Noise-induced flow in quasigeostrophic turbulence with bottom friction

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Abstract

Randomly-forced fluid flow in the presence of scale-unselective dissipation develops mean currents following topographic contours. Known mechanisms based on the scale-selective action of damping processes are not at work in this situation. Coarse-graining reveals that the phenomenon is a kind of noise-rectification mechanism, in which lack of detailed balance and the symmetry-breaking provided by topography play an important role.

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In the last decades much effort has been devoted towards understanding fluid motion in situations in which vertical velocities are small and slaved to the horizontal motion. Under these circumstances the flow can be described in terms of two horizontal coordinates, the vertical depth of the fluid becoming a dependent variable. The fluid displays many of the unique properties of two-dimensional turbulence, but some of the aspects of three-dimensional dynamics are still essential. Among the several quasi two-dimensional dynamics considered, barotropic quasigeostrophic turbulence [1,2] has focussed most of the interest. The reason for such interest lies on the relevance of this dynamics as a model to understand planetary atmospheres and ocean general circulations as well as to describe some plasma physics phenomena [3–5]. In the first case the twodimensionality of the flow is induced by the Coriolis force, whereas in the second it may arise from the action of magnetic fields. In the geophysical context, the topography over which the layer of fluid flows is the ingredient introducing a difference with respect to purely twodimensional turbulence.

The first works on topographic turbulence focused on its statistical properties in the absence of dissipation and forcing [6]. These studies established a tendency of the flow to reach a maximum entropy (Gibbs) state characterized by the existence of stationary mean currents following isobaths. Mean currents associated with topography are however not restricted to this inviscid and unforced case, as shown by [7], where viscosity was included. Statistical-mechanics equilibrium arguments can not be directly used to explain the appearance of mean currents correlated to topographic features, since we are now in a nonequilibrium situation of decaying turbulence. The explanation put forward was [7] that viscous damping, due to its stronger action at the smallest scales, dissipates enstrophy much faster than energy, a quantity more concentrated at large scales in twodimensional turbulence settings. Bretherton and Haidvogel [7] also showed that, for small viscosity, not only the enstrophy but also the potential enstrophy was decaying most of the time. As a consequence of this scale-selective behavior, the decaying turbulent state would be the one with the smaller potential enstrophy compatible with the (slowly decreasing in time) instantaneous value of the energy. It turns out that these minimum enstrophy states are closely related to the equilibrium maximum entropy ones and, as them, present distinct mean currents following isobaths. The natural tendency of topographic turbulence to generate currents along isobaths, under different situations, aroused a significant interest in the geophysical community. Specifically, studies were addressed to investigate the role played by these currents on the general circulation of the world's ocean [8–11].

It has been recently shown numerically [12] that mean currents following isobaths appear also in situations in which a scale-unselective dissipation, Rayleigh friction modeling an Eckman bottom drag, is used. This fact was observed in numerical simulations in which random forcing (also of a scale unselective nature) was present. Thus the state obtained is not one of decaying turbulence but a nonequilibrium statistically steady state. Statistical properties are not far numerically from those of a generalized canonical equilibrium, but neither the maximum entropy nor the minimum enstrophy mechanisms, the ones invoked in the previous situations, can here be used without modification, since the first requires statistical equilibrium and the second decaying turbulence. Instead [12] interpreted these currents as a noise rectification phenomenon [13]: the mean currents were generated by nonlinearity and sustained by noise, with topography providing the symmetry-breaking ingredient giving a particular sense to the flow. Later it was theoretically shown and numerically confirmed that noise rectification also appears in randomly forced topographic turbulence when a scale selective damping (viscosity) is considered [14,15]. However, the detailed understanding of the noise-rectification mechanism when a scale-unselective damping is present is still an open question. Addressing this question as well as to provide a rigorous theoretical framework to explain the numerical results shown in [12] are the main motivation of this Letter.

In the quasigeostrophic approximation, the motion of a single layer of fluid in a rotating frame or planet can be described in terms of the horizontal components of the velocity $(u(\mathbf{x}), v(\mathbf{x}))$ that can also be written in terms of a streamfunction verifying:

$$u = -\frac{\partial \psi}{\partial y} \ , \quad v = \frac{\partial \psi}{\partial x} \tag{1}$$

The streamfunction $\psi(\mathbf{x}, t)$, with $\mathbf{x} \equiv (x, y)$, is governed by the dynamics [2]:

$$\frac{\partial \nabla^2 \psi}{\partial t} + \lambda \left[\psi, \nabla^2 \psi + h \right] = -\epsilon \nabla^2 \psi + F \ . \tag{2}$$

ϵ is the friction parameter, modelling bottom friction, $F(\mathbf{x}, t)$ is any kind of relative-vorticity external forcing, and $h = f \Delta H / H_0$, with f the Coriolis parameter, H_0 the mean depth, and $\Delta H(\mathbf{x})$ the local deviation from the mean depth. λ is a bookkeeping parameter introduced to allow perturbative expansions in the interaction term. The physical case corresponds to $\lambda = 1$. The Poisson bracket or Jacobian is defined as

$$[A, B] = \frac{\partial A}{\partial x}\frac{\partial B}{\partial y} - \frac{\partial B}{\partial x}\frac{\partial A}{\partial y} \ . \tag{3}$$

Equation (2) gives the time evolution of relative vorticity subjected to forcing and dissipation. The form used by the dissipation term acts with the same strength at all spatial scales, in contrast with viscosity terms of the form $\nabla^4\psi$, excluded from our model (2), which would dissipate better the smaller scales. A convenient choice of F, able to model a variety of processes, is to assume it to be a Gaussian stochastic process with zero mean, and with a Fourier transform $\hat{F}_{\mathbf{k}}(\omega)$ having the following two-point correlation function:

$$\left\langle \hat{F}_{\mathbf{k}}(\omega)\hat{F}_{\mathbf{k}'}(\omega') \right\rangle = Dk^{-\mu}\delta(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega') \ . \tag{4}$$

Here, $\mathbf{k} = (k_x, k_y)$, and $k = |\mathbf{k}|$. Thus the process is white in time but has power-law correlations in space. Stochastic forcing has been used in fluid dynamics problems to model stirring forces [16], short scale instabilities [1], thermal noise [17,18], or processes below the resolution of computer models [19], among others [20].

Guided by the results in [14], we attribute the average currents to a large-scale rectification of the small-scale fluctuations introduced by the noise term. In consequence we resort to a coarse-graining procedure to investigate how the dynamics of long-wavelength modes in (2) is affected by the small scales. For our problem it is convenient to use the Fourier components of the streamfunction $\hat{\psi}_{\mathbf{k}\omega}$ or equivalently the relative vorticity $\zeta_{\mathbf{k}\omega} = -k^2\hat{\psi}_{\mathbf{k}\omega}$. This variable satisfies:

$$\begin{aligned}\zeta_{\mathbf{k}\omega} = {} & G^0_{\mathbf{k}\omega}F_{\mathbf{k}\omega} + \\ & \lambda G^0_{\mathbf{k}\omega}\sum_{\mathbf{p},\mathbf{q},\Omega,\Omega'} A_{\mathbf{kpq}}\left(\zeta_{\mathbf{p}\Omega}\zeta_{\mathbf{q}\Omega'} + \zeta_{\mathbf{p}\Omega}h_{\mathbf{q}}\right) \quad ,\end{aligned} \tag{5}$$

where the interaction coefficient is:

$$A_{\mathbf{kpq}} = (p_x q_y - p_y q_x)p^{-2}\delta_{\mathbf{k},\mathbf{p}+\mathbf{q}} \quad , \tag{6}$$

the bare propagator is:

$$G^0_{\mathbf{k}\omega} = (-i\omega + \epsilon)^{-1} \quad , \tag{7}$$

and the sum is restricted by $\mathbf{k} = \mathbf{p} + \mathbf{q}$ and $\omega = \Omega + \Omega'$. $\mathbf{p} = (p_x, p_y)$, $p = |\mathbf{p}|$, and similar expressions hold for $\mathbf{q}$. The wavenumbers are restricted to $0 < k < k_0$, with k_0 an upper cut-off. Following the method in Ref. [21], one can eliminate the modes $\zeta^{>}_{\mathbf{k}\omega}$ with k in the shell $k_0 e^{-\delta} < k < k_0$ and substitute their expressions into the equations for the remaining low-wavenumber modes $\zeta^{<}_{\mathbf{k}\omega}$ with $0 < k < k_0 e^{-\delta}$. The parameter δ measures thus the width of the band of eliminated wavenumbers. To second order in λ, the resulting equation of motion for the modes $\zeta^{<}_{\mathbf{k}\omega}$, written in terms of the large-scale streamfunction $\psi^{<}(\mathbf{x}, t)$ is:

$$\begin{aligned}\frac{\partial \nabla^2 \psi^<}{\partial t} &+ \lambda \left[\psi^<, \nabla^2 \psi^< + h^<\right] = \\ &- \epsilon \nabla^2 \psi^< - g \nabla^4 h^< + \nu' \nabla^4 \psi^< + F' \ ,\end{aligned} \tag{8}$$

where

$$\nu' = \frac{\lambda^2 D \mu \delta}{32 \pi \epsilon^2}, \tag{9}$$

$$g(\lambda, D, \delta, \epsilon, \mu) = \frac{\lambda^2 D (\mu + 2) \delta}{64 \pi \epsilon^2} \quad . \tag{10}$$

$F'(\mathbf{x}, t)$ is an effective noise which turns out to be also a Gaussian process but with mean value and correlations given by:

$$< F'(\mathbf{x}, t) >= -g \nabla^4 h^<, \tag{11}$$

$$\begin{aligned}\left\langle \left(\hat{F}'_k(\omega) - \left\langle \hat{F}'_k(\omega) \right\rangle\right) \left(\hat{F}'_{k'}(\omega') - \left\langle \hat{F}'_{k'}(\omega') \right\rangle\right) \right\rangle &= \\ D k^{-\mu} \delta(k + k') \delta(\omega + \omega')&\end{aligned} \tag{12}$$

Equations (8)-(12) are the main result in this Letter. They give the dynamics of long wavelength modes $\psi^{<}_{\mathbf{k}\omega}$ with $0 < k < k_0 e^{-\delta}$. They are valid for small λ or, when $\lambda \approx 1$, for small width δ of the elimination band.

The elimination of the small scales leads, as expected from physical grounds, to the appearance of an effective viscosity ν' at large scales. Depending on the sign of μ, this viscous term can be destabilizing. One should keep in mind however that Eq.(8) is only valid at large scales, so that such small-wavelength instabilities, when formally present, would be avoided by proper choice of the wavenumber cut-off. More importantly, a new term depending on the topography $g\nabla^4 h^<$ has been generated by the small scales. Another identical term is contained in the mean value of the effective large-scale noise F'. Both terms have the effect of pushing the large-scale motion towards a state of flow following the isolevels of bottom perturbations $h^<$. The energy in this preferred state is determined by the function $g(\lambda, D, \delta, \epsilon, \mu)$ which measures the influence of the different terms of the dynamics (nonlinearity, noise, friction). From relation (10) it is clear that while nonlinearities and noise increase the intensity of the mean currents in the preferred state, high values of the friction parameter would reduce them.

As in the situation with scale selective damping [14,15], (10) and (11) exhibit a characteristic dependence on the exponent μ of the random-forcing spectral power-law. In the present case, the dependence with $\mu + 2$ implies that the directed currents reverse sign as μ crosses the value -2, and that they vanish if $\mu = -2$. It is easy to check that the conditions for detailed balance between the fluctuation input and dissipation [22] are satisfied precisely for $\mu = -2$ (in general, the condition is $\mu = -n$, where n is the exponent of ∇ in the dissipation term). In this situation the steady-state probability distribution for ψ can be found exactly and is independent of topography. Thus the generation of flow along isobaths is a consequence of lack of detailed balance, a general feature of noise-rectifying mechanisms [13].

Concluding, relations (8)-(12) show that the origin of average circulation patterns in quasi-geostrophic turbulence over topography is related to nonlinearity and lack of detailed balance. The present approach provides a theoretical basis for the numerical observations presented in [12], for which mechanisms based on the scale-selective action of damping can not be applied. Nonlinear terms couple the dynamics of small scales to the large ones and provide the mechanism for energy transfer from the fluctuating component of the spectrum to the mean one. This mean spectral component, absent in purely two-dimensional turbulence [23], is controlled by the shape of the bottom boundary, which breaks the isotropy of the system, and characterizes the structure of the flow pattern.

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