Open- Versus Closed-Loop Performance of Synchronized Chaotic External-Cavity Semiconductor Lasers

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***Abstract*—We numerically study the synchronization or entrainment of two unidirectional coupled single-mode semiconductor lasers in a master–slave configuration. The emitter laser is an external-cavity laser subject to optical feedback that operates in a chaotic regime. The receiver can either operate at a chaotic regime similar to the emitter (closed-loop configuration) or without optical feedback and consequently under continuous-wave conditions when it is uncoupled (open-loop configuration). We compute the degree of synchronization of the two lasers as a function of the emitter-receiver coupling constant, the feedback rate of the receiver, and the detuning. We find that the closed-loop scheme has, in general, a larger region of synchronization when compared with the open loop. We also study the possibility of message encoding and decoding in both open and closed loops and their robustness against parameter mismatch. Finally, we compute the time it takes the system to recover the synchronization or entrainment state when the coupling between the two subsystems is lost. We find that this time is much larger in the closed loop than in the open one.**

***Index Terms*—Chaos, chaos encryption, optical chaos communications, semiconductor lasers, synchronization.**

I. Introduction

SYNCHRONIZATION of chaotic systems has attracted the attention of many researchers in the last decade. This interest was motivated by the pioneering work of Pecora and Carroll [1], an idea that was implemented in electronic circuits by Cuomo and Oppenheim [2]. After these successful papers, the possibility of applying such techniques to encode and decode information within a chaotic carrier has been developed. The first experiments were carried out using electronic circuits, such as Lorenz or Chua circuits. However, such systems present two disadvantages: on the one hand, the maximum frequency for the chaotic carriers is some tens of kilohertz and, on the other hand, the dimensionality of the generated chaos is low (typically less than three), allowing an easy interception and recovery of the message.

Most of these problems were overcame when working in the optical domain and by using delayed optical feedback to generate chaotic carriers. Based on these ideas, it was numerically shown that a message could be encoded and decoded within a high-dimensional chaotic carrier when using a pair of unidirectionally coupled semiconductor lasers subjected to coherent optical feedback [3], [4]. Experimental results were later obtained for erbium-doped fiber ring lasers [5] and semiconductor lasers [6]–[8]. More recently, it was also shown that the system would also work when using incoherent optical feedback [9] or optoelectronic feedback [10].

Many studies have already been carried out to check the robustness of the synchronized systems [11]–[14]. However, although most of these studies were done for two chaotic external-cavity semiconductor lasers (closed-loop scheme), many of the experimental studies where done on systems composed of an external-cavity semiconductor laser as an emitter while the receiver operates without any external feedback (open-loop scheme), i.e., under continuous-wave (CW) conditions when it is uncoupled to the emitter. In the latter, the receiver is entrained by the light coming from the emitter.

In this work, we numerically study the synchronization properties of the two unidirectionally coupled chaotic single-mode semiconductor lasers. We also analyze the message encoding/decoding for both closed- and open-loop configurations in order to determine the advantages and disadvantages. We can anticipate that in most cases, the closed-loop scheme has a better performance than the open one, although it requires a careful adjustment of both external cavities to operate correctly. In Section II, we present the model, in Section III, we present the results, and a summary and several conclusions are given in Section IV.

II. The Model

We study the synchronization between two single-mode semiconductor lasers in a master–slave configuration. We model the transmitter and receiver lasers by using the well-known Lang–Kobayashi rate equations [15] for the complex slowly varying amplitude of the electrical field $E_{t,r}$ and the carrier number inside the cavity $N_{t,r}$. With the assumption of a free link between both lasers and the introduction of the symmetric reference frame $\Omega = (\omega_t + \omega_r)/2$, $\Delta\omega = \omega_t - \omega_r$, these equations read [14]

$$\begin{aligned}\dot{E}_{t,r}(t) =& \pm \frac{i\Delta\omega}{2} E_{t,r} + \frac{(1+i\alpha)}{2}\left[G_{t,r} - \frac{1}{\tau_{ph}}\right] E_{t,r} \\ &+ \kappa_{t,r} E_{t,r}(t-\tau_f) e^{-i\Omega\tau_f} \\ &+ \kappa_c E_t(t-\tau_c) e^{-i\Omega\tau_c} + F_{E_{t,r}}(t) \end{aligned} \quad (1)$$

$$\dot{N}_{t,r}(t) = \frac{I_{t,r}}{e} - \frac{N_{t,r}}{\tau_n} - G_{t,r} P_{t,r}(t) \quad (2)$$

$$G_{t,r}(t) = \frac{g(N_{t,r} - N_o)}{1 + sP_{t,r}(t)} \quad (3)$$

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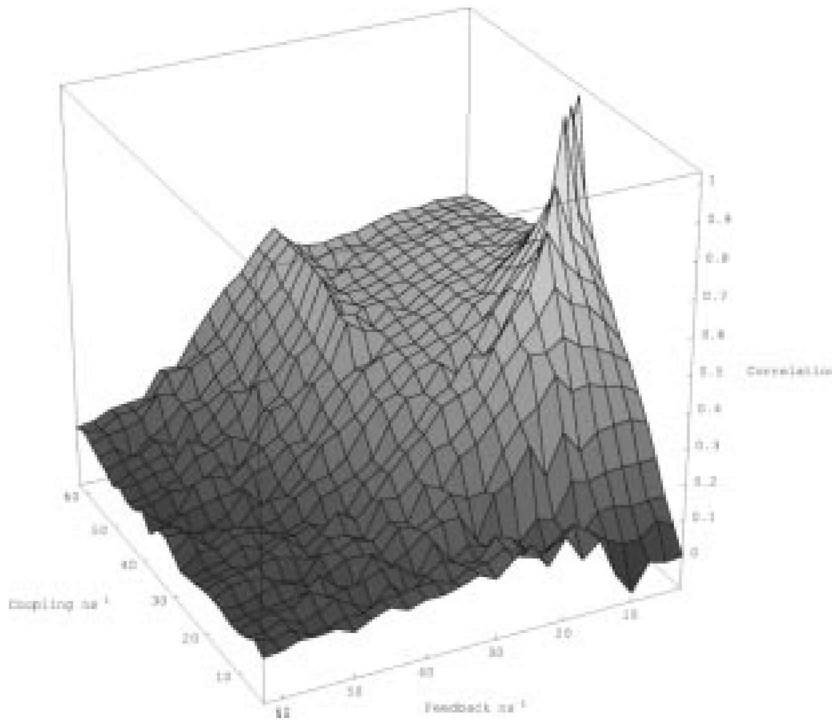
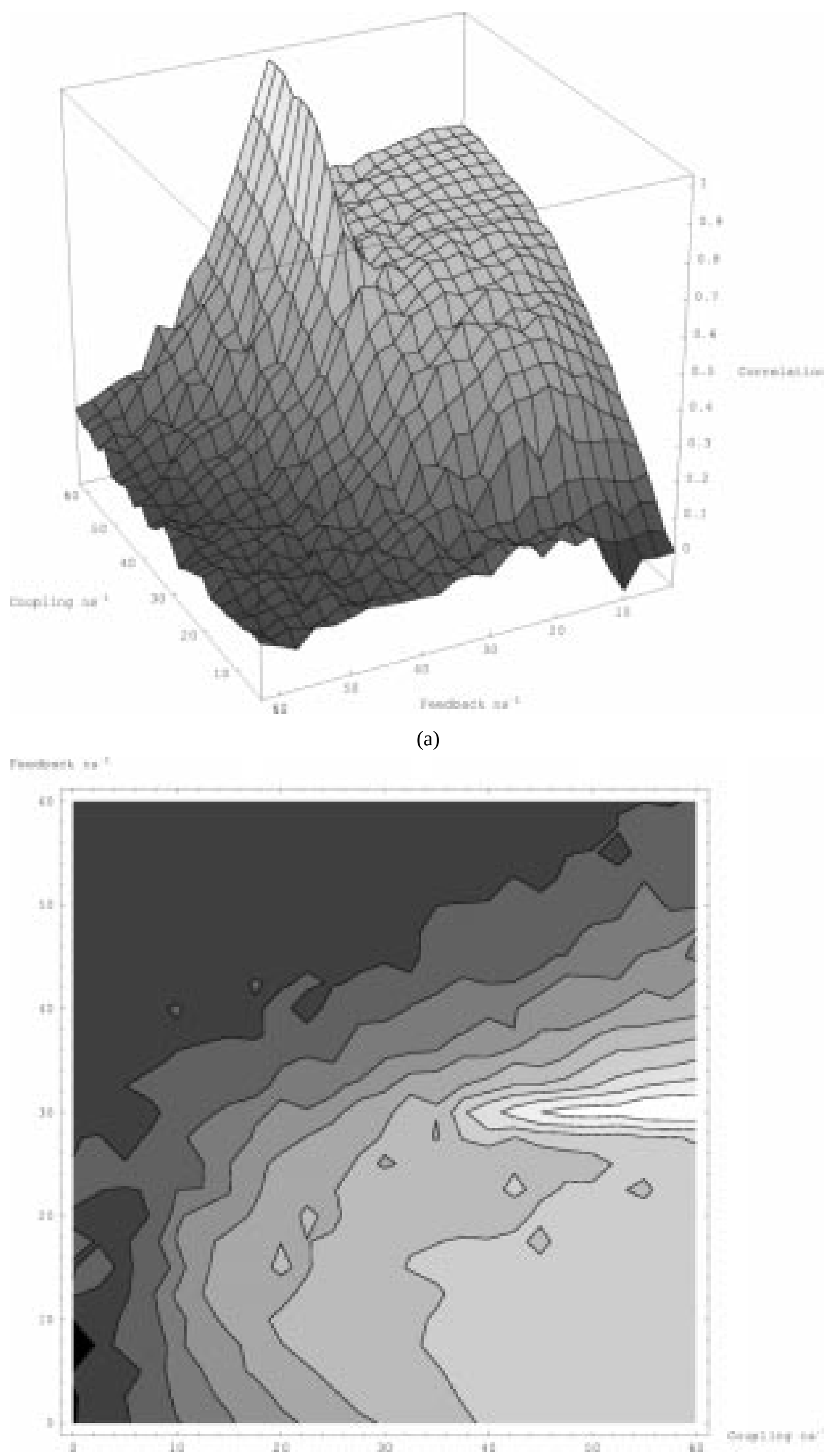


(a)

(b)

Fig. 1. $C(-\tau_c + \tau_f)$ as a function of the coupling constant and feedback rate of the receiver system. The feedback strength of the transmitter is fixed to 30 ns^{-1}. (a) 3-D and (b) contour plots are shown. A high (low) degree of synchronization is characterized by a light (dark) gray level.

where the subindices t, r correspond to the transmitter or master laser (ML) and receiver or slave laser (SL). The term $\kappa_c e^{-i\Omega\tau_c} E_t(t-\tau_c)$ only appears for the SL and accounts for the amount of ML output power that is injected into the SL. The last term in (1) represents Langevin noise sources that describe spontaneous emission processes. In this work, we will neglect their effect since it is already known that they slightly degrade the synchronization quality [16]. $P_{t,r}(t) = |E_{t,r}(t)|^2$ is the optical intensity or number of photons in the cavity. We consider both lasers to be very similar to each other and consequently we take the same parameter values: $\alpha = 5$ is the linewidth enhancement factor, $g = 1.5 \times 10^{-8}$ ps^{-1} is the gain parameter, $s = 5 \times 10^{-7}$ is the gain saturation coefficient, $\tau_{ph} = 2$ ps is the photon lifetime, $\tau_n = 2$ ns is the carrier



(a)

(b)

Fig. 2. $C(-\tau_c)$ as a function of the coupling constant and feedback rate of the receiver system. The feedback strength of the transmitter is fixed to 30 ns^{-1}. (a) 3-D and (b) contour plots are shown. A high (low) degree of synchronization is characterized by a light (dark) gray level.

lifetime, $N_o = 1.5 \times 10^8$ is the carrier number at transparency, $e = 1.602 \times 10^{-19}$ C is the electronic charge, $\omega_{t,r}$ is the frequency of the free-running laser, $\Delta\omega = \omega_t - \omega_r$ is the detuning between the optical frequencies of the lasers, $\kappa_{t,r}$ is the feedback coefficient, κ_c is the coupling rate, τ_f is the external-cavity round-trip time, and τ_c is the time the light takes to travel from the ML to SL. $I_{t,r} = 44$ mA is the bias current (the threshold current is $I_{th} \approx 14.7$ mA).

We consider two possible situations for the system: one in which the ML is subjected to a coherent optical feedback and operates in the coherence collapse regime while the SL operates under CW conditions (open-loop scheme) when they are uncopled. For the second situation, we consider both ML and SL subject to a coherent optical feedback (closed-loop scheme). In both schemes, only the light coming from the transmitter laser is injected into the receiver.

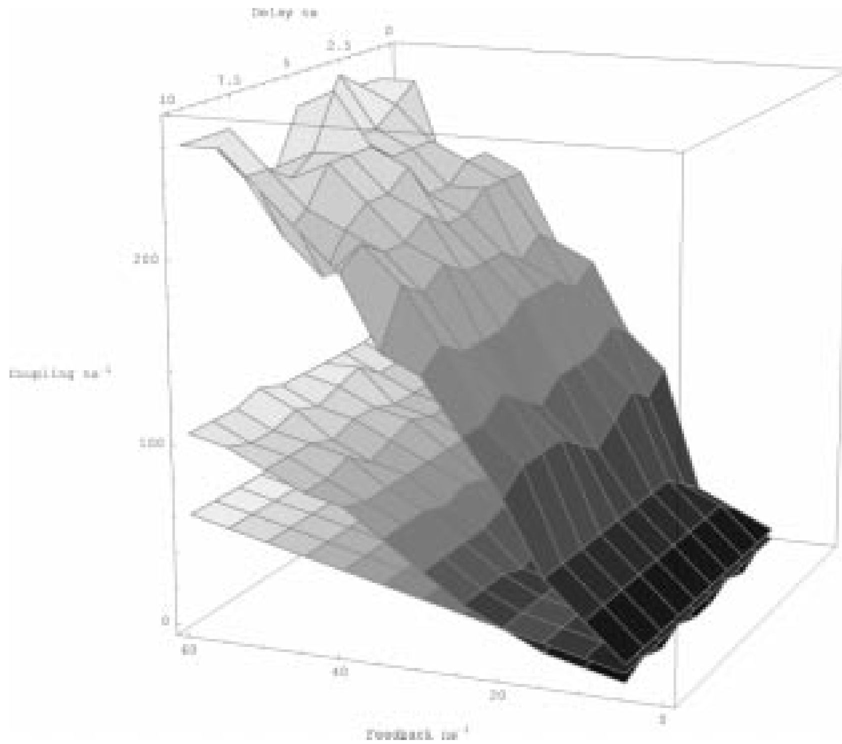


Fig. 3. Minimum coupling rate necessary to get a correlation coefficient of 0.9 as a function of the feedback delay time and feedback strength of the transmitter laser. Upper, middle, and lower surfaces correspond to the isochronous open-loop, isochronous closed-loop, and anticipating open-loop solutions, respectively. A high (low) degree of synchronization is characterized by a light (dark) gray level.

III. RESULTS

A. Synchronization Regions

Different types of synchronization have been found in coupled chaotic systems such as identical synchronization, generalized synchronization, phase synchronization, or lag synchronization [17]. Recently, two of these kinds of synchronization have been identified in unidirectionally coupled chaotic external-cavity semiconductor lasers [12]. The first type of solution is related to the so-called isochronous or generalized solution $P_r(t) = aP_t(t-\tau_c)$[13] while the second is related to the identical solution $P_r(t) = P_t(t-\tau_c+\tau_f)$ and is known as the anticipating solution [18]–[20]. These two type of synchronizations have been studied recently in terms of parameter mistmach between the emitter and receiver [21]. It has been found that while one can pass from one solution to the other when operating at low pump currents, they are well separated and it is not possible to switch from one to the other for high pump currents.

In this section, we numerically study the synchronization quality of both solutions in the parameter space of the coupling constant and feedback rate of the receiver (κ_c, κ_r), maintaining the frequency detuning at zero. The measurement of the degree of synchronization and the lag time is accomplished with the computation of the cross-correlation function

$$C(\Delta t) = \frac{\langle P_t(t+\Delta t)P_r(t)\rangle}{\sqrt{\langle P_t^2(t)\rangle\langle P_s^2(t)\rangle}}$$

between the ML and the SL output powers.

In the numerical simulations, the κ_c and κ_r coefficients are varied in the range 0-60 ns^{-1} at intervals of 2.5 ns^{-1}, while the rest of the external parameters ($I_{t,r}$, $\omega_{t,r}$, κ_t) are fixed. Fig. 1[1] shows the results obtained for $C(-\tau_c+\tau_f)$ in the parameter space (κ_c, κ_r). The synchronization region $C(-\tau_c+\tau_f) \geq 0.95$ for this anticipating solution is localized in a very narrow strip around the parameter condition $\kappa_t = \kappa_r + \kappa_c$, but, even when this condition is fulfilled , we note that a large coupling coefficient is necessary to obtain a good synchronization. In other words, the anticipating solution provided by this sufficient condition seems to lose its stability as the coupling decreases. Outside this small region, the value of the correlation coefficient does not exceed 0.6.

In Fig. 2, we plot $C(-\tau_c)$ versus the coupling and feedbacks rates. Now, the synchronization domain extends over the line $\kappa_t = \kappa_r$, and as in the previous case, a high injection rate is needed to guarantee the stability of the solution. It has to be noted that the length of the external cavities (external cavity round-trip times) has been perfectly matched to obtain a high degree of correlation. Even for lengths that differ by a fraction of the emission wavelength, the synchronization can be completely lost, emphasizing the necessity for careful control of these lengths [22]. When the system operates out of the optimal conditions ($\kappa_t = \kappa_r + \kappa_c$ for the anticipating solution and $\kappa_t = \kappa_r$ for the isochronous one), a strong degradation of the synchronization occurs in both cases. However, the isochronous solution presents a higher robustness (larger synchronization region) when comparing with the anticipating one. Thus, each solution has its own domain where the synchronization degree is high and the solution is stable. From the inspection of the cross-correlation function, we can also confirm that there are

[1]Color images are available at http://www.imedea.uib.es/~claudio/

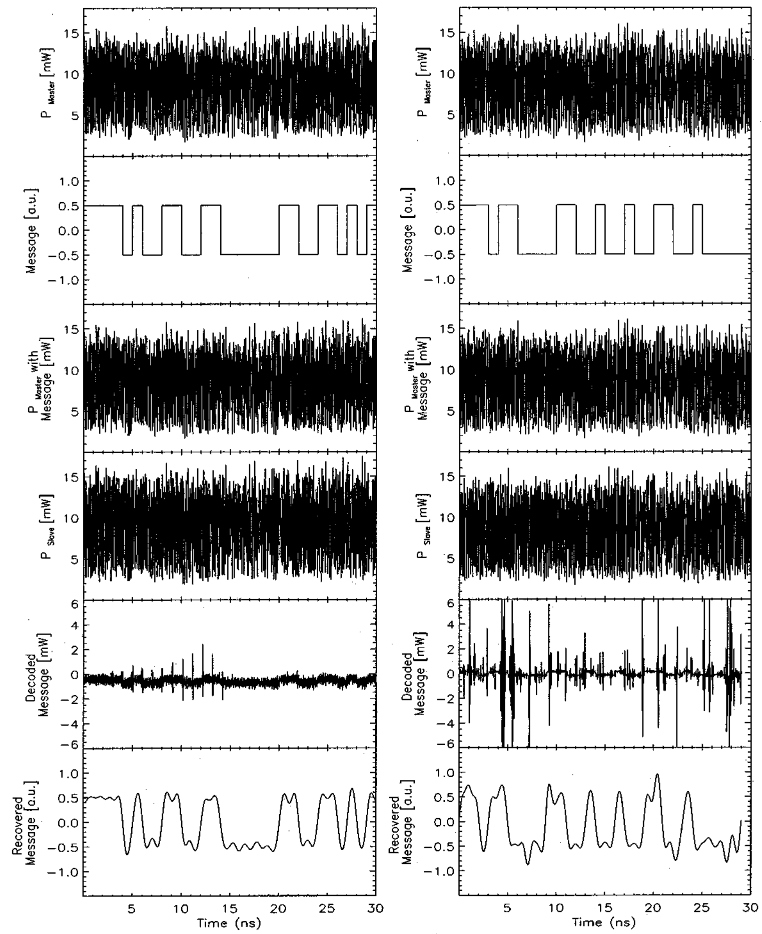


Fig. 4. Top to bottom: Output of the ML, encoded message, output of the ML with message, output of the SL, decoded message, and recovered message after filtering. Left panel: Closed loop with the isochronous solution. Right panel: Open loop with the anticipating solution.

no other lag solutions than the isochronous and the anticipated one in the regions of the parameter space we have studied. It is important to note that the isochronous solution, the one usually observed experimentally, also occurs for the open-loop case (line of SL feedback coefficient zero in Fig. 2). However, even for the maximum coupling considered in Fig. 2 (60 ns^{-1}), the value of $C(-\tau_c)$ is approximately 0.7 for this case. We have checked that a larger coefficient is necessary to reach a good degree of synchronization.

We have also computed the minimum coupling coefficient necessary to reach a correlation coefficient of 0.9 in both the open (for the anticipating and isochronous solution) and closed loop (for the isochronous solution) as a function of the feedback delay time and feedback strength of the transmitter laser. The feedback strength of the receiver laser is fixed to be $\kappa_r = \kappa_t$ for the closed loop and $\kappa_r = 0$ for the open loop. The Fig. 3 shows the results of the numerical simulations. The upper and lower surfaces correspond to the isochronous and anticipating solutions, respectively, in the open-loop regime. The middle one is obtained for the isochronous solution in the closed-loop case. We observe that this minimum coupling is in all cases independent of the delay time and it increases with feedback strength. In general, the coupling needs to be large for the isochronous solution in the open-loop configuration, while the same solution in the closed-loop case requires a much lower coupling strength. It is also observed that the anticipating solution in the open-loop scheme needs a smaller coupling coefficient than the previous ones.

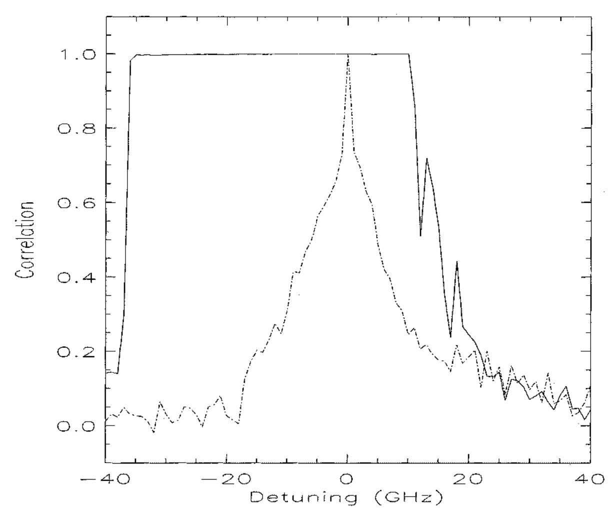


Fig. 5. Correlation coefficient as function of the detuning. Solid line: Closed-loop regime ($\kappa_t = \kappa_r = 30$ ns^{-1}, $\kappa_c = 60$ ns^{-1}). Dashed line: Open-loop regime ($\kappa_t = 30$ ns^{-1}, $\kappa_r = 0$ ns^{-1}, and $\kappa_c = 30$ ns^{-1}).

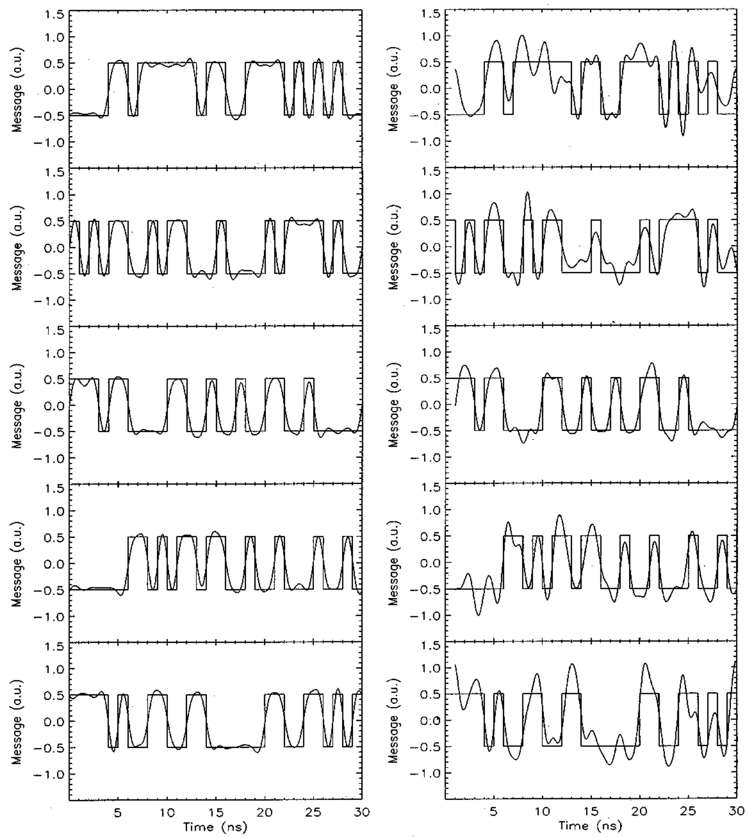


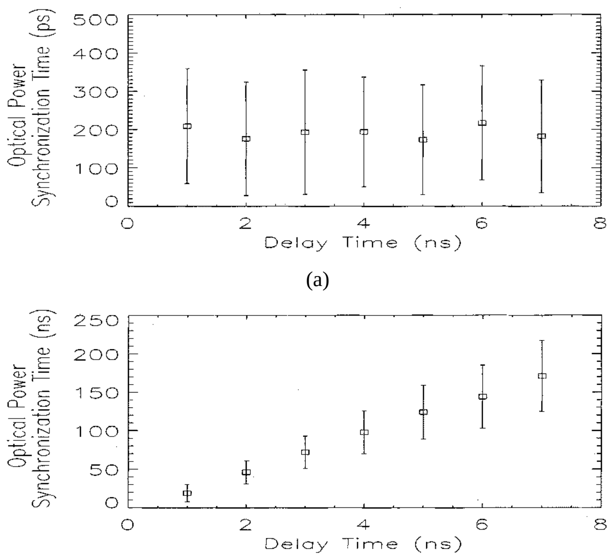
Fig. 6. Top to bottom: Encoded and recovered message after filtering for $\Delta\omega/2\pi = -2, -1$,0,1,2 GHz. Left panel: Closed loop with the isochronous solution. Right panel: Open loop with the anticipating solution.

B. Message Encoding and Decoding

Different schemes for secure message transmissions based on chaotic synchronization have been proposed up to now: chaos masking (CMA) [23], chaos modulation (CMO) [24], [25], chaos shift keying (CSK) [14], [11], and ON/OFF shift keying (OOSK) [3], [22]. In this section, we study the performance of the message encryption and recovery for the open- and closed-loop regimes using the CSK technique since, although it provides a similar quality of synchronization to that afforded by CMA, CMO, or OOSK, it is simpler to implement in a real communication system. The CSK method consists in the switching between two clearly different orbits, which define the bits "0" and "1." This switching can be obtained by means of the variation of an appropiate parameter, usually the pump current of the laser.

In the previous section, we found that the best conditions for obtaining a high degree of synchronization for the open-loop scheme (for the anticipating solution) are $\kappa_t = \kappa_c$ and $\kappa_r = 0$, while those for the closed loop (for the isochronous solution) are $\kappa_t = \kappa_r$ and a large value of κ_c. Then numerical simulations for both schemes were performed in such optimal regimes. In Fig. 4, we plot the results of the time traces of the optical power for the transmitter and receiver lasers in the open- and closed-loop configurations. The message modulation rate is 1 Gb/s with an amplitude of 1 mA. The output power emitted by the semiconductor lasers is calculated as $P = [hc\omega\alpha_m/4\pi\mu_g]|E|^2$ where h is Planck's constant, c is the speed of light in vacuum, $\alpha_m =$ 48 cm^{-1} is the facet loss, and $\mu_g = 4$ is the group refractive index. Although in both cases the messages can be recovered, from the time traces displayed in Fig. 4, a better quality of the decoded message is observed for the closed-loop (left panel) scheme than for the open-loop one (right panel). The quality of the recovered message is improved with the suppression of the fast oscillations in the decoded signal by the application of a fifth-order Butterworth filter. The time lags between the ML and SL outputs associated with the different synchronization solutions ($-\tau_c$ for the closed loop and $-\tau_c + \tau_f$ for the open loop) have been compensated for in the figures.

It is well known that one of the most important parameters to be considered is the detuning between the ML and SL. Then the possibility of a detuning between the free running laser frequencies is now taken into account. Fig. 5 shows the synchronization quality as a function of the detuning between both lasers for the open- (in the anticipating solution) and closed-loop (in the isochronous solution) schemes in the absence of a message. From the figure, we can conclude that for the open-loop case, a small detuning (hard to avoid in real systems) becomes a critical parameter and induces a dramatic loss of the anticipating synchronization while for the closed-loop scheme there exists a large range of detunings in which the synchronization remains almost perfect. For the conditions considered here, this range comprises $\sim$ 45 GHz and extends predominantly over the negative detuning region. The robustness of the synchronization is not only dependent on the kind of scheme used, but also on whether the synchronization phenomena are due to complete



(a)

(b)

Fig. 7. Representation of the synchronization recovery time for the optical power versus the external-cavity round-trip time. (a) Open-loop case. (b) Closed-loop case. Each point is averaged over 10^3 events. Error bars are also plotted.

chaos synchronization or nonlinear amplification as is discussed in [26].

Our results seem to indicate that the use of a closed-loop scheme for the transmission of an encoded message would be more robust than the open-loop one, under a detuning mismatch. Fig. 6 shows the encoded and recovered messages for the open (right panel) and closed loop (left panel) with different detuning values. A strong robustness against detuning for the closed-loop configuration is observed, while a notable degradation of synchronization appears in the open-loop case. In the latter, even for a detuning as small as 2 GHz, the recovery of the message is very difficult. Therefore, under the encryption technique we have considered (CSK), the closed-loop configuration seems to provide a better scenario for message transmission than the sensitive open-loop configuration.

C. Synchronization Recovery Time

An important point to be considered is the time it takes the system to resynchronize when a sudden cut in the link between the ML and SL occurs. We define the synchronization recovery time (or entrainment recovery time when the SL operates under CW conditions) as the time needed for the system to achieve a correlation coefficient of 0.95 from an initial uncoupled configuration where the correlation fluctuates around zero. In Fig. 7, we show the synchronization time as a function of the external-cavity round-trip time (τ_f) of the ML for the open- and closed-loop schemes while ensuring that the rest of the laser parameters are identical. The entrainment recovery time shows large fluctuations when it is measured repetitively, and induces the enormous error bars displayed in the former figure. Despite the size of the error bars, the entrainment recovery time seems to be independent of the delay time in the ML and it takes an average value of $\sim$ 200 ps for our parameter values to recover the synchronized state (Fig. 7, upper panel). On the other hand, when the system operates in the closed-loop regime, the synchronization time exhibits a linear dependence with the feedback time and is much larger (about two or three orders of magnitude) than the entrainment time. The different behavior of the synchronization recovery time is closely related to the configuration of the receiver system. When the latter operates in the closed-loop scheme, the system needs to adapt both to its actual and past states, due to the feedback term. Consequently, one would expect that the larger the feedback loop, the longer the synchronization recovery time. On the contrary, in the open-loop scheme, the receiver has no feedback and needs to adapt only to its present state and consequently one would not expect any dependence with the feedback time of the master system. Work related to the study of the synchronization and entrainment times of a delayed system is in progress.

IV. Conclusion

We have numerically studied the behavior of a chaotic communication system where the emitter laser operates subject to coherent optical feedback while the receiver can operate either with (closed loop) or without (open loop) feedback. We find, in general, that the performance of the closed-loop scheme has some advantages: it is more robust and easy to synchronize and less sensitive to the detuning between emitter and receiver, a quantity that is difficult to avoid in real systems. On the contrary, it requires a precise external cavity size of both the emitter and receiver in order to achieve synchronization. A small mismatch in the latter strongly degrades the synchronization quality [22]. A second disadvantage is that after a sudden cut in the link between emitter and receiver it requires quite long time to recover the synchronization state. On the other hand, the open-loop scheme is in general less robust and requires a perfect match between the feedback strength in the emitter and the light coupled into the receiver to achieve a good degree of entrainment that limits its operability. Also, a large coupling between ML and SL would be necessary if one looks for the isochronous solution. It has the advantage that it does not require any specification with respect to the external cavity of the emitter and that after a sudden cut in the emitter–receiver link, the recovery time for the entrainment is much shorter than in the closed-loop scheme.

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