pp. 242-249, 1990.

[6] J. C. Cartledge, "On the probability of side mode fluctuations in pulse-modulated nearly-single-mode semiconductor lasers," *IEEE J. Quantum Electron.*, vol. 26, pp. 2046-2051, 1990.

[7] K. Y. Lau, "Gain switching of semiconductor injection lasers," *Appl. Phys. Lett.*, vol. 52, pp. 257-259, 1988.

[8] A. Czylwik and W. Eberle, "Transient intensity noise of semiconductor lasers: Experiments and comparison with theory," *IEEE J. Quantum Electron.*, vol. 26, pp. 225-230, 1990.

[9] M. San Miguel, "Statistics of laser switch-on," in *Proc. SPIE*, vol. 1376, R. Roy, Ed., pp. 272-283, 1990; S. Balle, P. Colet, and M. San Miguel, "Statistics of transient response of a single-mode semiconductor laser gain switching," *Phys. Rev. A*, vol. 43, pp. 498-506, 1991; S. Balle, F. de Pasquale, N. B. Abraham, and M. San Miguel, "Statistics of transient frequency modulation in the switch-on of a single mode semiconductor laser," *Phys. Rev. A*, vol. 45, pp. 1955-1966, 1992.

[10] D. Marcuse, "Computer simulation of laser photon fluctuations: DFB lasers," *IEEE J. Quantum Electron.*, vol. QE-21, pp. 161-167, 1985.

[11] M. M. Choy, P. L. Liu, P. W. Shumate, T. P. Lee, and S. Tsuji, *Appl. Phys. Lett.*, vol. 47, pp. 448-449, 1985.

[12] A. J. Taylor, J. M. Wiesenfeld, G. Eisenstein, and P. S. Tucker, *Appl. Phys. Lett.*, vol. 49, pp. 681-683, 1986.

[13] S. E. Miller, "Turn-on jitter in nearly single-mode injection lasers," *IEEE J. Quantum Electron.*, vol. QE-22, pp. 16-19, 1986.

[14] P. Spano, A. D'Ottavi, A. Mecozzi, and S. Piazzola, "Experimental observation of time jitter in semiconductor laser turn-on," *Appl. Phys. Lett.*, vol. 52, pp. 2203-2204, 1988; A. Mecozzi, S. Piazzolla, A. D'Ottavi, and P. Spano, "Passage time statistics in semiconductor laser turn on," *Phys. Rev. A*, vol. 38, pp. 3136-3139, 1988.

[15] E. H. Böttcher, K. Ketterer, and D. Bimberg, "Turn-on delay time fluctuations in gain-switched AlGaAs/GaAs multiple-quantum-well lasers," *J. Appl. Phys.*, vol. 63, pp. 2469-2471, 1988.

[16] A. Mecozzi, P. Spano, A. D'Ottavi, and S. Piazzola, "Analysis of transients in pulse modulated semiconductor lasers biased near threshold," *Appl. Phys. Lett.*, vol. 55, pp. 769-771, 1989.

[17] S. E. Miller, "On the injection laser contribution to mode partition noise in fiber telecommunication systems," *IEEE J. Quantum Electron.*, vol. 25, pp. 1771-1781, 1989.

[18] P. L. Liu and M. M. Choy, "Modeling rare turn-on events of injection lasers," *IEEE J. Quantum Electron.*, vol. 25, pp. 1767-1770, 1989.

[19] P. Spano, A. Mecozzi, and A. Sapia, "Intensity noise structure in laser gain switching," *Phys. Rev. Lett.*, vol. 64, pp. 3003-3006, 1990; A. Mecozzi, P. Spano, and A. Sapia, "Quasi-deterministic theory of semiconductor laser gain switching," *Opt. Lett.*, vol. 19, pp. 1067-1069, 1990.

[20] T. M. Shen, "Timing jitter in semiconductor lasers under pseudorandom word modulation," *J. Lightwave Technol.*, vol. 7, pp. 1394-1399, 1989.

[21] C. R. Mirasso, P. Colet, and M. San Miguel, "Pulse statistics in single-mode semiconductor lasers modulated at gigahertz rates," *Opt. Lett.*, vol. 16, pp. 1753-1755, 1991.

[22] G. P. Agrawal and N. K. Dutta, *Long wavelength semiconductor lasers*. New York: Van Nostrand Reinhold, 1986.

[23] G. P. Agrawal, "Spectral hole burning and gain saturation in semiconductor lasers," *J. Appl. Phys.*, vol. 63, pp. 1232-1234, 1988.

[24] M. Lax, "Fluctuations from the Non-equilibrium steady state," *Rev. Mod. Phys.*, vol. 32, pp. 25-64, 1960; M. Lax and W. H. Louisell, "Quantum noise. XII. Density-operator treatment of field and population fluctuations," *Phys. Rev.*, vol. 185, pp. 568-591, 1969.

[25] C. W. Gardiner, *Handbook of stochastic methods*. Berlin: Springer-Verlag, 1983.

[26] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions*. New York: Dover, 1965.

Claudio R. Mirasso, photograph and biography not available at the time of publication.

Pere Colet, photograph and biography not available at the time of publication.

Maxi San Miguel, photograph and biography not available at the time of publication.