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BOUNDARY INDUCED FROZEN CHAOS*

V. M. EGUíLUZ†‡, E. HERNÁNDEZ-GARCíA†, O. PIRO† AND S. BALLE†

In the past few decades, considerable understanding of the phenomenon of temporal chaos in dynamical systems of few degrees of freedom has been achieved [1]. On the other hand, spatiotemporal chaos in extended dynamical systems with infinitely many degrees of freedom is currently under very active investigation[6]. It is remarkable however, that an area of problems laying somehow between the two extremes has not received so much attention, namely, purely spatial chaos as a stationary attractor of extended dynamical systems [2, 4, 5, 8, 9, 13].

In the context of fluid dynamics, the existence of spatially chaotic, but temporally steady solutions would also fill a conceptual gap between two well-studied complex phenomena: Lagrangian chaos, and Eulerian chaos or turbulence. The former refers to the chaotic motion of a fluid parcel which might occur in flows that are not necessarily chaotic in their Eulerian description. In fact, a laminar flow may induce chaotic motion for the fluid particles [3, 10, 11], and in three dimensions this is even possible if the flow is steady. On the other extreme, the road to turbulence is usually associated to a hierarchy of increasingly spatiotemporally chaotic Eulerian velocity fields $\mathbf{v}(\mathbf{r})$. Frozen spatial chaos would then refer in this context to a third possibility: a stationary velocity field $\mathbf{v}(\mathbf{r})$ spatially chaotic in the Euler description.

Stationary and uniformly propagating spatial structures in extended nonlinear dynamical systems in one spatial dimension have been considered in some detail in relatively recent investigations. Such systems are specially suitable to analysis because their steady state configurations, which depend only on the unique spatial coordinate, can be described using results of the theory of low dimensional dynamical systems, by simply interpreting the spatial coordinate as a ficticious time. In the cases previously studied, spatial chaos appears because the nonlinear dynamical system 'evolving' in the space coordinate has large enough dimensionality. The high dimensionality of the equations can be due to a) the presence of high-order spatial derivatives in a single evolution equation as in the cases of Kuramoto-Shivashinsky [7] and Swift-Hohenberg equation [8, 9], b) the coupling of several fields each one satisfying lower order differential equation as in the two real fields satisfying a complex Ginzburg-Landau equation[13] which supports chaotic travelling waves, or c) explicit space dependent forcing terms as in the driven KdV-Burgers [12] equation.

Stationary two-dimensional spatial chaos, on the contrary, is basically unexplored at present. In this case application of dynamical system concepts to the analysis of steady state solutions cannot be direct, since we have in principle two equally important spatial coordinates. In addition there is a much larger variety of non trivial boundary condition classes which surely lead to a greater richness in the expected properties of the steady field configurations. In this work we show, for a very simple

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†Instituto Mediterráneo de Estudios Avanzados IMEDEA (CSIC-UIB) and Departament de Física, Universitat de les Illes Balears, E-07071 Palma de Mallorca, Spain. URL: http://www.imedea.uib.es/PhysDept
‡victor@hp1.uib.es

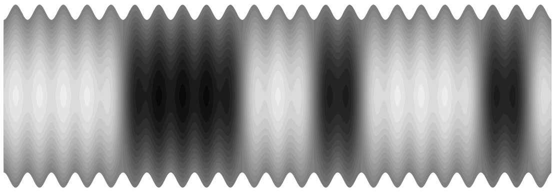

FIG. 1. Stationary, spatially chaotic solution of the Fisher-Kolmogorov equation. Different grey levels correspond to different values of the field ψ

model, that rather simple undulated strip-like domain shapes can induce the formation of patterns that are both *spatially chaotic* and *temporally attracting*. We believe that the kind of *modulated boundaries* we use, can be easily implemented in standard experimental pattern formation set-up's such as Faraday waves, convection cells, or open flows. In fact, our work was originally motivated by the observation, in a fluid dynamics experimental setup of a periodic array of pipe bents, that the transversal profile of the steady flow does not necessarily repeat itself with the same periodicity of the array [11].

We show in Figure 1 a typical spatially chaotic structure for the Fisher-Kolmogorov equation $\partial_t\psi = \nabla^2\psi + a\psi - \psi^3$ in a two dimensional undulated domain. The boundary conditions in the upper and lower boundaries are $\psi = 0$ and periodic in the longitudinal direction. This is a stationary attractor of the dynamics consisting of irregularly spaced domain walls.

A single transversal mode approximation permits a better understanding in some range of parameters where the geometry of the domain is slightly different from the rectangular and the system is close to threshold of instability of the laminar solution. In particular, it describes the onset of the spatially disordered structures, making a reduction from a system of infinite degrees of freedom to a low dimensional problem.

We are investigating the possibility that these spatially chaotic structures appear in low Reynolds number flows.

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