TRANSIENT EVOLUTION OF THE FIELD POWER SPECTRUM DURING THE SWITCH-ON OF A DETUNED LASER

S.Ciuchi* and M. San Miguel

Departament de Fisica, Universitat de les Illes Balears and

Instituto Mediterraneo de Estudios Avanzados, IMEDEA (CSIC-UIB)

E-07071 Palma de Mallorca, Spain

Abstract

We describe the time evolution of the finite window field power spectrum during the switch-on transient of a detuned laser. We derive an analytical expression for a stochastic map that generates a time series of the field over time steps of the order of the fast time scale of evolution. The different effects of static initial intensity noise and phase noise along the evolution are disentangled.

Initiation of laser emission can be viewed as a process of noise amplification. For this reason noise effects are particularly important during the transient dynamics. The statistics of laser switch-on processes have been considered in a variety of situations [1–11], including gas lasers [1,2,11], dye lasers [4,5], solid state lasers [11] and semiconductor laser [7,8,10,11].

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*permanent address Dipartimento di Chimica, Universitá della Basilicata, via Nazario Sauro 85, 85100 Potenza, Italy

In general it is possible to distinguish two time regimes during the switch-on process. The first is a regime of linear amplification up to a random switch-on time. During a second nonlinear regime large fluctuations of the laser field occur. These two regimes have been characterized either by ensemble averages of the intensity fluctuations or by the statistics of the switch-on time. Precise relations between these two characterizations have recently been described [11]. However, both characterizations refer to the evolution of the intensity, and much less attention has yet been paid to describing the transient stochastic dynamics of the frequency and phase of the laser field. A first study [9] along this line considered the coupling and interplay of phase and intensity fluctuations in the time domain. Such coupling is known to be particularly important in semiconductor lasers, where statistical properties of time resolved spectra during the transient regime have been discussed in some detail [10]. These studies have been numerical, making use of extensive simulation of stochastic equations. Up to now, no analytical description of the transient phase and frequency fluctuations in the frequency domain seems to be available. This is considered in this paper where we report a calculation of the transient Field Power Spectrum in a finite time window during laser switch-on. We consider the simple case of a class-A laser in which the laser field is the slowest relevant variable. Our analysis introduces a novel way of describing the transient evolution that recognizes clearly the different role of the intensity static departure noise and the phase noise. We obtain an analytical expression for a stochastic map which allows to integrate the time evolution of the field during the transient over a finite time step which is of the order of the fast time scale of the system. Our calculation evidentiates that the phase noise is relevant only in the final quasi-stationary phase diffusion regime while the stochastic transient frequency evolution is practically affected only by the random departure from the initial unstable state.

Within the third order Lamb approximation the model equations used to describe the transient dynamics of the slowly varying complex amplitude E of the electric field can be written as [9],

$$\dot{E(t)} = -(1+i\theta)\alpha_0 E(t) + \sqrt{\epsilon}\xi(t) \text{ for t} < 0 \tag{1}$$

$$\dot{E(t)} = (1+i\theta)(\alpha_1 - \beta|E(t)|^2)E(t) + \sqrt{\epsilon}\xi(t) \text{ for t} > 0 \tag{2}$$

where the switch-on from below to above threshold occurs at $t = 0$, and the equations are written in the reference frame in which the lasing frequency of the on-state is zero. The parameters $\alpha_1 > 0$ and $-\alpha_0 < 0$ are the net gain coefficients below and above threshold, respectively. Nonlinear saturation effects with parameter β are neglected below threshold. θ is the detuning parameter and ϵ measures the strength of spontaneous emission noise modeled by a complex Gaussian white noise of zero mean and correlation

$$\langle \xi^*(t)\xi(t')\rangle = 2\delta(t-t'). \tag{3}$$

For $t > 0$ the laser intensity grows up in a way which depends on the stochastic sequence of the spontaneous emission events in the cavity. Each of these sequences represents a different *realization* of the noise and gives rise to a different realization of the switch-on process. The Quasi Deterministic Theory (QDT) [3,5] gives an approximation to the stochastic dynamics. In its simpler formulation the QDT theory amounts to an approximation of the *individual realizations* of the process given by eqs. (1,2) by a deterministic evolution for the laser intensity which starts from some random condition whose statistical properties are self-consistently calculated. In the framework of the QDT for the laser intensity, noise effects are taken into account only in the initial unstable state while the transient is deterministic. To study the field transient we apply the QDT only to analyze the dynamics of the laser intensity while it is possible to solve exactly the equation for the phase. To do this let us rewrite eqs. (2) in terms of phase ϕ and amplitude R defined as $E(t) = R(t)\exp(i\phi(t))$. Using Ito stochastic calculus we get

$$\dot{R(t)} = (\alpha_1 - \beta R^2(t))R(t) + \frac{\epsilon}{2R(t)} + \sqrt{\epsilon}\xi_R(t) \tag{4}$$

$$\dot{\phi(t)} = \theta(\alpha_1 - \beta R^2(t)) + \sqrt{\epsilon}\frac{\xi_\phi(t)}{R(t)} \tag{5}$$

where again ξ_ϕ and ξ_R are uncorrelated white noise

$$\begin{aligned}\langle \xi_\phi(t)\rangle = \langle \xi_R(t)\rangle = 0 \\ \langle \xi_\phi(t)\xi_\phi(t')\rangle = \langle \xi_R(t)\xi_R(t')\rangle = \delta(t-t') \\ \langle \xi_\phi(t)\xi_R(t')\rangle = 0.\end{aligned} \tag{6}$$

From eqs. (4,5) it is seen that the motion of the amplitude is independent of the phase. We apply the QDT machinery to the determination of the amplitude motion [9]:

$$R^2(t) = \frac{h^2 e^{2\alpha_1 t}}{1 + \frac{\beta}{\alpha_1}(e^{2\alpha_1 t} - 1)h^2} \tag{7}$$

where h is a zero mean complex Gaussian variable with variance

$$\left\langle |h|^2 \right\rangle = \frac{\epsilon}{\alpha_1} + \frac{\epsilon}{\alpha_0}. \tag{8}$$

$h$is an effective initial condition which includes the effects of spontaneous emission events around the off state.

Once the motion of the amplitude is known the phase follows directly by integration of eq. (5)

$$\phi(t) = \phi(0) + \theta(\alpha_1 t - \beta \int_0^t dt' R^2(t')) + \int_0^t dt' \frac{\xi_\phi(t)}{R(t')} \tag{9}$$

We emphasize that the QDT gives a *process* approximation rather than an approximation of average properties i.e. for each realization of the stochastic noise we can compare the evolution as given by direct integration of eqs. (1,2) to the QDT approximation. From eq. (5) it is possible to define a *deterministic frequency* given by the drift term proportional to θ:

$$\omega_d(t) = \theta(\alpha_1 - \beta R^2(t)). \tag{10}$$

This frequency diminishes as soon as the intensity grows up approaching the on-state zero equilibrium value. It contains spontaneous emission contributions associated with the effective random initial condition h. On the other hand, the last term in (9) gives the contribution of the phase noise along the time evolution.

In order to characterize in the frequency domain the phase dynamics in the transition between the off and on states we will analyze the *finite window spectra* (FWS) defined as

$$S_{T_0}(\omega, T) = \left\langle |E_{T_0}(\omega, T)|^2 \right\rangle \tag{11}$$

where $E_{T_0}(\omega, T)$ is the Fourier transform of $E(t)$ over a finite window of width T_0 open at time T:

$$E_{T_0}(\omega) = \frac{1}{\sqrt{T_0}} \int_T^{T+T_0} dt e^{-i\omega t} E(t) \tag{12}$$

This function is the Fourier transform of the autocorrelation function only if the field $E(t)$ is stationary and $T_0 \to \infty$. During the transient the power spectrum is related to an integral of the autocorrelation function

$$S_{T_0}(\omega, T) = \frac{1}{T_0} \int_T^{T+T_0} dt_1 \int_T^{T+T_0} dt_2 e^{-i\omega(t_1 - t_2)} \langle E^*(t_1) E(t_2) \rangle \tag{13}$$

A qualitative analysis of this function identifies four different time regimes

a) For $t < 0$ the system is in a steady state which has a mean frequency identified with the maximum of $S_{T_0}(\omega, T)$ at $\omega = -\alpha_0 \theta$.

b) After the switch-on the laser is in an unstable state from which it departs exponentially after a random waiting time due to spontaneous emission processes. In the very early stage of this evolution the typical frequency suffers a sudden shift to the "unstable" value $\alpha_1 \theta$.

c) During the transient the non linear phenomena which contributes to the phase-amplitude correlations becomes important until the new equilibrium state is reached. The typical frequency shifts approaching zero.

d) As the new steady state is reached the laser intensity stabilizes and the frequency starts to diffuse while the typical frequency is zero.

An effect of the spontaneous emission noise is to broad the spectral line. This broadening can be described by a linearized calculation of the spectrum in stages a) and d). During the transient process the broadening varies with time and there is also a shift of the*mean* frequency (the center of the line).

The field spectrum in the steady state regime a) is easily calculated from eq. (1). The lineshape is lorentzian with linewidth is given by the inverse of the fluctuations correlation time

$$S_0(\omega) = \frac{\epsilon}{\alpha_0^2 + (\omega + \alpha_0\theta)^2} \tag{14}$$

Within the QDT approximation, the laser intensity given by eq.(7) reaches a constant value and the linewidth becomes, during regime d), solely due to the phase fluctuations. One easily obtains by linearizing the intensity process eq. (4) that

$$S_1(\omega) = \frac{\epsilon}{\Gamma^2 + \omega^2} \tag{15}$$

where the linewidth is now given by the inverse of the phase coherence time

$$\Gamma = \frac{\epsilon\alpha_1(1+\theta^2)}{2\beta}. \tag{16}$$

During the transient the mean frequency shifts from the value $\omega_0 = \alpha_1\theta$ to the value $\omega_1 = 0$ passing trough non lorentzian spectra. This is the *frequency chirping* phenomena. A further timescale relative to transient phenomena can be introduced: *the switch-on time* t_{sw}. i.e. the time at which the laser intensity actually becomes larger than the noise induced "off" intensity. The switch-on time is a stochastic variable its order of magnitude can be estimated from eq. 7 assuming h to be of the order of its variance

$$t_{sw} \simeq -\frac{1}{2\alpha_1}\log(\frac{\epsilon}{\alpha_0} + \frac{\epsilon}{\alpha_1}) \tag{17}$$

It is worth noting that the "off" state correlation and the phase diffusion timescales are very different and it is possible to choose an intermediate timescale of the order of the switch-on time t_{sw} which is long enough to contain several *microscopic* timescales $\alpha_0^{-1}, \alpha_1^{-1}$ but short compared to the phase diffusion time Γ^{-1}. Spanning the transient with spectra obtained opening a time window T_0 of this order will provide information about the evolution of th frequency and fluctuations during the transient.

Let us now turn to the transient region c). To evaluate the power spectrum through eq. (13) we must evaluate two time integrals and one ensemble average. In the QDT framework it is possible to formulate an efficient algorithm which allows to calculate $S_{T_0}(\omega, T)$ in the low frequency region. If we want the spectrum at low frequency we must sample the signal on a suitably wide time window. This will induce a limitation in the band. Since we are dealing with FWS this limitation is intrinsic. Moreover to get the low frequency spectrum we need only a *discrete* sampling of the field in time. Instead of directly evaluating the spectrum as given by eq. (13) we can take advantage of the fact that in the QDT approximation it is possible to generate a discrete sampling of the field $\{E(t_k)\}$ for any single transient having the *same statistical properties* of an infinite sampling $E(t)$. By this we mean that the sampling generated at a set of discrete times keeps the proper correlations of the continuous time QDT process. The algorithm follows by associating the electric field at a time t_{k+1} to the field at a previous time t_k, where $t_{k+1} - t_k = \Delta t$ and Δt is a *finite* time interval. From eq. (1) we have that for $t < 0$ the process describing the electric field is linear and we can easily integrate it on a finite time step

$$E(t_{k+1}) = e^{-(1+i\theta)\alpha_0 \Delta t}(E(t_k) + W_0(t_{k+1}, \Delta t)) \text{ for } t < 0 \tag{18}$$

where the new noise W_0 has correlations given by

$$\langle W_0(t_k, \Delta t) W_0^*(t_l, \Delta t)\rangle = \delta_{kl} \frac{\epsilon}{2\alpha_0}(1 - e^{-2\alpha_0 \Delta t}). \tag{19}$$

For $t > 0$ we write the field as

$$E(t_{k+1}) = e^{log(R(t_{k+1})/R(t_k)) + i\Delta\phi(t_k, \Delta t)} \tag{20}$$

with

$$\Delta\phi(t_k, \Delta t) = \int_{t_k}^{t_{k+1}} dt' \phi(t') \tag{21}$$

Using the QDT solution of eqs. (7,9) we have

$$E(t_{k+1}) = e^{(1+i\theta)\psi(t_k,\Delta t)+i\sqrt{\epsilon\chi(t_k,\Delta t)}W_1(t_{k+1})}E(t_k) \tag{22}$$

where ψ and χ are given by:

$$\psi(t_k,\Delta t) = \alpha_1\Delta t - \frac{1}{2}log\left(\frac{1+\frac{\beta}{\alpha_1}(e^{2\alpha_1(t_k+\Delta t)}-1)h}{1+\frac{\beta}{\alpha_1}(e^{2\alpha_1 t_k}-1)h}\right) \tag{23}$$

$$\chi(t_k,\Delta t) = \frac{\beta}{\alpha_1}\Delta t + \left(\frac{1}{2\alpha_1 h^2} - \frac{\beta}{2\alpha_1^2}\right)(1-e^{-2\alpha_1\Delta t})e^{-2\alpha_1 t_k} \tag{24}$$

The statistical properties of the noise W_1 are obtained by evaluating the properties of the integrated phase noise appearing in eq. (21)

$$\langle W_1(t_k,\Delta t)W_1(t_l,\Delta t)\rangle = \delta_{kl} \tag{25}$$

The point to be stressed is that eq. (22) gives an analytical expression for the stochastic map that generates a time series for the field. This map disentangles very clearly the two sources of randomness that contribute to the field spectrum: The effective random initial condition h for the laser intensity at $t=0$ which appears in ψ, and the phase noise along the dynamical evolution given by W_1. The calculation of the spectrum is then made in practice as follows. For each different transient a value of h is chosen from a distribution having the property given by eq. (8). At each step t_k random numbers W_0 (if $t_k<0$) and W_1 (if $t_k>0$) are updated. To get the power spectrum we simply Fourier transform the sample obtained in this way and then average over 2000 samples. We remark the fact that through the QDT theory it is possible to integrate analytically the process on a finite time step $\Delta t << t_{sw}$, but $\Gamma^{-1} >> T_0 >> \Delta t$ and $(\alpha_1)^{-1} \simeq \Delta t$, which samples correctly the field on intermediate timescales. We also notice that according to eq. (17) the number of frequencies present in every FWS scales as the logarithm of the inverse noise strength giving a quite detailed FWS provided the noise is low enough.

An example of the results that can be obtained with this approach are shown in Figs.1-3. In Fig.1 we show the average total intensity and fluctuations together with the opening time of our windows. We have taken windows of width ($T_0 = 12.8$) opened at nine times along the transient. Spectra obtained over these windows are shown in Fig. 2, in absence a) and presence b) of detuning. As in the time domain we can distinguish four region of the transient spectra according to the window opening time T. The FWS of the window # 1 practically coincides with the infinite-window stationary spectrum below threshold, therefore belonging to region a) of the transient. In fact during the first time window the laser is always in the "off" state, as we see from fig. 1. Moreover the window amplitude is anyway much larger that the equilibrium correlation time of the field α_0^{-1}. As soon as the laser is switched-on all the frequencies are initially amplified in the same way, (this happens along windows # 2-4). This initial equal amplification happens in the time region of linear amplification b) and it is a precursor of the intensity switch-on which happens on the time scale of $t_{sw} \simeq 8$. In the case of detuning we see that the deterministic frequency can be associated to the FWS peak which clearly shifts from the off-state deterministic frequency position $-\alpha_0\theta$ towards its "unstable" value $\alpha_1\theta$. Entering the region c) of the transient the lasing frequency is selected and shifts to the zero value (see fig. 2b time windows # 4-9). The relatively high large frequency tails for the time windows # 4-7 can be understood as a signature of the large number of intensity transients included in these samplings (see fig. 1 for comparison). The FWS obtained with opening times # 8-9 show decreasing large frequency tails and narrowing of the central peak due to approaching the "on" steady state.

The on-state lineshape is obtained only for very wide time windows (much larger than the large coherence time Γ^{-1} and cannot be reached with our finite time-window which is instead appropriate for the transient features. In the asymptotic steady state only phase fluctuations given by W_1 in eq. (22) are relevant. On the contrary, during the transient dynamics phase noise plays no important role in the spectral properties. This can be seen explicitly by fictiously removing from eq. (22) the term relative to phase-noise (eq. 24) after switch-on. The FWS sampled a 5 different frequencies is shown versus the time-window baricetral time in Fig.3. The calculations that do not take into account phase noise after switch-on give appreciably different results only in the last FWS sampling (# 9). As expected the phase noise increases the FWS at large frequencies.

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Figure Caption

Fig 1 The average intensity (solid line), and fluctuations of intensity (dashed line) vs time. Arrows mark the opening of the 9 time windows used to obtain FWS. The width T_0 of each window is equal to the distance from the leftmost arrow to the origin, $T_0 = 12.8$.

Fig 2 a) FWS without and b) with detuning ($\theta = 1.0$). Other parameters are $\alpha_0 = \alpha_1 = 1$, $\beta = 0.1$ and $\epsilon = 10^{-6}$. FWS corresponding to time windows # 1-3 and # 8-9 are indicated.

Fig 3 FWS at frequencies $\omega = 0, \pm 1, \pm 5$ vs the baricentral time of each window. Parameters are those of fig. 2b. Calculations which do not take into account phase noise are shown with bold symbols only for the window # 9 (the rightmost point). Phase noise does not give noticeable differences for other opening times.

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