Dynamical Entrainment of Unidirectionally Coupled Single Mode Diode Lasers

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ABSTRACT

We numerically study the entrainment of two unidirectional coupled single-mode semiconductor lasers in a master-slave configuration. The emitter laser is an external-cavity laser subject to optical feedback that operates in a chaotic regime while the receiver has no optical feedback and consequently operates under CW when it is uncoupled (open loop scheme). We compare the performance of this scheme with the close loop one in which both emitter and receiver are subjected to optical feedback and operate in a chaotic state. We compute the degree of entrainment or synchronization of the two lasers as a function of the detuning, the emitter-receiver coupling constant and the feedback rate of the receiver. We find that the close loop scheme has, in general, a larger region of synchronization when compared with the open loop. We also study the possibility of message encoding and decoding in the both open and close loops and their robustness against parameters mismatch. Finally we compute the time it takes the system to recover the synchronization or entrainment state when the coupling between the two subsystems is lost. We find that this time is much larger in the close loop than in the open one.

Keywords: Synchronization, chaos, chaos encryption, optical communications, semiconductor lasers.

1. INTRODUCTION

Synchronization of chaotic systems has attracted the attention of many researchers in the last decade. This interest was motivated by the pioneering work of Pecora and Carroll,[1] an idea that was implemented in electronic circuits by Cuomo and Oppenheim.[2] Since these successful papers, it develops the possibility of applying such techniques to encode information within a chaotic carrier, yielding an increase of the privacy in the communications. The first experiments were carried out using electronic circuits, such as Lorenz or Chua circuits. However, such systems present two disadvantages: on the one hand the maximum frequency for the chaotic carriers is some tens of KHz and, on the other hand, the dimensionality of the generated chaos is low (typically less than 3) allowing an easy interception and recovery of the message.

Most of these problems were overcame when working in the optical domain and by using delay optical feedback to generate chaotic carriers. Based on these ideas, it was numerically shown that a message could be encoded and decoded within a high dimensional chaotic carrier when using a pair of unidirectionally coupled single-mode semiconductor lasers subjected to coherent optical feedback.[3] Experimental results were later obtained for erbium doped fiber ring lasers[4] and semiconductor lasers.[5–7] More recently, it was also shown that the system would also work when using incoherent optical feedback[8] or optoelectronic feedback.[9]

Many studies have already been carried out to check the robustness of the synchronized systems.[10–13] However, although most of these studies were done for two chaotic external-cavity semiconductor lasers (close loop scheme) the experimental studies where done on systems composed of an external-cavity semiconductor laser as an emitter while the receiver operates without any external feedback (open loop scheme), i.e., under CW when it is uncoupled to the emitter. In the latter, the receiver is entrained by the light coming from the emitter.

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In this work we numerically study the synchronization and the message encoding in the open loop scheme, and its performance is compared with the close loop scheme in order to determine the advantages and disadvantages of each configuration. We can anticipate that in most of the cases the close loop scheme has a better performance than the open one although it requires a careful adjustment of both external cavities to operate correctly. In the section 2 we present the model, in section 3 we present the results and a summary and some conclusions are given in section 4.

2. THE MODEL

We study the coupling between two single-mode semiconductor lasers in a master-slave configuration. The single-mode rate equations are based on the well-known Lang-Kobayashi equations[14] and the assumption of a free link between both lasers. We define a symmetric reference frame in which $\Omega = \frac{\omega_t+\omega_r}{2}$ and the detuning $\Delta\omega = \omega_t - \omega_r$ between the optical frecuencies of the free running master laser (ω_t) and slave laser (ω_r). In this reference frame the equations read[13]:

$$\begin{aligned} \dot{E}_{t,r}(t) &= \frac{i\Delta\omega}{2}E_{t,r} + \frac{1}{2}(1+i\alpha)\left[\frac{g(N_{t,r}-N_o)}{1+s|E_{t,r}|^2} - \frac{1}{\tau_{ph}}\right]E_{t,r} + \kappa_{t,r}E_{t,r}(t-\tau_f)e^{-i\Omega\tau_f} \\ &\quad + \kappa_c E_t(t-\tau_c)e^{-i\Omega\tau_c} + F_{E_{t,r}}(t) \end{aligned} \tag{1}$$

$$\dot{N}_{t,r}(t) = \frac{I_{t,r}}{e} - \frac{1}{\tau_n}N_{t,r} - \left[\frac{g(N_{t,r}-N_o)}{1+s|E_{t,r}|^2} - \frac{1}{\tau_{ph}}\right]|E_{t,r}|^2 \tag{2}$$

where subindices t,r correspond to the transmitter or master laser (ML) and receiver or slave laser (SL). The term $\kappa_c E_t(t-\tau_c)e^{-i\Omega\tau_c}$ only appears for the SL and accounts for the amount of ML output power that is injected into the SL. The last term in equation (1) are Langevin noise sources that describe spontaneous emission processes. In this work we will neglect their effect since it is already known that they slightly degrade the synchronization quality.[15] We consider both lasers very similar to each other and consequently we take the same parameter values: $\alpha = 5$ is the linewidth enhancement factor, $g = 1.5 \times 10^{-8}$ ps^{-1} is the gain parameter, $s = 5 \times 10^{-7}$ is the gain saturation coefficient, $\tau_{ph} = 2$ ps is the photon lifetime, $\tau_n = 2$ ns is the carrier lifetime, $N_o = 1.5 \times 10^8$ is the carrier number at transparency, $e = 1.602 \times 10^{-19}$ C is the electronic charge, $\kappa_{t,r}$ is the feedback coefficient, κ_c is the coupling rate, τ_f is the external cavity round-trip time and τ_c is the time the light it takes to travels from the ML to SL. $I_{t,r} = 44$ mA is the bias current (the threshold current is $I_{th} \approx 14.7$ mA).

We consider two possible situations for the system: one in which the ML is subjected to a coherent optical feedback and operates in the coherence collapse regime while the SL operates under CW (open loop scheme) when they are uncopled. For the second situation we consider both ML and SL subject to a coherent optical feedback (close loop scheme). In both schemes, only the light coming from the transmitter laser is injected into the receiver one.

3. RESULTS

3.1. Synchronization Regions

Very recently, two types of synchronization[11] have been found in unidirectionally coupled chaotic external-cavity semiconductor lasers. The first type of synchronization is related to the isochronous solution $E_r(t) = aE_t(t-\tau_c)$[12] while the second is related to the solution $E_r(t) = E_t(t-\tau_c+\tau_f)$ and is called anticipating solution.[16–18]

In this section, we numerically study the synchronization quality of both solutions in the parameter space of the coupling constant and feedback rate of the receiver (κ_c, κ_r). To quantify the degree of synchronization, we compute the cross-correlation function

$$C(\Delta t) = \frac{\langle I_t(t+\Delta t) I_r(t) \rangle}{\left[\langle I_t^2(t)\rangle \langle I_s^2(t)\rangle\right]^{1/2}}$$

between the ML and the SL output powers. The κ_c and κ_r coefficients are varied from 0 ns^{-1} to 60 ns^{-1} at intervals of 2.5 ns^{-1}.

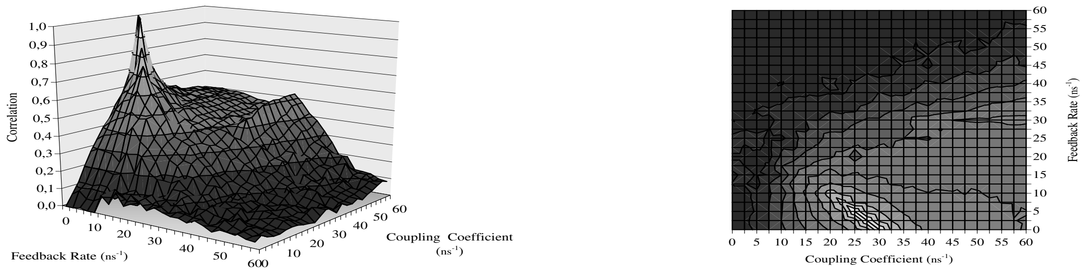

Figure 1. $C(-\tau_c + \tau_f)$ as a function of the coupling constant and feedback rate of the receiver system. The feedback rate of the master laser is 30 ns^{-1} while the coupling and external round-trip time is of 1 ns and 2 ns, respectively. Left side image is a 3-D plot as a function of κ_c and κ_r. The right side image is a contour plot that represents the correlation coefficient from 0 (black) to 1 (white).

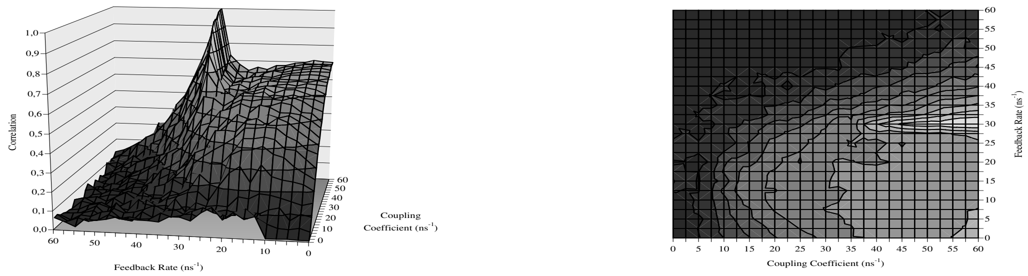

Figure 2. $C(-\tau_c)$ as a function of the coupling constant and feedback rate of the receiver system. The feedback rate of the master laser is 30 ns^{-1} while the coupling and external round-trip time is of 1 ns and 2 ns, respectively. Left side image is a 3-D plot as a function of κ_c and κ_r. The right side image is a contour plot that represents the correlation coefficient from 0 (black) to 1 (white).

Fig 1. shows the results obtained for $C(-\tau_c + \tau_f)$ in the parameter space (κ_c, κ_r). The synchronization region $(C(-\tau_c+\tau_f) \geq 0.9)$ for this anticipating solution is localized in a very narrow region around the condition

$\kappa_t = \kappa_r + \kappa_c$, but, even when this condition is satisfied, we find that a large coupling is necessary to obtain a good synchronization. In other words, the anticipating solution provided by this sufficient condition seems to loss its stability as the coupling decreases. Outside this small region, the correlation coefficient does not exceed the value 0.6.

In Fig. 2, we plot $C(-\tau_c)$ versus the coupling and feedbacks rates. For this case, the synchronization domain extends over the line $\kappa_t = \kappa_r$, and as in the previous case, a high injection rate is needed to guarantee the stability of the solution. It has to be noted that the length of the external cavities have been perfectly matched to obtain a high degree of correlation. If these length are not carefully controlled the synchronization is completely lost even for lengths that differ in a fraction of the emission wavelength.

When the system operates out of the optimal conditions ($\kappa_t = \kappa_r + \kappa_c$ for the anticipating solution and $\kappa_t = \kappa_r$ for the isochronous one) a strong degradation of the synchronization occurs in both cases. However, the isochronous solution presents a higher robustness when comparing with the anticipating one. Thus, each solution has its own domain where the synchronization degree is high and the solution is stable. From the inspection of the cross-correlation function we can also confirm that there are no other solutions than the isochronous and the anticipated one in the regions of the parameter space we have studied.

3.2. Message Encoding and Decoding

Differents schemes for secure transmissions based on the chaotic synchronization have been proposed: chaos masking (CMA),[19] chaos modulation (CMO) and chaos shift keying (CSK).[10,13] In this section, we study the performance of the message transmission and recovery for the open loop and close loop regimes using the CSK technique, since it provides a similar quality of synchronization than the CMA and CMO but it is simpler to implement in a real system. This method consists in switching between two different orbits, which define the bits '0' and '1' by varying a certain internal parameter.

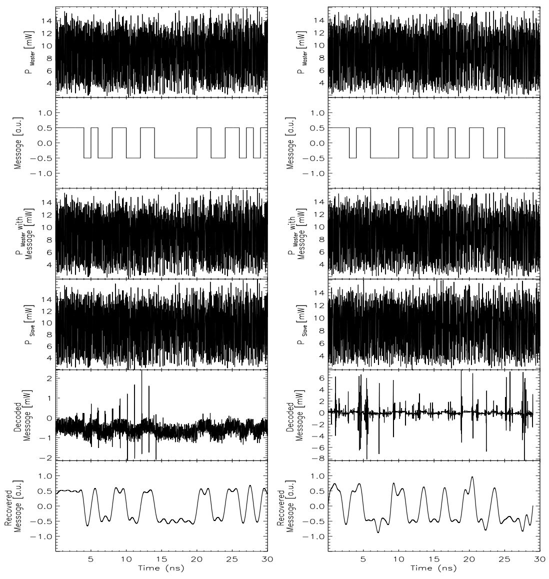

Figure 3. From top to bottom we plot the output of the ML, the encoded message, the output of the ML with message, the output of the SL, the decoded message, and the recovered message after filtering. The message modulation rate is 1 Gbit/s with an amplitude of 1 mA. The feedback strength of the ML is 30 ns^{-1}. The coupling rate and slave feedback coefficient are respectively, $\kappa_c = 60$ ns^{-1}, $\kappa_r = 30$ ns^{-1} (left panel) and $\kappa_c = 30$ ns^{-1}, $\kappa_r = 0$ ns^{-1} (right panel)

In Fig. 3, we plot the results of numerical simulations of the time traces of the optical power for the transmitter and receiver lasers in the open loop and close loop schemes. The output power emitted by the

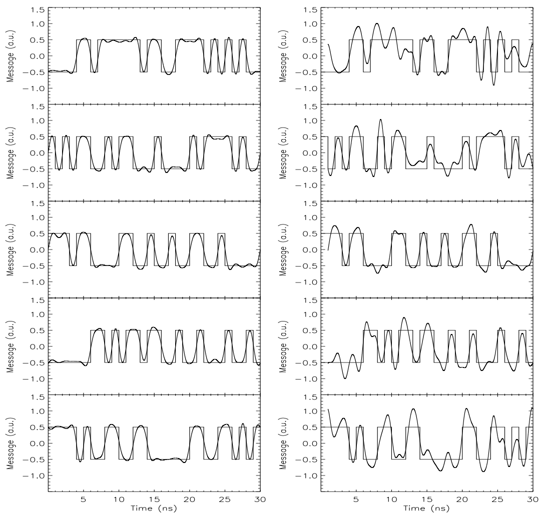

Figure 4. From top to bottom we plot the encoded message and the recovered message after filtering for $\Delta\omega = -2, -1, 0, 1, 2$ GHz. The message modulation rate is 1 Gbit/s with an amplitude of 1 mA. The feedback strength of the ML is 30 ns^{-1}. The coupling rate and slave feedback coefficient are respectively, $\kappa_c = 60$ ns^{-1} , $\kappa_r = 30$ ns^{-1} (left panel) and $\kappa_c = 30$ ns^{-1}, $\kappa_r = 0$ ns^{-1} (right panel)

semiconductor lasers is calculated as $P = [hc\omega\alpha_m/4\pi\mu_g]\,|E|^2$ where h is the Planck constant, c is the speed of light in vacuum, $\alpha_m = 48$ cm^{-1} is the facet loss and $\mu_g = 4$ is the group refractive index. Although in both cases the messages can be recovered, from the time traces displayed in the Fig. 3 it is observed a better quality of the decoded message for the close loop scheme than for the open loop one. The quality of the recovered message is improved with the suppression of the fast oscillations in the decoded signal by the application of a fifth-order Butterworth filter. The time lags between the ML and SL outputs associated with the different synchronization solutions ($-\tau_c$ for open loop and $\tau_f - \tau_c$ for the close loop) have been compensated in the figures.

In what follows, the possibility of a small detuning between the free-running laser frequencies is taken into account. Fig.4 shows the encoded and recovered messages for the open and close loop with different detuning values. A strong robustness against detuning for the close loop configuration is observed, while a notable degradation of synchronization appears in the open loop case. In this last regime, even for a detuning as small as 2 GHz the recovery of the message is very difficult.

To better show these behaviors, we plot in Fig. 5 the synchronization quality as function of the detuning between both lasers for the two configurations considered. From the figure we can conclude that for the open loop case a small detuning (hard to avoid in real systems) induces a dramatic loss of synchronization while for the close loop scheme there exists a large range of detunings where the synchronization is almost perfect. For the conditions here considered, this range is $\sim$45 GHz wide and is extended predominantly over the negative detuning region. Therefore, under the encryption technique we have considered (CSK), the close loop configuration seems to provide a better scenario for message transmission than the sensitive open loop.

3.3. Synchronization Recovery Time

An important point to be considered is the time it takes the system to re-synchronizes when a sudden cut in the link between the ML and SL occurs. We are interested in the time, that we call synchronization recovery time,

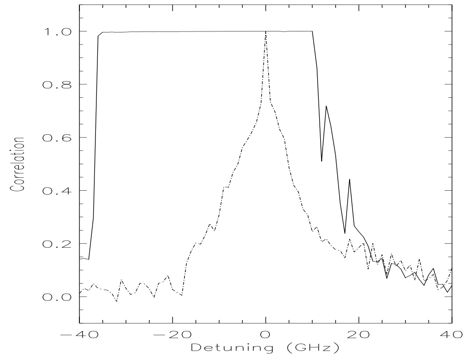

Figure 5. Correlation coefficient as function of the detuning. With solid line we plot the results obtained in the close loop regime ($\kappa_t = \kappa_r = 30$ ns^{-1}, $\kappa_c = 60$ ns^{-1}). With the dashed line we plot the results for the open loop regime ($\kappa_t = 30$ ns^{-1}, $\kappa_r = 0$ ns^{-1} and $\kappa_c = 30$ ns^{-1}).

when the coupling is switched on after an unexpected loss of synchronization. We define the synchronization time (or entrainment time when the SL operates in CW) as the time needed for the system to achieve a correlation coefficient of 0.95 from an initial uncoupled configuration.

In the Fig. 6 we show the entrainment time as a function of the external cavity round-trip time (τ_f) of the ML for the open loop scheme while maintaining the rest of the laser parameters identical

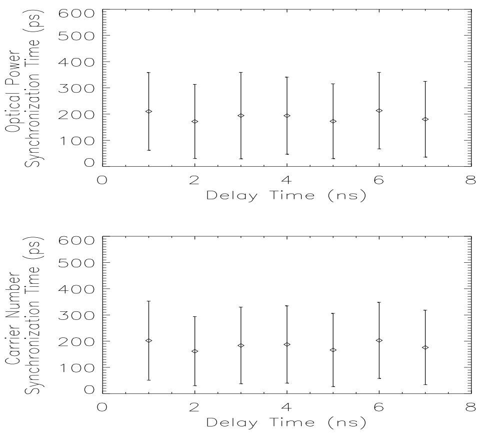

Figure 6. Representation of the entrainment time for the optical power (upper graphic) and carrier number (lower graphic) versus the delay time in the open loop configuration. Each point is averaged with 1000 events. The error bars ($t_{syncho} \pm \sigma$) also are plotted

The entrainment recovery time shows large fluctuations when it is measured repetitively, and induces the enormous error bars displayed in the former figure. Despite of the size of the error bars, the synchronization recovery time seems to be independent of the delay time in the ML and it takes in average a value of ~200 ps for our parameter values, to recover the synchronized state. On the other hand, when the system operates in the close loop regime our preliminary results show that the synchronization recovery time is much larger (about two orders of magnitude larger) than in the precedent case. Moreover, this time strongly depends on the feedback time of the ML. Work is in progress related with the characterization of mean values and variance for this case. From our results we can conclude that the open loop scheme offers a faster recovery of the synchronization or entrainment than the close one, a fact that has to be taken into account when designing chaotic communication systems.

4. CONCLUSIONS

In conclusions, we have numerically studied the behavior of a chaotic communication system where the emitter laser operates subject to coherent optical feedback while the receiver can operate either with (close loop) or without (open loop) feedback. We find, in general, that the performance of the close loop scheme has some advantages: it is more robust and easy to synchronize and less sensitive to the detuning between emitter and receiver, a quantity that is hard difficult to avoid in real systems. On the contrary it requires a very precise external cavity size of both emitter and receiver in order to achieve synchronization. A small mismatch in the latter strongly degrades the synchronization quality. A second disadvantage is that after a sudden cut in the link between emitter and receiver it requires quite long time to recover the synchronization state. On the other hand, the open loop scheme is in general less robust and requires a perfect match between the feedback strength in the emitter and the light coupled into the receiver to achieve a good degree of entrainment that limits its operability. It has the advantage that does not require any specification with respect to the external cavity of the emitter and that after a sudden cut in the link emitter-receiver the recovery time for the entrainment is much shorter than in the close loop scheme.

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