Domain Growth in a Multivariable non Potential System

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Abstract

We present a study of dynamical scaling and domain growth in a non potential system that models Rayleigh–Bénard convection in a rotating cell. In $d = 1$, dynamical scaling holds, but the non potential terms modify the characteristic growth law with a crossover from logarithmic to linear in time. In $d = 2$ the non potential terms prevent coarsening for values of the angular rotation speed below the Küppers-Lortz instability.

Proceedings of the V Latin American Workshop on Nonlinear Phenomena

PACS: 75.60.Ch, 75.40.Gb
Keywords: domain growth, dynamical scaling, interface dynamics

1 Introduction and model

Over the last decades, a large effort has been devoted to study the coarsening processes that drive a system back to the equilibrium state [1,2]. One of the main points of interest is to investigate the existence of dynamical scaling in the late stages of the evolution. Briefly stated, dynamical scaling means that there exists a single characteristic length, $R(t)$, such that the domain structure is independent of time (in a statistical sense) when all the lengths are scaled by $R(t)$. In other words, the system evolves in a self-similar manner. It is known that for potential dynamics, i.e., for systems whose dynamical evolution involves the minimization of a potential (free energy) with two equivalent minima, and after a transient time in which domains are formed, there appear well defined interfaces separating the equivalent states. The subsequent dynamics is governed by interface motion. The mechanism for interface motion, and therefore the characteristic length $R(t)$, strongly depend on the dimensionality, d, as well as whether the (scalar) order parameter is conserved or not during the dynamical evolution. For non conserved order parameter the results are as follows: in $d = 1$, the domain boundaries (hereafter to be called *kinks*) move due to the interaction between them, leading to a logarithmic growth with time of the characteristic length, $R(t) \sim \log t$. In $d \geq 2$ the mechanism for domain growth is curvature driven and the characteristic length behaves as $R(t) \sim t^{1/2}$.

It is the purpose of this paper to study the influence of non potential effects (those that can not be derived from the minimization of a potential function) on the coarsening process and, namely, on the growth law of the characteristic domain size as well as the validity of the scaling description of the dynamics. To this end, we have used a theoretical model proposed in the context of fluid dynamics by Busse and Heikes [3] to which spatial dependent terms of the simplest diffusive form have been added:

$$
\begin{aligned}
\partial_t A_1 &= \nabla^2 A_1 + A_1 \left(1 - A_1^2 - (\eta + \delta)\, A_2^2 - (\eta - \delta)\, A_3^2\right) \\
\partial_t A_2 &= \nabla^2 A_2 + A_2 \left(1 - A_2^2 - (\eta + \delta)\, A_3^2 - (\eta - \delta)\, A_1^2\right) \\
\partial_t A_3 &= \nabla^2 A_3 + A_3 \left(1 - A_3^2 - (\eta + \delta)\, A_1^2 - (\eta - \delta)\, A_2^2\right)
\end{aligned}
\tag{1}
$$

This models aims to represent the appearance of convection rolls in a Rayleigh-Bénard fluid subject to Coriolis forces due to rotation. A_1, A_2 and A_3 are the (real) amplitudes of the convection rolls in three different space directions oriented at 120^0 from each other. The parameters η and δ are related to physical properties of the fluid. In particular δ is linked to the rotation angular velocity in such a way that $\delta = 0$ means no rotation of the fluid. We can split (1) into potential and non potential contributions [4]: $\partial_t A_i = -\delta \mathcal{F} / \delta A_i + \delta \cdot f_i$ $(i = 1, 2, 3)$. Therefore when $\delta = 0$, the system adopts a potential form with a potential $\mathcal{F}$. When $\delta \neq 0$, the dynamics is said to be non potential or non variational.

The set of equations (1) admits two kinds of homogeneous stationary solutions, namely: three "roll" solutions ($A_i = 1,\ A_j = 0,\ i \neq j$) and one "hexagon" solution ($A_1 = A_2 = A_3 = (1 + 2\eta)^{-1/2}$). When the angular velocity is smaller than some critical value, $\delta < \delta_c = \eta - 1$, the rolls are the only stable solutions and the dynamics leads, after a short transient time, to a situation in which there exist well-defined interfaces connecting two roll states. When the angular velocity is greater than the critical value, $\delta > \delta_c$, the system switches to a time dependent dynamics known as the Küppers-Lortz (KL) instability [5]. The KL instability introduces a chaotic dynamics that prevents coarsening and the typical domain size $R(t)$ saturates to a finite value [6]. In this paper we show that for angular rotation velocities smaller than the critical value (hence, far away from the KL instability) the system coarsens in $d = 1$ but not necessarily in $d = 2$. The difference lays on the fact that in $d = 2$ coarsening can be stopped due to the existence of points where three different front lines meet. The non potential dynamics makes the front lines rotate around the vertices.

2 Fronts and domain growth in $\mathbf{d = 1}$

As mentioned before, for small enough angular velocities, $\delta < \delta_c$, there are stable kink solutions connecting two different roll states, say A_i, A_j, the third amplitude being zero everywhere. In the potential case, $\delta = 0$, a solitary kink does not move because of the symmetry of the connected roll states. When $\delta \neq 0$ this symmetry is broken and the kink moves at a constant velocity, $v(\delta)$. This can be computed by means of a perturbation analysis through a solvability condition. The resulting expression at leading order in δ is [7]:

$$v(\delta) = \delta \, \frac{\int_{-\infty}^{\infty} dx \, A_i^0 A_j^0 \, (A_j^0 \, \partial_x A_i^0 - A_i^0 \, \partial_x A_j^0)}{\int_{-\infty}^{\infty} dx \, \left[(\partial_x A_i^0)^2 + (\partial_x A_j^0)^2 \right]} \tag{2}$$

where A_i^0, A_j^0 are the amplitudes of the stationary potential problem. With the help of this expression it is possible to know not only the magnitude of the velocity but also the direction of the motion which is related to the sign of v. Out of the six possible types of kinks connecting different roll states, three move to the right and three to the left.

We describe now the coarsening process that occurs when we start from random initial conditions for the three amplitudes. After a short transient time, a pattern emerges with well defined domains separated by rather sharp kinks. Those move in such a way that neighboring kinks traveling in opposite directions annihilate each other and, therefore, the number of domains decreases as time increases. The final state of the system is a homogeneous roll solution filling up the whole system. This sequence of events happens both for $\delta = 0$ (potential case) or $\delta \neq 0$. The difference being that in the potential case, kinks move due to attractive forces whereas in the non potential regime there is an additional mechanism that brings on the kink motion with a constant velocity given by (2). By combining both effects and in the case of a single domain and two varying amplitudes it is possible to derive an equation for the rate of change in the single domain size $R(t)$:

$$\partial_t R(t) = 2v(\delta) - \gamma e^{-\sqrt{\eta - 1} R(t)}, \quad \gamma > 0 \tag{3}$$

Here $v(\delta)$ is the solitary kink velocity (2) and the second term in the rhs is the attractive force between the kinks (γ is a constant independent of δ at the lowest order). When $\delta = 0$ only this second term is present. If $\delta \neq 0$ there is a competition between interaction and non potential effects. For the shortest times, when the kinks are very close to each other, kink interaction will be the dominant effect providing that δ is small enough. This leads to a growth law logarithmic with time. As the system coarsens, the average domain size grows and the system reaches a situation in which both effects are of the same order. Finally, when the average domain size is large enough, the non potential kink motion will dominate. In this regime we can consider each kink as moving at constant velocity as a result of which the growth law is linear with time . This prediction is supported by numerical simulations [7]. In figure 1 we show the evolution of the average characteristic length $R(t)$ for the small value of $\delta = 0.001$. In accordance with the previous discussion, we see that the initial logarithmic growth crosses over to linear at intermediate times. Notice that, for very late times, $R(t)$ saturates due to finite size effects. For larger values of δ, the initial logarithmic regime is very fast because of the fast kink annihilation at the very beginning and it can barely be observed in the simulations, which show instead a linear growth from the very beginning. Since δ turns out to be a relevant variable for the growth law, we expect that the scaling functions will depend on δ. We have indeed checked that the equal time correlation function $C(\mathbf{r}, t) = \langle \sum_{\mathbf{x}} A_i(\mathbf{x} + \mathbf{r}, t) A_i(\mathbf{x}, t) \rangle$ satisfies the scaling hypothesis $C(r, t) = f(r/R(t))$, although the accuracy of the data and the fast appearance of finite size effects for the smaller values of δ prevent us from making a satisfactory comparison between the different scaling functions.

3 Two–dimensional coarsening

The mechanisms responsible for interface dynamics in a two dimensional system differ from the ones holding in $d = 1$. In a two dimensional potential system whose order parameter is non conserved, interface motion is driven by curvature. The normal velocity of a front is given by $v_n = -\kappa$ (Allen-Cahn law, see [1]), where κ is the local curvature of the front line connecting two equivalent states. In the absence of other effects, the system tends to reduce the total interface area and as consequence it coarsens. The characteristic length grows as $R(t) \sim t^{1/2}$.

In our non potential system, there will be another contribution to the interface motion coming from the fact that the interfaces connecting roll states move at a constant velocity, namely

$$v_n = v(\delta) - \kappa \tag{4}$$

where $v(\delta)$ is the velocity of the planar front which is simply equal to the velocity of a one dimensional interface (eq. 2) and is zero when δ is zero. In the case of a circular drop of radius R, eq. (4) transforms into $v_n = -R^{-1} + v(\delta)$. We conclude that for a radius $R = v(\delta)^{-1} \equiv R_c$ the drop neither grows nor shrinks. Notice that this critical radius $R_c(\delta)$ does not appear in the potential problem for which a drop will always collapse in order to reduce the surface tension [8]. However, the most notorious fact on the dynamical evolution in the two-dimensional system is that coarsening can be stopped, even for values of δ smaller than the critical one or, equivalently, when the system does not undergo the KL instability. We remind that in the KL regime, the system does not coarsen independently of the dimensionality. This unexpected result is due to the fact that three amplitudes are considered in this model. This allows the presence in the system of "vertex points" at which three front lines meet (see fig 2). The role of the non potential dynamics is mainly to rotate the front lines around these points, preventing the system from coarsening (provided that the system is large enough). This is different to the situation in $d = 1$ in which the topology does not allow the meeting of three different domains. At least three amplitudes are necessary to stop the coarsening process. If, for instance, one amplitude were absent, we would observe drops of one mode immersed in a sea of the other. These drops, as in a usual nucleation theory, either grow (spreading over the whole surface) or shrink (disappearing), and the system coarsens.

As an evidence of the mechanisms leading to the absence of coarsening in the two dimensional system, we present in figure 2 snapshots of the evolution of the system. We observe that the system has reached a stationary state in which the average domain size remains approximately constant. In a short time scale, the front lines simply rotate around the vertex points. For larger time variations, the vertex points themselves diffuse. The exact nature of the interactions amongst the vertices and their effect on the dynamics will be addressed in future work.

We acknowledge financial support from DGICYT (Spain) projects numbers PB94-1167 and PB94-1172.

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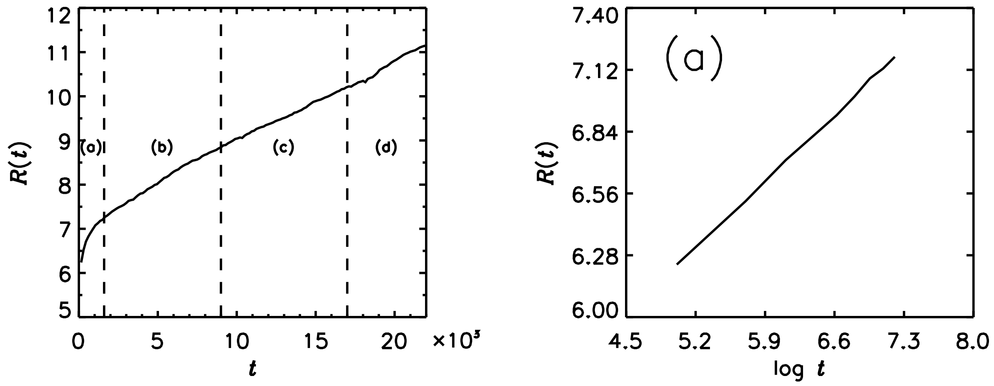

Fig. 1. Time evolution of the characteristic domain size for the one dimensional case. We have started from random initial conditions and used periodic boundary conditions. Parameter values: $\eta = 3.5$, $\delta = 0.001$. The initial logarithmic profile (region (a)) becomes linear (region (c)) after a crossover (region (b)). The region (d) is related to finite size effects.

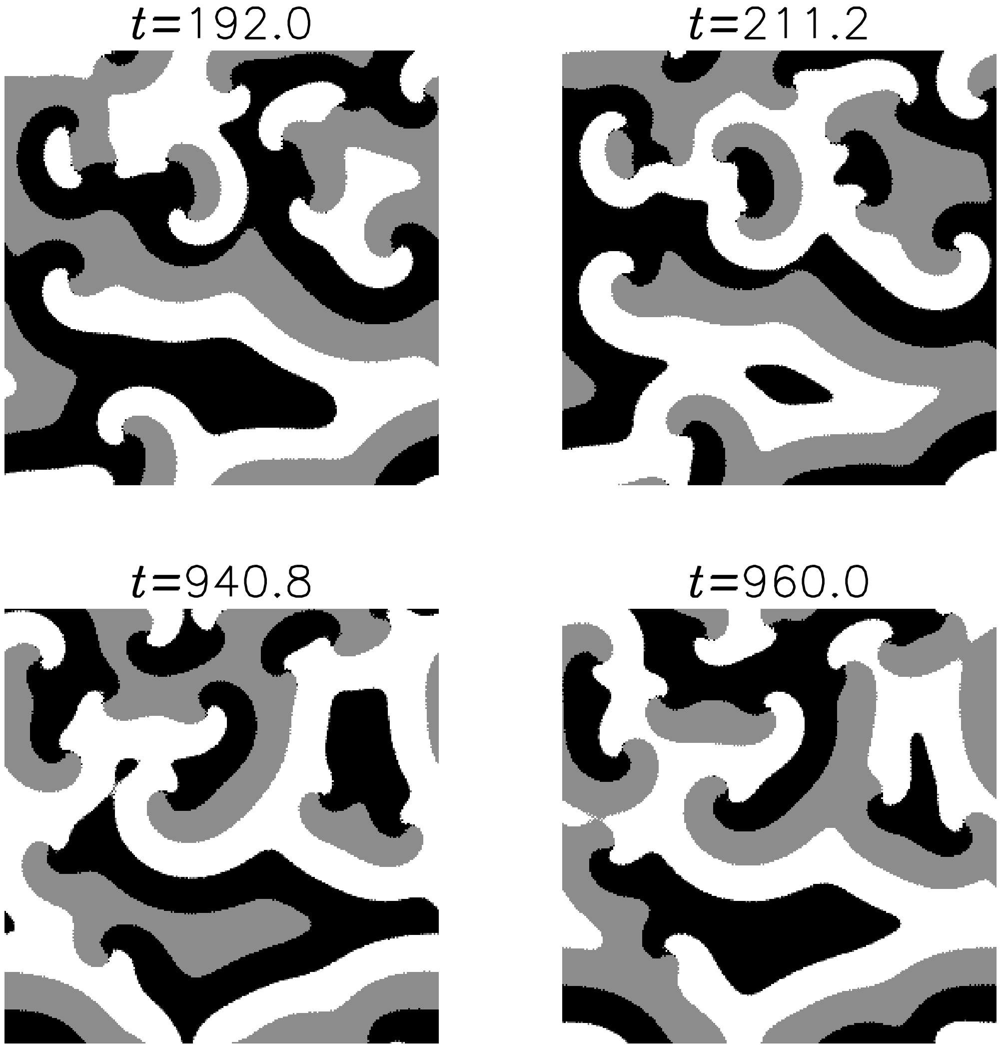

Fig. 2. Four snapshots corresponding to the numerical simulation of the system (1) in $d = 2$. Parameter values: $\eta = 3.5$, $\delta = 0.5$. The black, grey and white regions represent the regions occupied by A_1, A_2 and A_3 respectively.