Polarization and Transverse Mode Selection in Quantum Well Vertical-Cavity Surface-Emitting Lasers: Index- and Gain-guided Devices.

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Abstract

We study polarization switching and transverse mode competition in Vertical Cavity Surface Emitting Lasers in the absence of temperature effects. We use a model that incorporates the vector nature of the laser field, saturable dispersion, different carrier populations associated with different magnetic sublevels of the conduction and heavy hole valence bands in quantum well media, spin-flip relaxation processes and cavity birefringence and dichroism. We consider both index-guided and gain-guided VCSELs, and we find that spin-flip dynamics and linewidth enhancement factor are crucial for the selection of the polarization state corresponding to a given injection current. For index-guided VCSELs the effect of spatial hole burning on the polarization behavior within the fundamental mode regime is discussed. For gain-guided VCSELs, transverse mode and polarization selection is studied within a Maxwell-Bloch approximation which includes field diffraction and carrier diffusion. Polarization switching is found in the fundamental mode regime. The first order transverse mode starts lasing orthogonally

polarized to the fundamental mode. At larger currents polarization coexistence with several active transverse modes occurs.

I. INTRODUCTION

Vertical Cavity Surface Emitting Lasers (VCSELs) [1,2] present special optical and electrical properties (single-longitudinal mode emission, low-threshold current, low-divergence circular output beam, simplicity for building 2D arrays, etc) which make these devices extremely attractive for several applications such as optical-fiber and free-space communications, optical information storage, laser printers, etc. They also offer a number opportunities to study fundamental aspects of semiconductor laser dynamics. Among the latter, the issue of laser polarization state selection and the competition of polarization state and transverse mode profile are particularly interesting. Polarization selection in conventional edge-emitting lasers is largely determined by the geometry of the laser, but the cylindrical symmetry of the VCSEL gives, in principle, no preference for any polarization direction in the plane of the wafer. This makes very relevant the question of polarization selection by the nonlinear gain dynamics. Such a question was examined at large in the early days of laser physics, where preference for linear or circular polarization for several gas lasers was understood in terms of symmetries associated with different lasing transitions [3]. Pattern formation in the transverse profile of a laser beam has received considerable attention in the general framework of nonlinear dynamics [4]. VCSELs are interesting in this aspect because they easily have a rather large Fresnel number which favors the appearance of transverse structures [5,6]. Polarization control and transverse mode control are two linked practical problems which call for a better fundamental understanding of the physical mechanisms of polarization and transverse mode selection.

Light emitted from VCSELs is typically linearly polarized with the vector field oriented along one of two orthogonal crystal axis which are perpendicular to the emission direction. The two linearly polarized modes have also slightly different emission frequencies ($\approx$ 3-22 GHz) [7] due to the birefringence of the crystal [8,9]. Even though several VCSELs display stable linear polarization emission at any value of the applied current [10–12], many of these devices show a polarization instability, a relative abrupt switch in the orientation of the optical field known as Polarization Switching (PS). This occurs as the injection current is increased further above threshold [5,11–15]. Emission in both linearly polarized modes with different emission frequencies [10,11] as well as in both linearly polarized modes with the same emission frequency (elliptically polarized light) [16] have been reported. These polarization phenomena occur close to threshold within the fundamental transverse mode regime. For higher injection currents they may be accompanied by changes in the emission profile due to the excitation of higher order transverse modes. The onset of the first order transverse mode occurs for an applied current which strongly depends on the dimensions of the stripe contact. In addition, it is commonly observed that the first order transverse mode starts lasing orthogonally polarized to the fundamental mode [5,6,10,11,13,17,18].

An explanation offered to polarization state selection in VCSELs is based on the different rates of the thermal wavelength shift of the gain curve and the cavity resonances combined with the uncontrolled strain present in the growth process which may cause VCSEL anisotropies (birefringence and dichroism) [11]. As a consequence of the birefringence of the crystal, the two linearly polarized states experience slightly different material gains in such a way that the mode with higher gain to loss ratio dominates at threshold. When the injected current is increased above the lasing threshold, self-heating of the active region produces a faster red shift of the gain peak relative to the cavity resonances so that the gain difference between the linearly polarized modes changes. Within this framework *(i)* stable polarization emission occurs when the gain difference favors the same polarization mode at any current value, *(ii)* PS is expected when there is an exchange of the relative gain of the two modes, *(iii)* coexistence of both linearly polarized states occurs if the gain difference is too small.

Several points are worth noting in the above explanation. In the first place, since the frequency splitting between orthogonal linearly polarized states is often very small as compared to the width of the gain curve (below or of the order of 20 GHz as compared to 20 nm or more, respectively), the gain differences may be minute, so PS in the fundamental mode should be difficult to observe. Second, it does not explain the preference for linearly polarized emission in front of, i.e., circularly polarized light in VCSELs with very small anisotropies. Finally, it predicts stable polarization emission if thermal effects are minimized on the device, while PS is a well known phenomenon in gas lasers in which thermal effects are not important. In fact, recent experiments in gain-guided VCSELs by indicate that PS is still observed when the active region temperature is kept constant by using fast pulse low duty cycle operation [19]. Such PS does not fit in the scheme just discussed, but it is consistent with the mechanisms invoked for PS in the model introduced in Ref. [20] (SFM model).

The SFM model takes into account the vector nature of the laser field, saturable dispersion associated with the linewidth enhancement factor, and the carrier dynamics associated with the different magnetic sublevels of the conduction and heavy hole valence bands in a quantum-well. It predicts the preference of quantum well material for linearly polarized emission due to the finite coupling (via spin-flip relaxation processes) between the carrier populations involved in the two natural circularly polarized emission channels. For a perfect isotropic cavity, the linearly polarized laser light is randomly oriented in the plane of the active region, while for an anisotropic cavity, the cavity anisotropies fix the direction of linearly polarized emission [20]. A detailed study of the stability of the solutions of the SFM model incorporating cavity anisotropies reveals that the polarization state selection within the fundamental mode regime in VCSELs is governed by the coupling between the two carrier populations in the presence of amplitude-phase coupling (linewidth enhancement factor) [21–24]. As the injection current is scanned above the lasing threshold, stable polarization emission, polarization switching, polarization coexistence, or elliptically polarized emission, can be predicted depending on the VCSEL anisotropies.

The studies in [21–24] were based on rate-equations for the amplitudes of the two circularly polarized modes, disregarding spatial effects. However, Spatial-Hole-Burning and carrier diffusion may have an important role in the interplay between polarization and transverse mode selection. Two different situations can be distinguished according to the transverse structure of the VCSEL. In index-guided devices, the modal profiles and frequencies are determined by the built-in refraction index distribution of the VCSEL, thus allowing for a description in terms of modal amplitudes [25,26]. Instead, for gain-guided devices a modal description is not strictly valid because the field profile is fully determined by the carrier distribution, which in turn is dynamically coupled to the field through stimulated emission.

In this paper we consider an extension of the SFM model to incorporate spatial effects in both index-guided and gain-guided VCSELs. Concerning index-guided VCSELs, polarization properties have already been analyzed in Ref. [25] by using a model that includes birefringence and spatial hole burning effects. In this paper we incorporate saturable dispersion and polarization sensitive dynamics of the carriers in semiconductor quantum well media, and we restrict our analysis to the lowest order transverse mode. Spatial effects and transverse mode competition in gain-guided VCSELs are incorporated within a spatially continuous two dimensional approach which takes into account light diffraction, carrier diffusion and frequency dependent gain in the context of a Maxwell-Bloch type approximation for a quantum well laser [20]. A partial account of the main results of this model was reported in Ref. [27].

The paper is organized as follows: In Sect. II we summarize the SFM model which is based on the energy band structure of quantum-well VCSELs and we review the main results for polarization selection and switching in the rate-equation description for the fundamental transverse mode. In Sect. III we study polarization stability and PS within the fundamental transverse mode for a weakly index-guided birefringent VCSEL. In this case, birefringence induces different confinement factors for the linearly polarized modes. As a consequence, the linearly polarized modes experience different gains, which leads to polarization selection at threshold. Polarization switching is observed as the current is increased due to the combined effect of saturable dispersion and spin dynamics. In Sect. IV we discuss a continuous two-dimensional model for gain-guided lasers in the context of Maxwell-Bloch equations. The spatio-temporal and polarization properties of wide-area gain-guided VCSELs are discussed in Sect. V. We first study the polarization properties during the turn-on of the VCSEL. Polarization stability and PS within the fundamental transverse mode are shown as the current is scanned for two situations where the relative gain difference between the linearly polarized modes is different. New polarization instabilities are observed at higher injection currents related to the onset of higher order modes. The first order transverse mode always starts lasing orthogonally polarized to the fundamental one. At larger currents polarization coexistence with several active transverse modes occurs. We show that these results are sensitive to the strength of coupling between carrier populations associated with different circular polarizations of light (spin relaxation rate). A summary of our results and conclusions is given in Sect. VI.

II. LIGHT POLARIZATION DYNAMICS IN VCSELS

The model proposed in Ref. [20] to describe polarization dynamics in a VCSEL takes into account the coupling of the vector field amplitude to the dominant allowed optical transitions $\pm\frac{1}{2}\leftrightarrow\pm\frac{3}{2}$(see Fig. 1) corresponding to the spin sublevels of the conduction and heavy hole valence bands in surface emitting quantum well semiconductor lasers. Transitions to the light hole valence band ($\pm\frac{1}{2}$) are neglected. In a first approximation we consider a two-level type of approach in which transitions only occur at the center of the band gap. Consideration of lasing transitions at different wavenumbers leads to band mixing between the heavy and light hole band (which are nondegenerate at the center of the band gap). A detailed analysis of these effects indicates that they do not produce qualitative changes in the results that we will describe [28]. The dynamical variables considered are the two slowly varying circularly polarized components of the (scaled) vector optical field, $F_\pm$, and the (scaled) carrier populations of the two emission channels, $D_\pm$, which are coupled via spin-flip relaxation processes [29,30]. Defining the normalized total carrier number as $D = (D_+ + D_-)/2$ and the difference in the carrier numbers of the two magnetic sublevels as $d = (D_+ - D_-)/2$, the equations appropriate for narrow contact (single mode) VCSELs operating at constant active region temperature read

$$\dot{F}_\pm = -\kappa F_\pm - i\omega_o F_\pm + \kappa(1+i\alpha)(D\pm d)F_\pm - (\gamma_a - i\gamma_p)F_\mp \; , \tag{1}$$

$$\dot{D} = -\gamma_e(D-\mu) - \gamma_e(D+d)|F_+|^2 - \gamma_e(D-d)|F_-|^2 \; , \tag{2}$$

$$\dot{d} = -\gamma_s d - \gamma_e(D+d)|F_+|^2 + \gamma_e(D-d)|F_-|^2 \; , \tag{3}$$

where κ is the field decay rate, α is the linewidth enhancement factor, $\omega_0 = \kappa\alpha$ is a change of the frequency reference frame, γ_e is the decay rate of the total carrier population, and μ is the injection current normalized to threshold.

The parameter γ_s is the decay rate of the carrier population difference through spin-flip relaxation processes [30]. Polarization spectroscopy measurements of spin relaxation times in quantum wells estimate the value of the spin-flip decay time (γ_s^{-1}) in tens of picoseconds [29].

This gives an order of magnitude of the value of this parameter of the theory. However, precise determination of effective spin-flip relaxation rate in different operating conditions of a VCSEL is still lacking. Indirect evidence of the coupling between D_+ and D_-, as implied by γ_s, might be given by anticorrelations of the two independent polarization components in RIN spectra [31]. An indirect estimation of γ_s in the context of the predictions of this model has also been reported by measuring the magnetoelipticity of the VCSEL output light in the presence of a weak magnetic field [16]. Also in the context of this model, there are indications that the value of the spin-flip relaxation rate can be obtained from birefringency measurements of the spontaneous emission optical spectra [32,33] as well as from polarization fluctuations spectra of the VCSEL output [34]. Measurements of γ_s could also be obtained in experiments with optically pump VCSELs with circularly polarized pumps.

It is worth noting that the equation for the population difference cannot be adiabatically eliminated since the "spin decay time", γ_s^{-1}, is intermediate between the decay rate of the electric field and the one of the total carrier population D. As we will show next, the carrier variable d plays an important role in the polarization properties of VCSELs. In particular, the preference for linearly polarized light arises —even in the absence of anisotropies— from the finite value of γ_s [20,22]. In the limit of infinitely fast spin relaxation ($\gamma_s^{-1} = 0$), and in the absence of cavity anisotropies, there is no preference for linearly or circularly polarized light emission. Finally, we recall that the physics of the usual semiconductor laser rate equations for the intensity of a linearly polarized mode (given by $I = |E_+|^2 + |E_-|^2$) is recovered by setting $d = 0$ in Eqs. (1)-(3).

The meaning of the VCSEL anisotropies is most clearly displayed by rewriting Eq. (1) in the linear basis ($F_x = (F_+ + F_-)/\sqrt{2}, F_y = -i(F_+ - F_-)/\sqrt{2}$)

$$\dot{F_x} = -(\kappa + \gamma_a)F_x - i(\omega_o - \gamma_p)F_x + \kappa(1 + i\alpha)(DF_x + idF_y) \ , \tag{4}$$

$$\dot{F_y} = -(\kappa - \gamma_a)F_y - i(\omega_o + \gamma_p)F_y + \kappa(1 + i\alpha)(DF_y - idF_x) \ , \tag{5}$$

$$\dot{D} = -\gamma_e[D(1 + |F_x|^2 + |F_y|^2) - \mu + id(F_yF_x^* - F_xF_y^*)] \ , \tag{6}$$

$$\dot{d} = -\gamma_s d - \gamma_e[d(|F_x|^2 + |F_y|^2) + iD(F_yF_x^* - F_xF_y^*)] \ . \tag{7}$$

The frequency splitting between the linearly polarized modes is given by $2\gamma_p$ (birefringence). The strength of their anisotropic gain/losses is given by $2\gamma_a$ (dichroism). VCSEL anisotropies have been introduced phenomenologically in the model assuming, for simplicity, that the principal axes for birefringence and dichroism coincide, so that both are diagonalized by the same basis states. The axis mismatch between γ_a and γ_p yields elliptically, instead of linearly, polarized light emission [35,36], the ellipticity depending on the angle between the linear anisotropies. Reported measurements of elliptically polarized emission in VCSELs show that the ellipticity is very small [16,35], which indicates that the axes of anisotropy are nearly aligned.

A thorough discussion of the polarization states found in the rate equation model given by Eqs. (1)-(3), is given in Ref. [22]. Here we just review the main results. The model reveals the existence of different polarization states: (*i*) two orthogonal linearly polarized modes with different thresholds (D_i^{th}) and different emission frequencies (ω_i), given by

$$F_x = \sqrt{\frac{\mu - D_x^{th}}{D_x^{th}}} e^{i\omega_x t} \; , D_x^{th} = 1 + \frac{\gamma_a}{\kappa} \; , \tag{8}$$

$$F_y = \sqrt{\frac{\mu - D_y^{th}}{D_y^{th}}} e^{i\omega_y t} \; , D_y^{th} = 1 - \frac{\gamma_a}{\kappa} \; , \tag{9}$$

where $\omega_x = \gamma_p + \alpha\gamma_a = -\omega_y$ and $d_x^{th} = d_y^{th}$ =0. Notice that we have defined $\hat{x}$ ($\hat{y}$) polarization as the mode with high (low) frequency for positive γ_p but this definition is arbitrary and does not influence the results (see Ref. [22]); (*ii*) elliptically polarized states, where there is coexistence of two frequency degenerated linear polarization states but with different power strengths; and (*iii*) unstable states of circularly polarized light, which only exist in the absence of birefringency and dichroism [20].

The linear stability of the linearly polarized solutions for a fixed value of the α-factor is studied in terms of two control parameters commonly measured in PS experiments, the injection current normalized to the threshold current (μ), and the frequency splitting between the linearly polarized modes (γ_p) induced by birefringence. We recall that typical polarized light-current measurements correspond to vertical scannings in ($\mu - \gamma_p$) diagrams since the frequency splitting is a fixed characteristic of a given VCSEL, which is nearly independent of the injected current [7]. The results of the linear stability analysis are summarized in Fig. 2 for two general situations in which the relative gain-to-loss ratio between the modes is different.

Fig. 2(a) corresponds to the stability diagram for VCSELs where the gain difference favors the $\hat{y}$-polarized mode at threshold ($\gamma_a > 0$). The $\hat{y}$-polarized mode is stable on the left and below the dashed line. The $\hat{x}$-polarized mode is stable on the right and above the solid line. The stability diagram is divided into several regions: there are zones in which only one mode is stable, a zone in which none of the linearly polarized modes is stable, and a bistable domain. Stable $\hat{y}$-polarized emission occurs close to threshold for any birefringence value. For VCSELs where $\gamma_p > \gamma_s/(2\alpha)$, an abrupt $\hat{y}\rightarrow\hat{x}$ switching occurs as the dashed line is crossed during the current scanning [21,22]. The PS occurs by destabilization of the mode with the higher gain-to-loss ratio in favor of the weaker mode. In addition, the switching current linearly depends on the gain/loss anisotropy

$$\frac{\mu_{sw}}{\mu_{th}} = 1 + \frac{2(\gamma_s^2 + 4\gamma_p^2)}{\kappa(2\alpha\gamma_p - \gamma_s)} \frac{\gamma_a}{\gamma_e} \, . \tag{10}$$

Such a dependence is consistent with recent experimental results in gain-guided VCSELs operated at constant temperature of the active medium [19].

For gain difference favoring $\hat{x}$-polarization at any injection current, $\gamma_a < 0$, the $\hat{x}$-polarized mode is stable below the solid line, while $\hat{y}$-polarization is stable inside the zone bounded by the dashed curve (see Fig. 2(b)). Regions of linearly polarized monostability, bistability and total instability are again obtained. Stable $\hat{x}$-polarized emission is expected close above threshold for any birefringence value. For VCSELs in which $\gamma_p < \gamma_s/(2\alpha)$, $\hat{x} \rightarrow \hat{y}$ PS can occur if the injection current is increased sufficiently so that the solid line is crossed. The switching current is given by

$$\frac{\mu_{sw}}{\mu_{th}} = 1 + \frac{(\gamma_p^2 + \gamma_a^2)}{\kappa(\gamma_a + \alpha\gamma_p) - \gamma_p^2} \frac{\gamma_s}{\gamma_e} \, , \tag{11}$$

This switching is not abrupt. Rather it occurs through one of two frequency-degenerate (orthogonal orientations) stable elliptically polarized states. Consequently, elliptically polarized light can be understood as intermediate states reached in the destabilization by a steady bifurcation of the linearly polarized solution as the current is increased. Hysteresis in the switching current, as reported in some experimental observations [37], is also predicted as the injection current is raised and lowered.

We recall that the stability diagrams in Fig. 2 are a consequence of the combined effect of saturable dispersion associated with the α-factor, and of the spin dynamics associated with the finite value of γ_s. This statement is checked by studying the stability diagrams in the limits $\alpha \to 0$ or $\gamma_s \to \infty$. In any of these limits, the only region that survives is the bistable one, and laser emission occurs in the lowest threshold mode at any current value. The fact that PS will never occur in this situation indicates that the polarization state selection in VCSELs is largely determined by both the AM-FM coupling through the α-factor and the carrier population coupling of the natural circularly polarized emission channels via spin-flip relaxation processes.

III. POLARIZATION DYNAMICS OF INDEX-GUIDED VERTICAL-CAVITY SURFACE EMITTING LASERS

Studies of polarization state selection in index-guided VCSELs have been reported by *Valle et al.* [25,26,38,39]. For these devices, there is a modal gain difference between transverse modes with orthogonal polarization as a consequence of the combined effect of birefringence and spatial hole burning. Within this framework, polarization switching within the fundamental transverse mode as well as polarization switching in higher order transverse modes is attributed to spatial hole burning. However, the role of a finite spin-flip relaxation rate together with the $\alpha-$factor were disregarded.

In this section we extend the model in [39] to account for spin dynamics and AM-FM coupling, which we have shown in the previous section to be important mechanisms in the selection of the polarization state in VCSELs. We consider a cylindrically symmetric weak index-guided VCSEL. Birefringence is taken into account by assuming that the core refraction index in the x-direction is smaller than in the y-direction, while the cladding refractive index is the same in both directions. We will consider an index step Δn smaller than 0.01, Then the appropriate modes are the LP_{mn} modes [40]. Here we treat the case of competition between the two polarization states of the fundamental transverse mode LP_{01}. Several mechanisms of selection of this mode have been discussed in the literature [41,42]. The model can, if required, be extended to account for competition between polarization states of higher order transverse modes [38]. However, the polarization behavior due to spatial hole burning effects is similar in both the fundamental and multi-transverse mode operation regimes [39].

Spatial effects can be incorporated in Eqs. (1)-(3) by considering carrier diffusion, the optical mode transverse profiles and the carrier transverse distribution. Due to the cylindrical symmetry of the VCSEL structure, a cylindrical coordinate system is appropriate to describe spatial effects. In the basis of linearly polarized modes, the transverse optical field can be written in the following form

$$\mathbf{E}(r;t) = [E_x(t)\psi_x(r)\mathbf{x} + E_y(t)\psi_y(r)\mathbf{y}]e^{i\kappa\alpha t} + cc, \tag{12}$$

where $\psi_x(r)$ and $\psi_y(r)$ are the profiles for the $\hat{x}$ and $\hat{y}$ modes. They are obtained by solving the Helmholtz equation [25]. These profiles are slightly different due to birefringency. As the $\hat{y}$ mode has greater core refraction index (lower emission frequency), it is better confined than the mode $\hat{x}$. The equations describing the optical fields $E_i(t)$ $(i = x, y)$, the total carrier distribution $N(r,t)$, and the difference of the carrier distributions associated with the difference magnetic band sublevels $n(r,t)$ read

$$\dot{E}_x = \kappa(1+i\alpha)[E_x(g_x^m - 1) - iE_y g_{xy}^m] - (\gamma_a - i\gamma_p)E_x + \sqrt{\frac{\beta}{2}}[\sqrt{\overline{N}+\overline{n}}\;\xi_+(t) + \sqrt{\overline{N}-\overline{n}}\;\xi_-(t)], \tag{13}$$

$$\dot{E}_y = \kappa(1+i\alpha)[E_y(g_y^m - 1) + iE_x g_{yx}^m] + (\gamma_a - i\gamma_p)E_y - i\sqrt{\frac{\beta}{2}}[\sqrt{\overline{N}+\overline{n}}\;\xi_+(t) - \sqrt{\overline{N}-\overline{n}}\;\xi_-(t)], \tag{14}$$

$$\partial_t N = j(t)C(r) + \mathcal{D}\nabla_\perp^2 N - \gamma_e[N(1 + |E_x|^2\psi_x^2 + |E_y|^2\psi_y^2) + in(E_x E_y^* - E_y E_x^*)\psi_x\psi_y], \tag{15}$$

$$\partial_t n = -\gamma_s n + \mathcal{D}\nabla_\perp^2 n - \gamma_e[n(|E_x|^2\psi_x^2 + |E_y|^2\psi_y^2) + iN(E_x E_y^* - E_y E_x^*)\psi_x\psi_y], \tag{16}$$

where $g_i^m(t)$ $(i = x, y)$ is the modal gain normalized to the threshold gain

$$g_i^m = \frac{\int_0^\infty N(r;t)\psi_i^2(r)rdr}{\int_0^\infty \psi_i^2(r)rdr}, \tag{17}$$

and $g_{ij}^m(t)$ is given by

$$g_{ij}^m(t) = \frac{\int_0^\infty n(r,t)\psi_i(r)\psi_j(r)rdr}{\int_0^\infty \psi_i^2(r)rdr}. \tag{18}$$

Note that the modal gains for the $\hat{x}$ and $\hat{y}$ modes are different due to the different optical mode profiles. However, we neglect the material gain difference since the frequency splitting is very small as compared to the width of the gain curve. The total injected current is uniformly distributed across a circular disc contact of diameter ϕ, and the prefactor $j(t)$ allows for the generation of current ramps. Spontaneous emission processes are modeled by the terms $\xi_\pm$ taken as complex Gaussian white noise sources of zero mean value and δ-correlated in time. In the noise terms, the carrier distribution is integrated over the active region of radius a:

$$\overline{N}(t) = \frac{\int_0^a N(r,t) r dr}{a^2} \, , \quad \overline{n}(t) = \frac{\int_0^a n(r,t) r dr}{a^2}. \tag{19}$$

The numerical integration of Eqs. (13)-(16) is performed with an integration step in the radial direction of 0.1 μm, and an integration time step of 10^{-2} ps. The boundary condition for the carrier distribution is taken as $N(\infty, t) = n(\infty, t) = 0$. The initial conditions correspond to the stationary solution in the state below threshold with $\mu = 0.9$ [43]. The parameters values involved are: $\kappa = 300$ ns^{-1}, $\gamma_e = 1$ ns^{-1}, $\alpha = 3$, and $\mathcal{D} = 3$ cm^2/s. Different values of the spin-flip relaxation rate γ_s and of the index step Δn will be considered to study the combined effect of spin dynamics and spatial hole burning on the polarization properties of index-guided VCSELs. When Δn increases, the difference between the optical transverse profiles for the $\hat{x}$ and $\hat{y}$ modes decreases, because the ratio of birefringence to index step decreases. Then the modal gain anisotropy also decreases [38,39], and spatial hole burning effects have a smaller impact.

We first consider the case without external gain/loss anisotropy $\gamma_a = 0$, with a small birefringence $\gamma_p = 5.85$ ns^{-1} ($\Delta\nu \sim 2$ GHz), $\gamma_s = 50$ ns^{-1} and $\Delta n = 0.005$. The current is injected using a circular disc contact of diameter $\phi = 6$ μm equal to the diameter of the active region. The inset of Fig. 3 shows the stability diagram for $\gamma_a = 0$, calculated as in Fig. 2 (spatial effects are not taken into account). Just above threshold and for small values of the birefringence there is bistability of the two linearly polarized modes. However, the $\hat{y}$-polarized mode is always selected at threshold in the numerical simulations. This selection is due to the modal gain anisotropy between the fundamental modes induced by birefringence. Since at low current values the carriers accumulate near the center of the active region, the modal gain of the better confined polarization, the $\hat{y}$-mode, is always larger than that of the orthogonal polarization mode. Fig. 3 shows the polarized L-I characteristics obtained by linearly increasing the applied current from $I = 0.9I_{th}$ to $I = 2I_{th}$ in 200 ns. After the selection of the $\hat{y}$-polarization at threshold, a $\hat{y}\rightarrow\hat{x}$ PS is observed at $I \sim 1.45I_{th}$. This PS is due to the combined effect of spin dynamics and $\alpha-$factor, since the mode with less modal gain is selected when the current is increased (see Fig. 3); however, the switching current is higher than predicted by Eqs. (1)-(3) because of spatial effects.

In order to force emission in the $\hat{x}$-polarization at threshold, we introduce an external gain/loss anisotropy favoring $\hat{x}$-polarization (γ_a = -1 ns^{-1}). We also consider a higher birefringence $\gamma_p = 53\ ns^{-1}$ (around 15 GHz), and a higher spin-flin relaxation rate $\gamma_s = 100\ ns^{-1}$. In this situation, even though the modal gain for the better confined $\hat{y}$-mode is higher than for the $\hat{x}$-mode, the total net gain for the $\hat{x}$-mode is higher than for the $\hat{y}$-mode, thus favouring $\hat{x}$-polarization at threshold (see Fig. 4). This situation corresponds to the stability diagram shown in the inset of Fig. 4, which has been calculated by neglecting spatial effects as in Sect. II. From the inset, a $\hat{x}\rightarrow\hat{y}$ PS is expected at $I \sim 1.7I_{th}$. However, this switching is delayed by spatial effects, since the modal gain of the better confined $\hat{y}$-mode decreases when the injection current increases. The spatial hole burnt near the center of the active region in the carrier profile is deeper, which leads to an accumulation of carriers towards the cladding region, and hence to a decrease in the modal gain of the better confined polarization. However, for the parameters considered in our previous case, equal radius for the contact and the cavity and an index step $\Delta n = 0.005$, the delay in the PS is rather small, and the switching occurs at $I \sim 2I_{th}$. The delay increases for smaller Δn and larger radius of the disc contact even though the sweep rate across the bifurcation is not changed. Fig. 4 shows the L-I characteristics for a device with a contact diameter of 10 μm and an index step of 0.002. Similar results are obtained when the contact and the cavity have the same radius, but an effective diffusion length of 2 μm for the charge carriers is considered to take into account current spreading [44]. Since extra carriers are injected in the cladding region, spreading effects help spatial hole burning in creating a carrier profile concentrated in the near cladding region [39].

The L-I characteristics in Fig. 4 is obtained by linearly increasing the applied current from $I = 0.9I_{th}$ to $I = 7.7I_{th}$ in 200 ns. At threshold, the VCSEL switchs-on in the $\hat{x}$-polarized mode because of the effect of the external gain/loss anisotropy. Due to the spin dynamics and linewidth enhancement factor, an abrupt $\hat{x}\rightarrow\hat{y}$ PS is observed at $I \sim 5I_{th}$, even though the modal gain of the better confined $\hat{y}$ polarization is smaller than that of the $\hat{x}$ polarization due to spatial hole burning.

IV. TWO-LEVEL MAXWELL-BLOCH MODEL FOR GAIN-GUIDED VERTICAL-CAVITY SURFACE EMITTING LASERS

In this section we study the spatiotemporal properties of gain-guided (proton implanted) VCSELs. The SFM model can be extended to account for spatial effects in the transverse plane by considering in Eqs. (1)-(3) the optical field diffraction, carrier diffusion, and the transverse dependence of the variables $F_{\pm}$, D, d and the injection current μ. The continuous description includes in a natural way a very high number of transverse modes. In addition one has to take into account that the dynamics of several transverse modes in VCSELs evolves in time scales of the order of the inverse of their frequency difference (typically 100 GHz). Hence, to properly describe such a mode competition we should include in the model an equation incorporating the material susceptibility providing a frequency-dependent gain and dispersion. In this paper we do this in a simple approximation for the material polarization of the laser medium by considering it as a set of two-level coupled systems (one for each of the two independent polarization components of the field). Whitin this framework, the system can be described in terms of two sets of semiclassical Maxwell-Bloch equations, coupled through spin relaxation processes, which describe the dynamics of the carrier densities in each spin channel, the slowly-varying amplitudes (SVA) of the circularly polarized optical fields and their associated material dipole densities (see i.e. [20,27]).

After an appropriate rescaling of the dynamical variables, the equations describing the spatio-temporal and polarization properties of a single-longitudinal mode gain-guided VCSEL read

$$\partial_t E_{\pm}(x,y;t) = -\kappa(1+i\theta)E_{\pm} + P_{\pm} - i\frac{c^2}{2\Omega {n_e}^2}\nabla_{\perp}^2 E_{\pm} - (\eta_a - i\eta_p)E_{\mp} \ , \quad (20)$$

$$\partial_t P_{\pm}(x,y;t) = -\gamma_{\perp}(1-i\theta)P_{\pm} + \gamma_{\perp} g_N (1+\theta^2)(N - N_0 \pm n)E_{\pm} + \sqrt{\beta(N\pm n)}\chi_{\pm}(x,y;t) \ , \quad (21)$$

$$\partial_t N(x,y;t) = j(t)C(x,y) - \gamma_e N + \mathcal{D}\nabla_{\perp}^2 N - [(E_+P_+^* + E_-P_-^*) + c.c.] \ , \quad (22)$$

$$\partial_t n(x,y;t) = -\gamma_s n + \mathcal{D}\nabla_{\perp}^2 n - [(E_+P_+^* - E_-P_-^*) + c.c.] \ , \quad (23)$$

where $P_{\pm}$ are the SVA of the material dipole densities corresponding to the left and right circularly-polarized optical fields $E_\pm$, $\theta = (\omega_{mat} - \Omega)/\gamma_\perp$ is the detuning of the carrier frequency (Ω) from the material dipole resonance frequency (ω_{mat}), $N - N_0$ is the total carrier distribution referred to the transparency value N_0, and n is the difference of the carrier distributions associated with E_+ and E_- respectively. Optical diffraction and carrier diffusion are taken into account in a longitudinal mean field approximation by the transverse Laplacian ($\nabla_\perp^2$) terms in Eq. (20) and Eq. (22)-(23) respectively. In our gain-guided situation diffraction leads to the selection of the transverse mode profiles and the associated frequencies [45]. The total injected current, I, is assumed to be uniformly distributed across a circular region of diameter ϕ (VCSEL contact) defining the transverse current density distribution, $C(x,y)$, while the prefactor $j(t)$ allows for the generation of current ramps. Spontaneous emission processes are modeled by the terms $\chi_\pm$ taken as complex Gaussian white noise sources of zero mean value and δ-correlated in space and time in the usual way (see i.e. Ref. [46]). The parameters η_a and η_p are related to the linear dichroism and birefringence of the VCSEL respectively, but notice that, as discussed in the next section, they do not directly correspond to γ_a and γ_p because of the nonlinear susceptibility. Other parameters in the equations are: the index of refraction n_e, the carrier frequency Ω (corresponding to the emission wavelength $\lambda = c/(\Omega n_e)$), the differential gain g_N, the carrier diffusion constant $\mathcal{D}$, and the spontaneous emission factor β. We recall that, neglecting the transverse dependence, Eqs. (1)-(3) are recovered after the adiabatic elimination of Eq. (21), using the scaled variables $D = g_N(N - N_0)/\kappa$, $d = g_N n/\kappa$, and $F_\pm = \sqrt{2\kappa/\gamma_e} E_\pm$.

We next discuss the limitations of the two-level Maxwell-Bloch model for semiconductor lasers. The active-material dipole densities $P_\pm$ are sources of the optical fields, providing both material gain and changes in the background refraction index of the system (dispersion) through the complex electrical susceptibility. The complex nonlinear susceptibility resulting from our macroscopic nonlinear polarization is obtained by Fourier transform of Eq. (21). In the frequency domain, it reads

$$\begin{aligned}\mathcal{P}_\pm = \epsilon_o \chi_\omega(N) \mathcal{E}_\pm = \\ \frac{g_N(1+\theta^2)(N-N_0)}{1+(\theta-\omega/\gamma_\perp)^2}(1+i(\theta-\omega/\gamma_\perp))\mathcal{E}_\pm \,, \end{aligned} \tag{24}$$

where we have taken into account that n=0 in the steady state for linearly polarized light.

The real part of the complex susceptibility yields the gain spectrum, which in the Two-Level approach has a symmetric, Lorentzian profile. The dispersion spectrum, associated with the imaginary part of the complex susceptibility, is antisymmetric. The gain peak is located at the dipole resonance frequency (ω_{mat}), where dispersion vanishes. It is also worth noting that the ratio of the carrier-induced changes in the imaginary and the real parts of the susceptibility, which would correspond to Henry's α-factor in semiconductor lasers [47], is given by

$$\alpha = \frac{\partial_N \ \chi_\omega{}''}{\partial_N \ \chi_\omega{}'} = \theta - \frac{\omega}{\gamma_\perp} \ . \tag{25}$$

where ω is the emission frequency referred to the carrier frequency. Hence, the detuning in the two-level model plays the role of the α-parameter in semiconductor lasers. Besides providing AM-FM coupling, the effect of the detuning, θ, in a two-level model is twofold: (i) it determines the change in refraction index induced by carrier density: for positive (negative) detuning, an increase in carrier density is accompanied by a decrease (increase) in the index of refraction, thus yielding carrier-induced index anti-guiding (guiding) and opposing (reinforcing) the gain-guiding mechanism for the field; and (ii) since higher order modes have always higher frequencies than the fundamental one, the sign of the detuning affects the stability of the fundamental mode: for negative detuning, the gain of the fundamental mode is always the highest, while for positive detuning a higher order mode closer to the gain peak can exist [48]. Typical values of the detuning in two-level lasers are $-0.25 < \theta < 0.25$ while, for semiconductor lasers, $\alpha \sim 2 - 6$. Such large values of α as compared to those of θ in two-level lasers imply that the system must be artificially enforced to operate very far away from the gain peak resonance.

From the above discussion, we see that the Two-Level-Model does not account for some properties of a semiconductor medium, which include: i) a strongly asymmetric gain spectrum, ii) an operation wavelength close to the gain peak, and iii) large α-factor in the vicinity of the gain peak. However, an accurate description of the material polarization in semiconductors is really complicated since radiative transitions occur between band states with uneven, temperature dependent occupation; moreover, the band structure and transition probabilities of the system can be affected by many-body effects like bandgap shrinkage and Coulomb enhancement. Several microscopic models have been formulated for the calculation of the material susceptibility [49,50], but their high complexity makes difficult to interpret the dynamical features of the system when aiming to identify the physical dominant mechanisms. In addition, there is no simple way to write time-dependent dynamical equations since the nonlinear, frequency dependent, susceptibility is given. For this reason, there is ongoing research towards developing simpler models which incorporate the main results of the microscopic theories in a phenomenological way [51,52].

Having spelled out clearly the difficulties of modeling the nonlinear dynamics of semiconductors gain-guided VCSELs, we use here a Two-Level model in order to provide the system with a frequency dependent susceptibility (gain and dispersion). It incorporates, in good comparison with experimental measurements, a proper description of the spatio-temporal dynamics of semiconductor gain-guided lasers which are strongly dominated by transverse mechanisms as the modal gain, the carrier diffusion and optical field diffraction [45,46]. We choose to operate the VCSEL on the negative detuning side of the gain spectrum (negative α) in order to preserve the observed property that higher order transverse modes have, typically, lower gain that the fundamental one. We note that the effects of such a negative α-value — which leads to carrier-induced-guiding instead of antiguiding — occur in real VCSELs due to thermal effects since the index distribution depends on both the carrier distribution, which leads to carrier-induced index-antiguiding through the α-factor, and the temperature profile in the active region associated to the current distribution. The latter leads to thermally induced index-guiding. Typical values for these mechanisms are dn/dN=-1.2 $\times$ 10 $^{-8}\mu m^3$ [53] and dn/dT=3 $\times$ 10 $^{-4}K^{-1}$ [54]. Thus, both mechanisms act in opposite directions so the carrier-induced index antiguiding can be compensated or even overcome by the thermally-induced guiding. From measurements of the wavefront curvature in gain-guided Quantum Well VCSELs it is shown that thermal-guiding is stronger than carrier-antiguiding [17].

V. POLARIZATION AND TRANSVERSE MODE DYNAMICS OF GAIN-GUIDED VERTICAL-CAVITY SURFACE EMITTING LASERS

In this Section, the polarization and transverse mode dynamics of gain-guided VCSELs is studied by numerical integration of Eqs. (20)-(23). The integration scheme we follow has four steps: (*i*) the diffraction and diffusion terms are calculated by Fast Fourier Transform (FFT) from the previous integration step (initially from noise initial conditions); (*ii*) the complex spatial distributions for the spontaneous emission in both material polarizations are generated; (*iii*) the spatial distributions for the optical fields $E_\pm$, the carrier densities N and n, and the material polarizations $P_\pm$ are updated via the Euler method for the integration of stochastic differential equations [55]; and (*iv*) in order to obtain the light-intensity (LI) characteristics, the injected current is updated by increasing $j(t)$, and the scheme is repeated for the next time step. We point out that the choice of the time integration step is strongly influenced by the optical diffraction term, since otherwise the algorithm becomes numerically unstable. In order to achieve a compromise between spatial resolution, equation stability and computation time, we use 32x32 grid points for a transverse width of 40x40 μm^2 and an integration time step of $10^{-4} ps$.

Several things are worth noticing from the numerical scheme. On the one hand, the calculation of the Laplacian terms by the FFT method requires the use of periodic boundary conditions for the dynamical variables. However, such a problem is solved by considering a transverse integration region much wider than the injection area, so that the dynamical variables decay to values of the order of the noise level at the integration boundaries, thus avoiding spurious reentering waves. On the other hand, since the thermal response of the VCSEL is around 1 μs [56] and the carrier-field interaction is around 1 ns, the use of short current ramps (40 ns) to generate the LI characteristics ensures that the measurements are taken in a quasi-steady situation, while there is no thermally induced red shift of both the cavity mode and gain spectrum [19].

The equations of the model are rescaled so that $|E_i(x_o, y_o)|^2$ $(i = x, y)$, represents a magnitude proportional to the photon number going out from the VCSEL at the point (x_o, y_o) of the transverse plane. The total emitted power is calculated by integrating the optical intensity transverse distribution for each polarization and assuming a quantum efficiency of 100 % and a 1.5 μm effective cavity length. The rest of the parameter values involved in the equations are [16,46,57]: κ =300 ns^{-1}, $\gamma_\perp$ =20 ps^{-1}, γ_e =1 ns^{-1}, γ_s =50 ns^{-1}, θ=-3, N_0 =1.3 $10^6 \mu m^{-3}$, λ =750 nm, n_e=3.55, $g_N = 10^{-6} \mu m^3 ps^{-1}$, and $\mathcal{D} = 3\ cm^2/s$.

At this point, it is important to discuss the mechanisms which can lead to gain/loss anisotropies for the linearly polarized modes in the present model. These are the external gain/loss anisotropies, modal gain anisotropies, and material gain anisotropy. External anisotropies, which are described by η_a, can have different origins; they can be either intentional, i.e. introduced in either the cavity geometry [10,58,59] or in the gain medium [60,61], or unintentional, i.e. due to the imperfections in the fabrication process [16,62]. Modal gain differences are due to the different overlap of the optical mode profiles and the material gain distribution. This effect is naturally included in the model. The evaluation of this source of gain anisotropy in gain-guided VCSELs, and generally in gain-guided lasers, is a difficult task since the transverse mode profiles are intimately related to the carrier density transverse distribution, which strongly depends on the carrier diffusion, the leakage current, and on the power distribution of the optical mode (spatial hole burning). Finally, the material gain anisotropy is related to both the birefringence and the material gain spectrum: since the two linearly polarized modes are frequency split, they also have slightly different material gain coefficients.

An approximate estimation of the material gain anisotropy in the VCSEL can be performed rewriting Eqs. (20)-(21) in the frequency domain, neglecting the transverse terms, and considering the steady state carrier population values for linearly polarized light (n^{ST} =0, $N^{ST} = N_i^{th}$). These equations, in the linear basis, read

$$i\omega\mathcal{E}_i = -\kappa(1+i\theta)\mathcal{E}_i + \mathcal{P}_i \mp (\eta_a - i\eta_p)\mathcal{E}_i \ , \tag{26}$$

$$i\omega\mathcal{P}_i = -\gamma_\perp(1-i\theta)\mathcal{P}_i + \gamma_\perp g_N(1+\theta^2)(N_i^{th} - N_0)\mathcal{E}_i \ , \tag{27}$$

where the negative (positive) sign stands for the $i = x(y)$ polarized TEM$_{00}$ mode. Replacing Eq. (27) into Eq. (26) we end up with the fundamental mode emission frequencies

$$\omega_x = \frac{\eta_p + \theta\eta_a}{1+\kappa/\gamma_\perp} \ , \qquad \omega_y = -\frac{\eta_p + \theta\eta_a}{1+\kappa/\gamma_\perp} \ ,$$

and the carrier threshold values for each linearly polarized mode

$$(N_i^{th} - N_0) = \frac{(1 + (\theta - \omega_i/\gamma_\perp)^2)}{g_N(1+\theta^2)}(\kappa \pm \eta_a) \quad (i = x, y) \ , \tag{28}$$

where the positive (negative) sign stands for $\hat{x}$ ($\hat{y}$) polarization. Above threshold, the carrier density remains clamped to its threshold value N^{th} (smallest value of N_x^{th} and N_y^{th}), which will depend on the particular choice of the η_a and η_p values. Then, the material gain for each linearly polarized mode, with associated frequency ω_i, is provided by the real part of the nonlinear susceptibility

$$g_i = g(\omega_i) = \frac{g_N(1+\theta^2)(N^{th} - No)}{1 + (\theta - \omega_i/\gamma_\perp)^2} \quad (i = x, y) \ . \tag{29}$$

Hence, the material gain difference between the linearly polarized modes is

$$\Delta g = (g_y - g_x) \ . \tag{30}$$

while the real frequency splitting between the linearly polarized modes is

$$\Delta\nu = \frac{\Delta\omega}{2\pi} = \frac{\omega_x - \omega_y}{2\pi} = \frac{\eta_p + \theta\eta_a}{\pi(1 + \kappa/\gamma_\perp)} \ . \tag{31}$$

The devices we study are circular contact VCSELs with different diameters, a fixed value of η_p, and different η_a values. The first device we consider, VCSEL A, has a diameter of 12.5 μm, and we chose η_p= 3 γ_e and η_a=0. For these parameters (see the inset in Fig. 5), $\omega_y < \omega_x$ ($\Delta\nu \sim$ 0.95 GHz) so $N^{th} = N_y^{th} < N_x^{th}$. As a consequence, the material gain favors $\hat{y}$-polarization at threshold ($\Delta g \approx 0.053\ \gamma_e > 0$). In addition, the threshold current, calculated from the threshold carrier density, is $I_{th} \sim 5\ mA$.

Fig. 5 shows the polarization dynamics during the laser turn-on. The laser has a pre-bias current of 0.98 I_{th} and is abruptly biased to 1.05 I_{th} at $t = 0$. Initially, the laser suffers a delay of 0.5 ns in the switch-on time because the prebias current is below threshold. After the switch-on, the intensity in both polarizations shows the typical relaxation oscillations, the power in the $\hat{y}$-polarized mode being larger than in the $\hat{x}$-polarized mode. The reason is that, during the switch-on, the total carrier density overcomes the threshold carrier density for each linearly polarized mode, and therefore, the modal gain for both polarizations overcomes its threshold value. However, 2 ns after the switch-on, the output power in the $\hat{x}$-TEM$_{00}$ mode goes to zero since the total carrier density is reaching $N_y^{th}(x, y)$. As a consequence, the $\hat{x}$- polarized mode switches off, and the $\hat{y}$-polarization mode remains as expected. A similar behavior has been experimentally found in Ref. [63], but in this case the relaxation oscillations are more damped because the pumping conditions (pre-bias current of I_{th} and biased to 2 I_{th}). Nevertheless, the characteristic time for the selection of the polarization state is in good agreement with the one found numerically.

Fig. 6 shows the polarized L-I characteristics for VCSEL A obtained by linearly increasing the applied current from $I = 1.05I_{th}$ to $I = 2.16I_{th}$ in 40 ns. The total output power emitted by the VCSEL increases linearly with the current and stable $\hat{y}$-polarized emission in the fundamental transverse mode is observed up to $I \sim 1.50I_{th}$. This polarization behavior is predicted in Fig. 2(a) for low birefringences, as it is the case. For current values larger than $1.50I_{th}$ higher order modes start lasing in both polarizations. Here we observe a general feature of the polarization instabilities seen in several VCSELs: the first order transverse mode starts lasing orthogonally polarized to the fundamental mode [12,13,17].

The insets in Fig. 6 show the instantaneous transverse near-field profiles at the indicated currents (the plotted area corresponds to a square of $10 \times 10\mu m^2$). During fundamental mode operation, the width of the $\hat{y}$-polarized Gaussian mode is about 60% of the current contact, and slightly increases with increasing current. The off-polarization mode only emits amplified spontaneous emission from the entire VCSEL contact. In the multitransverse mode regime, the insets in Fig. 6 show that, while the $\hat{y}$-polarized component still consists on a single lobe with the peak moving around the VCSEL contact as the current increases, the $\hat{x}$-polarized beam profile dynamically adopts several shapes: (i) two lobes oscillating in time around the center of the VCSEL at $I \sim 1.55I_{th}$, (ii) a doughnut-shape at $I \sim 1.79I_{th}$, (iii) an off-centered lobe distribution at $I \sim 2.08I_{th}$.

In order to force $\hat{x}$-polarized emission at threshold we should introduce an external gain/loss anisotropy ($\eta_a < 0$) to overcome the material gain anisotropy. In addition, to increase the current range where fundamental mode operation occurs, we have to decrease the diameter of the pump region. For these reasons, we consider VCSEL B with a contact diameter of 10 μm and the following parameters values $\eta_p = 3.0\gamma_e$ and $\eta_a = -5.0\gamma_e$. The threshold current is now lower, $I_{th} = 3.25\ mA$ because of the smaller active region volume. The real frequency splitting is $\Delta\nu \sim 5.6\ GHz$ (so $\omega_y < \omega_x$). However, the threshold carrier density remains clamped to $N^{th} = N_x^{th}$ because of the negative value of η_a. Hence, even though the material gain for the fundamental $\hat{y}$–polarized mode is higher than for the $\hat{x}$–polarized mode ($\Delta g \approx 0.314\ \gamma_e > 0$), the total net gain for the $\hat{x}$–polarized mode is higher than that for the $\hat{y}$–polarized mode.

Fig. 7 shows the L-I characteristics for VCSEL B. At threshold, the VCSEL switches-on in the $\hat{x}$-polarized TEM_{00} mode because of the effect of the external anisotropy. For increasing current, an abrupt $\hat{x}\rightarrow\hat{y}$ PS is observed at $I \sim 1.45 I_{th}$ while the mode profile does not change. The switching current depends on the value of η_a, (i.e. for $\eta_a = -2.5\gamma_e$ PS occurs at $I \sim 1.30 I_{th}$). For VCSEL B, the single mode regime extends up to 1.85 I_{th} to be compared with VCSEL A, for which the onset of higher order modes occurs at 1.5 I_{th}. For this reason, we expect that PS will disappear in wider VCSELs with the same parameter values. The first order transverse mode is orthogonally polarized to the fundamental one, as for VCSEL A. During the multitransverse mode regime, the $\hat{x}$-polarized total power increases almost linearly while the $\hat{y}$-polarized total power saturates. Such general behavior corresponds to the scenario found in [11,13,18].

The modal behavior of the VCSEL emission can be obtained by integrating Eqs. (20)-(23) at a fixed current value instead of using a current ramp. Fig. 8 shows the optical spectra and the transverse mode profiles obtained at four different injection current values. These spectra are equivalent to those obtained by a Fabry-Perot interferometer with a free spectral range of 1000 GHz and a frequency resolution of 2 GHz. At $I \sim 1.29 I_{th}$, the polarized spectrum in Fig. 8(a) shows that the laser mainly emits in the $\hat{x}$-TEM_{00} mode. However, the orthogonal polarization shows a strongly suppressed peak (~-40 dB) while the frequency difference is below the frequency resolution, so it may correspond to elliptically polarized light. Beyond the switching current, at $I \sim 1.67 I_{th}$, laser emission occurs in the $\hat{y}$-polarized Gaussian mode (Fig. 8(b)). For increasing current, $I \sim 1.87 I_{th}$, two transverse modes, the TEM_{00} (α) and the TEM_{10} (β), coexist, but with different polarizations (Fig. 8(c)). The onset of the TEM_{10} can be explained through the change in its modal gain due to the competition between spatial hole burning and carrier diffusion [40,64], but an explanation of why it is orthogonally polarized to the fundamental mode requires invoking polarization-dependent mechanisms. Also notice that, because of the two-level nonlinear susceptibility model, there is a small red shift of the modal frequencies for increasing current, whereas a blue shift is expected for a better description of semiconductor susceptibility.

Fig. 8(d) shows the spectrum at $I \sim 2.26 I_{th}$. Several transverse modes are active in each polarization. It is remarkable that the two linear polarizations choose to operate in modes of different parity. Even order modes are $\hat{y}$-polarized (a dominant TEM_{00} mode (α), and some strongly suppressed second order modes (δ and ϵ)), while $\hat{x}$-polarized modes have odd order profiles (TEM_{10} (β) and TEM_{01} (γ) modes). In opposition to the cases shown in Fig. 8(a)-(c), where the total intensity emitted in each polarization is constant, at this current value the output of the laser oscillates in time. The total $\hat{x}$-polarized power is modulated at twice the beat note of the $\hat{x}$-polarized first-order transverse modes ($\approx 17\ GHz$). Periodic modulation at twice the beat frequency is also observed in the total $\hat{y}$-polarized power but, in this case, as a consequence of the nonlinear coupling between the two linearly polarized field components and the total carrier population.

For even larger current values, $I \sim 2.55 I_{th}$, coexistence of several transverse modes in both linear polarizations is observed in the optical spectrum (Fig. 9(b)). The time dependence of the total $\hat{x}$- and $\hat{y}$-polarized output powers, shown in Fig. 9(a), is as previously described for $I \sim 2.26 I_{th}$, but a fast additional modulation (ripples) in both linearly polarized powers is observed. This fast modulation corresponds to twice the beat notes between the fundamental mode and the frequency non-degenerated first order modes ($\sim$ 180 and 200 GHz respectively). Although the total emitted power has a similar behavior for both polarizations, the spatiotemporal dynamics is different. The $\hat{y}$-polarized beam keeps a single-lobe distribution (almost Gaussian) with the peak slightly moving around the center of the VCSEL contact. In the orthogonal polarization, the beam profile oscillates periodically (it does not twist) between two positions where the beam consists on two-lobes oriented along the diagonals of the $x-y$ plane (see the insets in Fig. 9), and where the maximum power is emitted. Between these two positions, we observe doughnut-like emission. Similar spatiotemporal behavior is observed in the case of Fig. 8(d).

We have also studied the relevance of the coupling mechanism between the circularly polarized emission channels in the transverse and polarization properties of gain-guided VCSELs by taking a fast spin-flip relaxation rate ($\gamma_s = 500\gamma_e$). In these conditions, there is very fast mixing of the carrier population between the two channels, so that n quickly relaxes to zero for linearly polarized light. Hence, the two linearly polarized fields are coupled to a single carrier population N. The most relevant difference with respect to previous cases is that now the dynamics is not sensitive to the polarization. In either VCSEL A or B, polarization stability is observed in the fundamental mode regime and transverse modes start lasing in the polarization of the fundamental mode. For VCSEL A, the onset of higher order modes occurs at 1.60 I_{th}, that is similar to the value found when the spin-flip dynamics is not too fast (see Fig. 6). However, the fundamental mode regime extends to $I \sim 2.25 I_{th}$ for VCSEL B as evidenced from the L-I characteristics in Fig. 10. These features indicate that the selection of a particular transverse mode does not *only* depend on the modal gain when the spin-flip dynamics is not too fast. Therefore, we infer that physical mechanisms associated with the spin-flip relaxation rate are crucial in determining the transverse and polarization properties of gain-guided VCSELs.

VI. CONCLUSIONS

Our results show that physical mechanisms associated with the spin dynamics are crucial in determining the polarization behavior of both index-guided and gain-guided VCSELs. A finite spin-flip relaxation rate together with the linewidth enhancement factor are crucial in determining the transverse and polarization state of the VCSEL. Polarization switching from the less to the better confined polarization in the fundamental mode of index-guided VCSELs is not observed when these mechanisms are not taken into account. As compared to the rate equation description, the location of the boundary between the different regions in the stability diagram of the linearly polarized fundamental mode is modified by spatial effects. For gain-guided devices, we have used a Two-Level model which properly describes most polarization and spatio-temporal properties of real VCSELs. The time necessary for the selection of the polarization state during a turn-on event is in good agreement with the experimental observations. For Light-Intensity measurements, either polarization stability or polarization switching occur within the fundamental mode regime depending on the VCSEL anisotropies. In addition, the onset of the first order transverse mode occurs in a polarization orthogonal to the lasing fundamental transverse mode, as observed in the experiments. Finally, we notice that the occurence of polarization switching in the fundamental mode may be inhibited by the excitation of higher order transverse modes.

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FIGURES

FIG. 1. Four level model for polarization dynamics in QW-VCSELs.

FIG. 2. Stability diagrams of the linearly polarized solutions for (a) γ_a= 0.1 γ_e ($\hat{y}$-favored), and (b) γ_a= -0.1 γ_e ($\hat{x}$-favored). The rest of the parameters are: κ = 300 ns^{-1}, γ_e = 1 ns^{-1}, γ_s= 50 ns^{-1}, and α =-3.

FIG. 3. (a) L-I characteristic for the linear polarizations. The stability diagram obtained from Sect. II appears in the inset. (b) Modal gain versus injection current. Curves related to $\hat{x}$ ($\hat{y}$) polarization are plotted with solid (dashed) line.

FIG. 4. Same as in Fig. 3 but for γ_s = 100 ns^{-1}, γ_a =-1 ns^{-1}, γ_p = 53 ns^{-1}, Δn = 0.002, and ϕ = 10 μm.

FIG. 5. Turn-on event for VCSEL A. The inset shows the Maxwell-Bloch two-level gain spectrum and the location of the linearly polarized TEM_{00} modes for the set of parameters chosen.

FIG. 6. VCSEL A. L-I characteristic for the linearly $\hat{x}$ (solid) and $\hat{y}$ (dashed) polarizations.

FIG. 7. Same as in Fig. 6 but for VCSEL B.

FIG. 8. VCSEL B. Optical spectra of the linearly polarized field components E_x (solid) and E_y (dashed) for fixed current values indicated in Fig. 7 by arrows: (a) $I = 1.29 I_{th}$, (b) $I = 1.67 I_{th}$, (c) $I = 1.87 I_{th}$, (d) $I = 2.26 I_{th}$.

FIG. 9. VCSEL B. (a) Time evolution of the total emitted power, and (b) optical spectrum at $I = 2.55 I_{th}$. Solid (dashed) line stands for the $\hat{x}$ ($\hat{y}$) polarized field component.

FIG. 10. Fast spin-flip relaxation rate. Same as in Fig. 7 but for γ_s=500 γ_e.