Role of dimensionality in Axelrod's model for the dissemination of culture

Konstantin Klemm

Interdisciplinary Centre for Bioinformatics, University of Leipzig, Kreuzstr. 7b, D-04103 Leipzig, Germany

Víctor M. Eguíluz, Raúl Toral, Maxi San Miguel

Instituto Mediterráneo de Estudios Avanzados, IMEDEA (CSIC-UIB) E-07071 Palma de Mallorca, Spain

Abstract

We analyze a model of social interaction in one and two-dimensional lattices for a moderate number of features. We introduce an order parameter as a function of the overlap between neighboring sites. In a one-dimensional chain, we observe that the dynamics is consistent with a second order transition, where the order parameter changes continuously and the average domain diverges at the transition point. However, in a two-dimensional lattice the order parameter is discontinuous at the transition point characteristic of a first order transition between an ordered and a disordered state.

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1 Introduction.

There has been some recent work addressing the issue of consensus formation [1–3]. In the context of cultural globalization, Axelrod [4] proposed a simple model for the analysis of how cultural features disseminate. The model is based on the premise that by an interaction the similarity of the two interacting individuals is increased. A more detailed analysis has shown that depending on the initial diversity there is a phase transition between an ordered monocultural state and a disordered multicultural state [5]. Further analyses of this model have been devoted to checking the robustness of the transition in the presence of noise [6], changing the range and topology of interactions (e.g., in small-world and scale-free networks) [7,8], and modifying the interaction probability [9]. In the present paper we show that the order of the phase transition depends on the spatial dimensionality of the problem.

2 The model

The model we study is defined by considering N agents as the sites of a lattice [4]. The state of agent i is a vector of F components (cultural features) $(\sigma_{i1}, \sigma_{i2}, \cdots, \sigma_{iF})$. Each σ_{if} can take any of the q integer values (cultural traits) $1, \ldots, q$, initially assigned independently and with equal probability $1/q$. The time-discrete dynamics is defined as iterating the following steps:

(1) Select at random a pair of neighboring sites of the network connected by a bond (i, j).
(2) Calculate the *overlap* (number of shared features) $l(i,j) = \sum_{f=1}^{F} \delta_{\sigma_{if},\sigma_{jf}}$.
(3) If $0 < l(i,j) < F$, the bond is said to be *active* and sites i and j interact with probability $l(i,j)/F$. In case of interaction, choose g randomly such that $\sigma_{ig} \neq \sigma_{jg}$ and set $\sigma_{ig} = \sigma_{jg}$.

By definition, adjacent sites i and j cannot interact if either they share all traits ($l(i,j) = F$) or none of them ($l(i,j) = 0$). Then we call the bond between i and j inactive. A given configuration is absorbing if all bonds are inactive. The q^F completely homogeneous configurations, where $l(i,j) = F$ for all i and j are absorbing. In large systems, most absorbing configurations contain both kinds of inactive bonds. The saturated bonds ($l(i,j) = F$) connect sites belonging to a homogeneous cluster whereas the non-overlapping bonds ($l(i,j) = 0$) form the borders between clusters. In order to characterize the absorbing configurations, we introduce the order parameter

$$L = 2(zNF)^{-1} \sum_{\text{bonds}\,(i,j)} (F - l(i,j)) \;, \tag{1}$$

where z is the coordination number of the lattice and the sum is performed over all bonds. L assumes values in the interval $[0, 1]$. The minimum value $L = 0$ is reached only by the completely homogeneous configurations. Note that in one-dimensional lattices with $z = 2$, the dynamics always decreases the value of L. This role of L as a Lyapunov function has been proven rigorously [10]. Furthermore, L can be written as $L \propto \sum_{f=1}^{F} \sum_{\text{bonds}\,(i,j)} \delta_{\sigma_{if},\sigma_{jf}}$ which is the negative energy of F uncoupled Potts models.

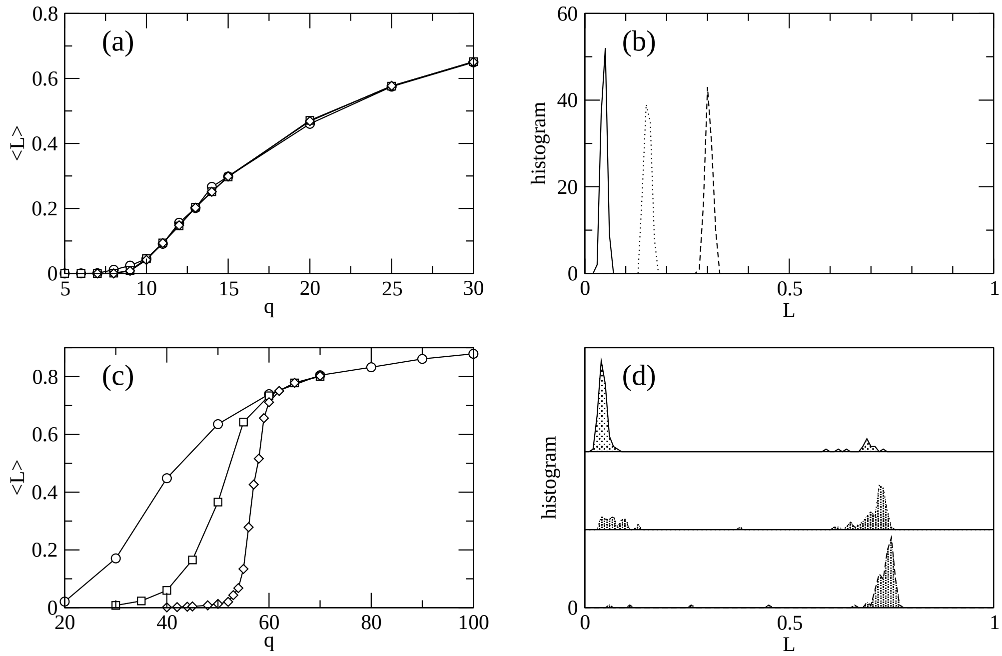

Fig. 1. (a) Dependence of the average order parameter $\langle L \rangle$ on the parameter q in one-dimensional lattices of sizes $N = 100$ (circles), 1000 (squares), and 10000 (diamonds), using F=10. (b) Distributions of L using the same setup as in (a) with $N = 10000$ and $q = 10$ (solid), $q = 12$ (dotted), and $q = 15$ (dashed line). (c) Dependence of the average order parameter $\langle L \rangle$ on the parameter q in two-dimensional lattices of sizes $N = 10^2$ (circles), 30^2 (squares), and 100^2 (diamonds), using F=10. (d) Distribution of L using the same setup as in (c) with $N = 10000$ and $q = 55, 58, 60$ (top to bottom). Averages and distributions in (a)-(d) are based on 100 independent realizations for each value of q.

3 Transitions in one and two dimensions

Figure 1 compares the behavior of the order parameter in one- and two-dimensional systems. For the one-dimensional lattice with bonds between nearest neighbors (Fig. 1(a)), the averaged order parameter remains zero for parameter values $q < q_c$, then for q growing beyond q_c increases *continuously*. For a given value of q, values of L for different realizations are narrowly distributed around the mean value, with unimodal distributions for L.

For the two-dimensional lattices (coordination number $z = 4$) quite a different behavior is observed. The order parameter undergoes a discontinuity at a transition point q_c (Fig. 1(c)). At this parameter value, the cumulative distribution of L shows peaks at two distinct values (Fig. 1(d)). The distribution of the order parameter is bimodal amounting to bistability of the system.

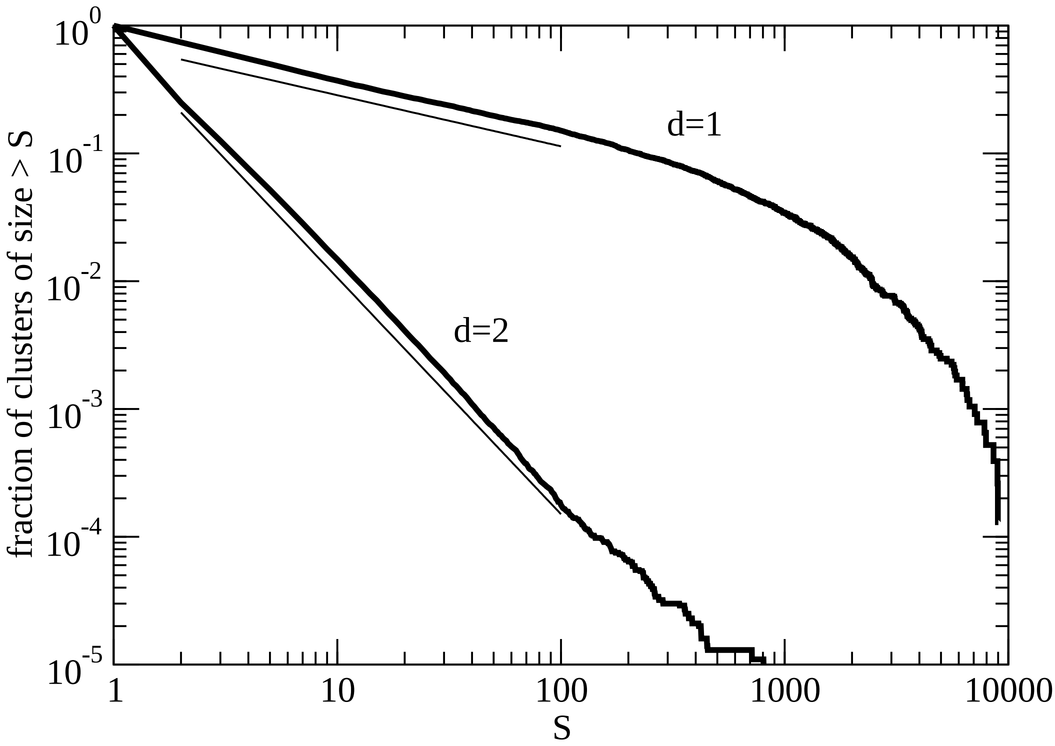

Fig. 2. Cumulative distributions of cluster sizes at the transition point (thick curves). For the one-dimensional system ($d = 1$) $q = 9$, for the two-dimensional system ($d = 2$) $q = 62$ has been used. In both cases $N = 10^4$ and $F = 10$. The thin lines have slopes -0.4 and -1.85.

The transitions in $d = 1$ and $d = 2$ are furthermore distinguishable by the distributions of cluster sizes at the transition, plotted in Fig. 2. While in both dimensions these distributions decay as a power law $p(S) \sim S^{-\tau_d}$, the observed exponents τ are clearly different. In the one-dimensional system, $\tau_1 \approx 1.4$ is found. For $d = 2$, we have $\tau_2 \approx 2.85$.

The values of the exponents hint at qualitatively different scenarios for $d = 1$ and $d = 2$. In the one-dimensional case the transition is similar to percolation [11]. Then we have an exponent $\tau_1 < 2$ for the power law regime of the cluster size distribution such that—in the limit of infinite system size—the first moment $\langle S \rangle$ diverges with the cut-off $S_{\max}$ as q approaches q_c from above. At the critical value of the parameter q the system is still disordered. In the case $d = 2$ there is no divergence of $\langle S \rangle$ with $S_{\max}$ because here the cluster size distribution has the exponent $\tau_2 > 2$. It is worth noting that a similar behavior has been observed in the case $F = 2$ and $d = 2$ where the exponent is smaller than 2 in contrast to the case $F > 2$ studied here [5].

Previous studies have employed the average fraction of sites occupied by the largest cluster $\langle S_{\max} \rangle / N$ as an order parameter [5,6,8,12]. We have observed that the distribution of $S_{\max}$ shows bistability only for the two-dimensional system, while it is unimodal in $d = 1$.

4 Conclusions.

We have analyzed Axelrod's model for cultural dissemination by extensive numerical simulations. We have introduced the average relative overlap between neighbors as a measure of the the order in the system. We have found that while in a one-dimensional chain the system shows a second order transition, in a two dimensional lattice the transition is first order. We have also obtained numerically the distribution of domains close to the transition points, showing that in the one dimensional case the average size of a cultural domain diverges. These results stress the role of the topology of the interaction network [8].

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