FUZZY CONTROL OF CHAOS

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Int. J. Bifurcation and Chaos **8**, 1743–1747, 1998

We introduce the idea of the fuzzy control of chaos: we show how fuzzy logic can be applied to the control of chaos, and provide an example of fuzzy control used to control chaos in Chua's circuit.

1. Introduction

Chaos control exploits the sensitivity to initial conditions and to perturbations that is inherent in chaos as a means to stabilize unstable periodic orbits within a chaotic attractor. The control can operate by altering system variables or system parameters, and either by discrete corrections or by continuous feedback. Many methods of chaos control have been derived and tested [Chen & Dong, 1993; Lindner & Ditto, 1995; Ogorzałek, 1993]. Why then consider fuzzy control of chaos?

A fuzzy controller works by controlling a conventional control method. We propose that fuzzy control can become useful together with one of these other methods — as an extra layer of control — in order to improve the effectiveness of the control in terms of the size of the region over which control is possible, the robustness to noise, and the ability to control long period orbits.

In this paper, we put forward the idea of fuzzy control of chaos, and we provide an example showing how a fuzzy controller applying occasional proportional feedback to one of the system parameters can control chaos in Chua's circuit.

2. Fuzzy Control

Fuzzy control [Driankov *et al.*, 1993; Terano *et al.*, 1994] is based on the theory of fuzzy sets and fuzzy logic [Yager & Zadeh, 1991; Bezdek, 1993]. The principle behind the technique is that imprecise data can be classified into sets having fuzzy rather than sharp boundaries, which can be manipulated to provide a framework for approximate reasoning in the face of imprecise and uncertain information. Given a datum, x, a fuzzy set A is said to contain x with a degree of membership $\mu_A(x)$, where $\mu_A(x)$ can take any value in the domain $[0, 1]$. Fuzzy sets are often given descriptive names (called linguistic variables) such as *FAST*; the membership function $\mu_{FAST}(x)$ is then used to reflect the similarly between values of x and a contextual meaning of *FAST*. For example, if x represents the speed of a car in kilometres per hour, and *FAST* is to be used to classify cars travelling fast, then *FAST* might have a membership function equal to zero for speeds below 90 km/h and equal to one for speeds above 130 km/h, with a curve joining these two extremes for speeds between these values. The degree of truth of the statement *the car is travelling fast* is then evaluated by reading off the value of the membership function corresponding to the car's speed.

Logical operations on fuzzy sets require an extension of the rules of classical logic. The three fundamental Boolean logic operations, intersection, union, and complement, have fuzzy counterparts defined by extension of the rules of Boolean logic. A fuzzy expert system uses a set of membership functions and fuzzy logic rules to reason about data. The rules are of the form 'if x is *FAST* and y is *SLOW* then z is *MEDIUM*', where x and y are input variables, z is an output variable, and *SLOW*, *MEDIUM*, and *FAST* are linguistic variables. The set of rules in a fuzzy expert system is known as the rule base, and together with the data base of input and output membership functions it comprises the knowledge base of the system.

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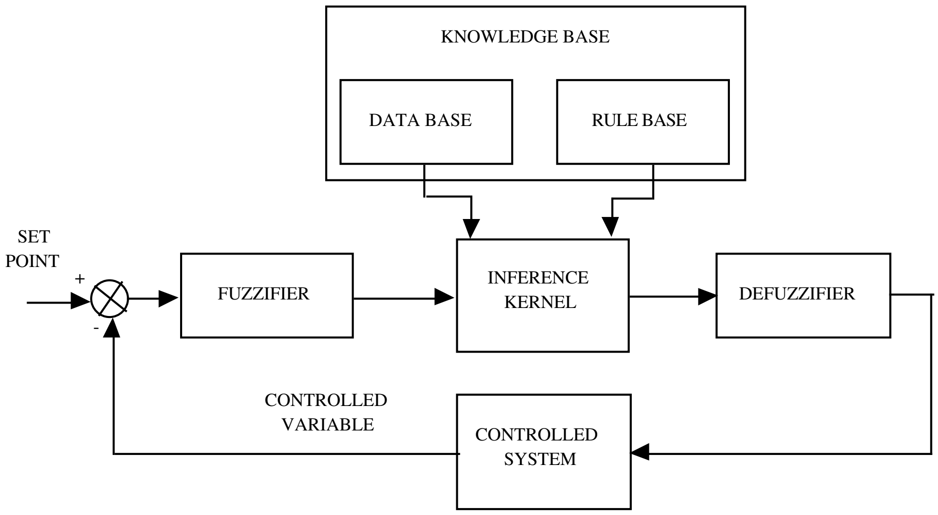

Figure 1: Fuzzy logic controller block diagram.

A fuzzy expert system functions in four steps. The first is *fuzzification*, during which the membership functions defined on the input variables are applied to their actual values, to determine the degree of truth for each rule premise. Next under *inference*, the truth value for the premise of each rule is computed, and applied to the conclusion part of each rule. This results in one fuzzy set to be assigned to each output variable for each rule. In *composition*, all of the fuzzy sets assigned to each output variable are combined together to form a single fuzzy set for each output variable. Finally comes *defuzzification*, which converts the fuzzy output set to a crisp (non-fuzzy) number.

A fuzzy controller may then be designed using a fuzzy expert system to perform fuzzy logic operations on fuzzy sets representing linguistic variables in a qualitative set of control rules — see Figure 1.

As a simple metaphor of fuzzy control in practice, consider the experience of balancing a stick vertically on the palm of ones hand. The equations of motion for the stick (a pendulum at its unstable fixed point) are well-known, but we do not integrate these equations in order to balance the stick. Rather, we stare at the top of the stick and carry out a type of fuzzy control to keep the stick in the air: we move our hand slowly when the stick leans by a small angle, and fast when it leans by a larger angle. Our ability to balance the stick despite the imprecision of our knowledge of the system is at the heart of fuzzy control.

3. Techniques for Fuzzy Chaos Control

To control a system necessitates perturbing it. Whether to perturb the system via variables or parameters depends on which are more readily accessible to be changed, which in turn depends on what type of system is to be controlled — electronic, mechanical, optical, chemical, biological, etc. Whether to perturb continuously or discretely is a question of intrusiveness — it is less intrusive to the system, and less expensive to the controller, to perturb discretely. Only when discrete control is not effective might continuous control be considered.

Ott, Grebogi, and Yorke [Ott *et al.*, 1990] invented a method of applying small feedback perturbations

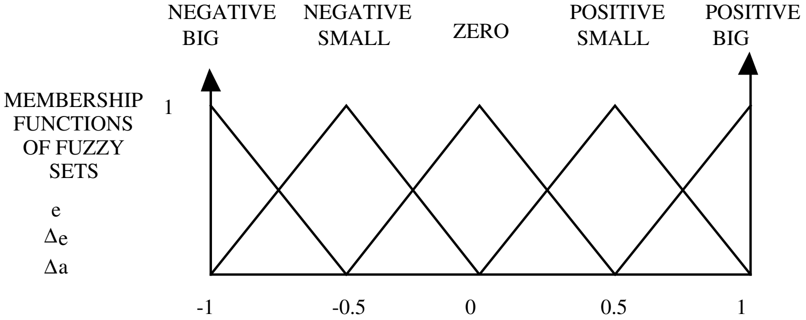

Figure 2: Membership functions of the input and output variables e, Δe, and Δa.

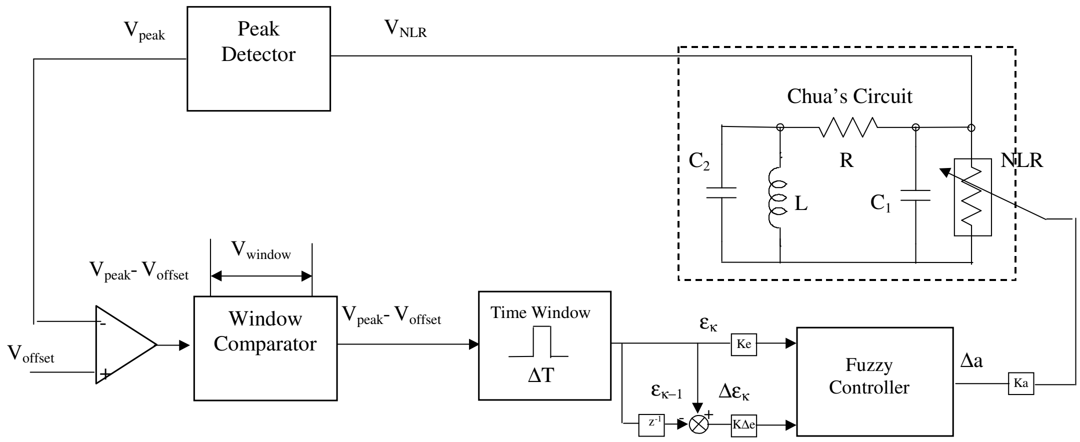

Figure 3: The whole controller and control system in the form of a block diagram, including the fuzzy controller, the peak detector, the window comparator, and the Chua's circuit system being controlled.

Table 1: Quantification levels and membership functions.

Error, e	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
Change in error, Δe	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
Control, Δa	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
Quantification level	-4	-3	-2	-1	0	1	2	3	4
Linguistic Variables	Membership Functions								
Positive Big, PB	0	0	0	0	0	0	0	0.5	1
Positive Small, PS	0	0	0	0	0	0.5	1	0.5	0
Approximately Zero, AZ	0	0	0	0.5	1	0.5	0	0	0
Negative Small, NS	0	0.5	1	0.5	0	0	0	0	0
Negative Big, NB	1	0.5	0	0	0	0	0	0	0

to an accessible system parameter in order to control chaos. The OGY method uses the dynamics of the linearized map around the orbit one wishes to control. Using the OGY method, one can pick any unstable periodic orbit that exists within the attractor and stabilize it. The control is imposed when the orbit crosses a Poincaré section constructed close to the desired unstable periodic orbit. Since the perturbation applied is small, it is supposed that the unstable periodic orbit is unaffected by the control.

Occasional proportional feedback [Hunt, 1991; Lindner & Ditto, 1995] is a variant of the original OGY chaos control method. Instead of using the unstable manifold of the attractor to compute corrections, it uses one of the dynamical variables, in a type of one-dimensional OGY method. This feedback could be applied continuously or discretely in time; in occasional proportional feedback it is applied discretely. Occasional proportional feedback exploits the strongly dissipative nature of the flows often encountered, enabling one to control them with a one-dimensional map. The method is easy to implement, and in many cases one can stabilize high period unstable orbits by using multiple corrections per period. It is a suitable method on which to base a fuzzy logic technique for the control of chaos, since it re-

Table 2: Rule table for the linguistic variables in Table 1.

Δe \ e	NB	NS	AZ	PS	PB
NB	NB	NS	NS	AZ	AZ
NS	NB	NS	AZ	AZ	PS
AZ	NS	NS	AZ	PS	PS
PS	NS	AZ	AZ	PS	PB
PB	AZ	AZ	PS	PS	PB

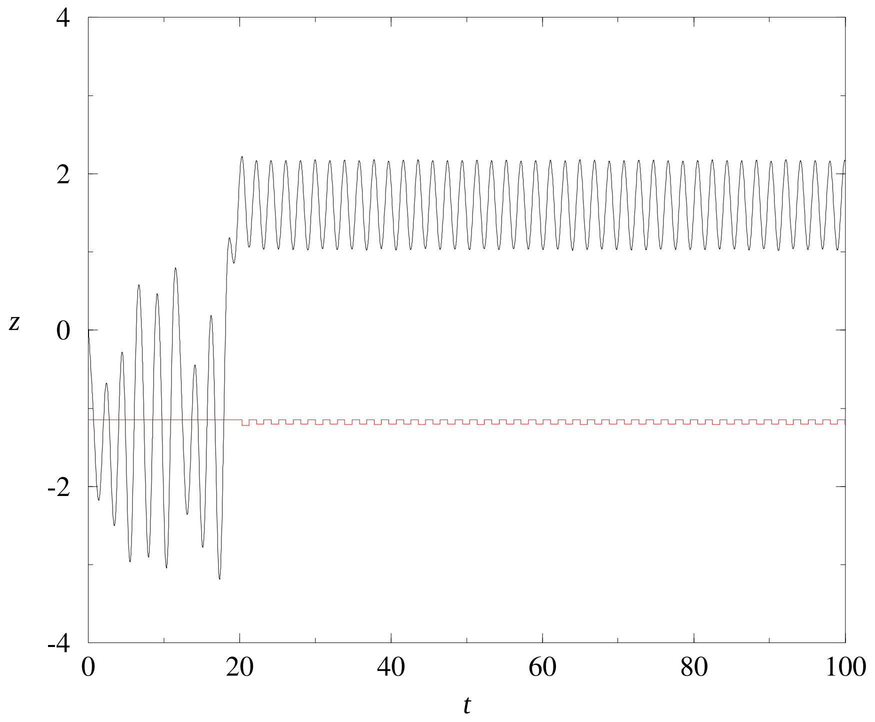

Figure 4: The fuzzy controller stabilizes a previously unstable period-1 orbit. The control is switched on at time 20. The lower trace shows the correction pulses applied by the controller.

quires no knowledge of a system model, but merely an accessible system parameter.

4. An Example: Fuzzy Control of Chaos in Chua's Circuit

Chua's circuit [Matsumoto, 1984; Kennedy, 1993] exhibits chaotic behaviour that has been extensively studied, and whose dynamics is well known [Madan, 1993]. Recently, occasional proportional feedback has been used to control the circuit [Johnson *et al.*, 1993]. The control used an electronic circuit to sample the peaks of the voltage across the negative resistance and if it fell within a window, centred about a by a set-point value, modified the slope of the negative resistance by an amount proportional to the difference between the set point and the peak value. The nonlinear nature of this system and the heuristic approach used to find the best set of parameters to take the system to a given periodic orbit suggest that a fuzzy controller that can include knowledge rules to achieve periodic orbits may provide significant gains over occasional proportional feedback alone.

We have implemented a fuzzy controller to control the nonlinearity of the nonlinear element (a three segment nonlinear resistance) within Chua's circuit. The block diagram of the controller is shown in Figure 1. It consists of four blocks: knowledge base, fuzzification, inference and defuzzification. The knowledge base is composed of a data base and a rule base. The data base consists of the input and output membership functions (Figure 2). It provides the basis for the fuzzification, defuzzification and inference mechanisms. The rule base is made up of a set of linguistic rules mapping inputs to control actions. Fuzzification converts the input signals e and Δe into fuzzified signals with membership values assigned to linguistic sets. The inference mechanisms operate on each rule, applying fuzzy operations on the antecedents and by compositional inference methods derives the consequents. Finally, defuzzification converts the fuzzy outputs to control signals, which in our case control the slope of the negative resistance Δa in Chua's circuit (Figure 3). The fuzzification maps the error e, and the change in the error Δe, to labels of fuzzy sets. Scaling and quantification operations are applied to the inputs. Table 1 shows the quantified levels and the linguistic labels used for inputs and output. The knowledge rules (Table 2) are represented as control statements such as 'if e is *NEGATIVE BIG* and Δe is *NEGATIVE SMALL* then Δa is *NEGATIVE BIG*'.

The normalized equations representing the circuit are

$$\dot{x} = \alpha \left(y - x - f(x) \right),$$

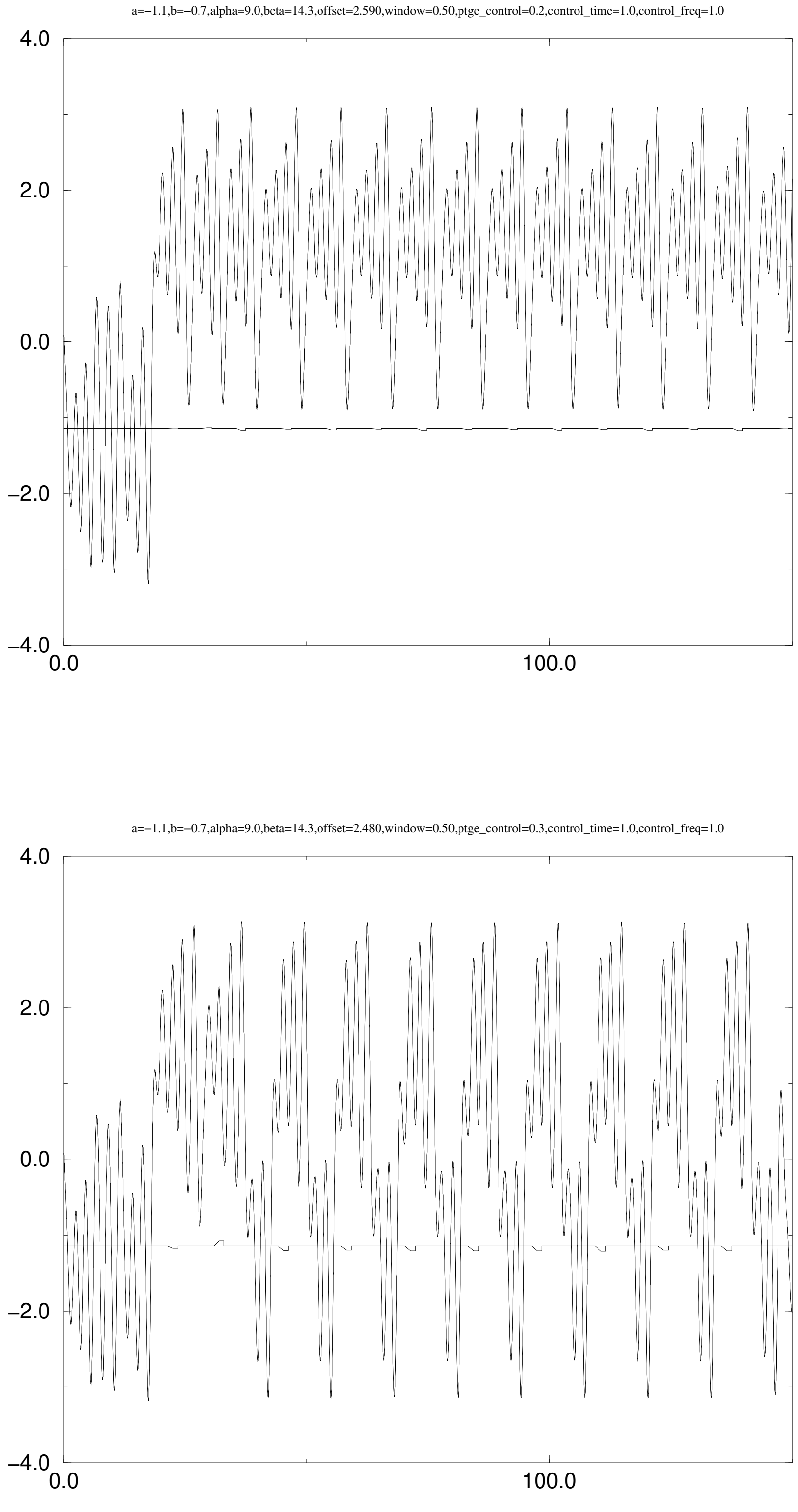

Figure 5: Trajectory traces show higher period orbits stabilized by the controller. As before, the lower trace shows the correction pulses applied by the fuzzy control.

$$\begin{aligned}\dot{y} &= x - y + z,\\ \dot{z} &= -\beta y, \end{aligned} \quad (1)$$

where $f(x) = bx + \frac{1}{2}(a-b)(|x+1| - |x-1|)$ represents the nonlinear element of the circuit. Changes in the negative resistance were made by changing a by an amount

$$\Delta a = \text{Fuzzy Controller Output} \times \text{Gain} \times a. \quad (2)$$

We have performed numerical simulations, both in C and in Simulink, of Chua's circuit controlled by the fuzzy logic controller. Figure 3 shows the whole control system in the form of a block diagram, including Chua's circuit, the fuzzy controller, the peak detector, and the window comparator. Figure 4 gives a sample output of the fuzzy controller stabilizing an unstable period-1 orbit by applying a single correction pulse per cycle of oscillation. By changing the control parameters we can stabilize orbits of different periods. In Figure 5 we illustrate more complex higher period orbits stabilized by the controller. One can tune the fuzzy control over the circuit to achieve the type of response required in a given situation by modifying some or all of the rules in the knowledge base of the system.

Of course, in the case of Chua's circuit the system equations are available and fuzzy logic is thus not necessary for control, but this simple example permits us to see the possibilities that fuzzy control provides, by allowing a nonlinear gain implemented in the form of knowledge based rules.

5. Conclusions

We have introduced the idea of using fuzzy logic for the control of chaos. Fuzzy logic controllers are commonly used to control systems whose dynamics is complex and unknown, but for expositional clarity here we have given an example of its use with a well-studied chaotic system. We have shown that it is possible to control chaos in Chua's circuit using fuzzy control. Further work is necessary to quantify the effectiveness of fuzzy control of chaos compared with alternative methods, to identify ways in which to systematically build the knowledge base for fuzzy control of a particular chaotic system, and to apply the fuzzy controller to further chaotic systems.

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