# Stretching structures in the ocean surface: Transport and biological impacts

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V. Rossi, V. Garçon, J. Sudre, E. Tew Kai, …

*IFISC

# OUTLINE

- Introduction: Dynamical systems and fluid transport
- Finite-size Lyapunov exponents in the ocean
- Impact of coherent flow structures on
  - Phytoplankton
  - Frigatebirds
- Towards three dimensions

# The dynamical systems approach to fluid transport

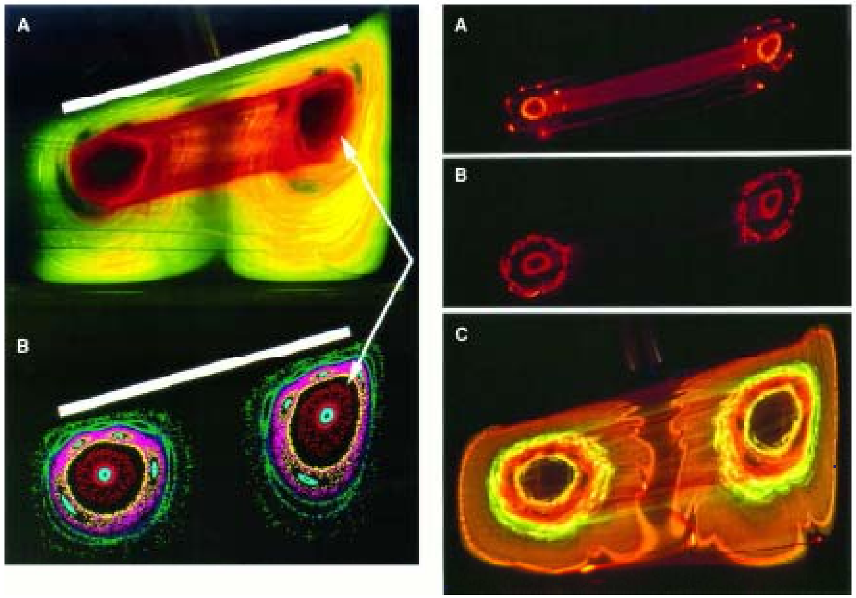
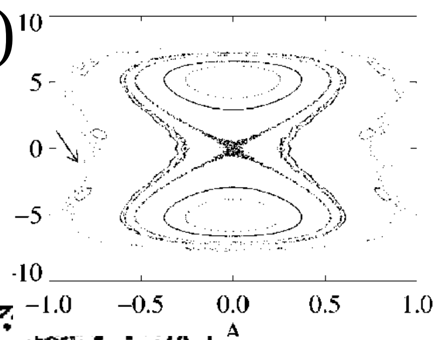
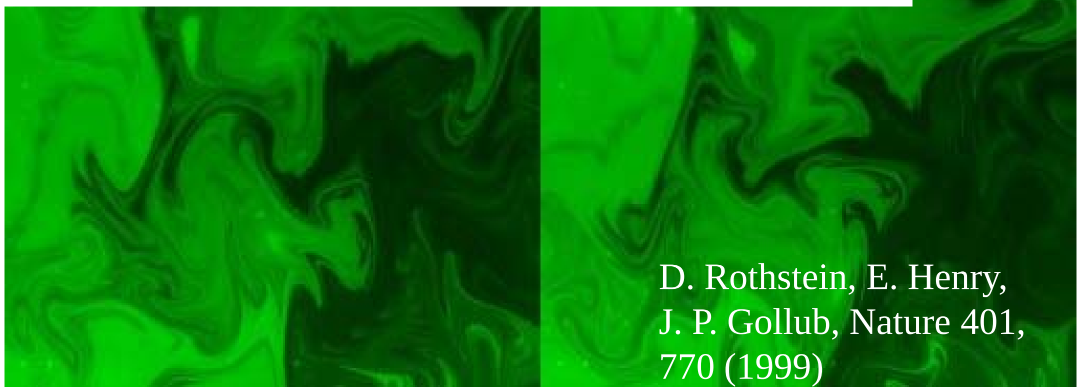



**Figure 2.7:** KAM tori and elliptic islands visualized by fluorescent dye in an experiment with a steady three-dimensional flow in a viscous fluid (from Fountain et al. (1998)).

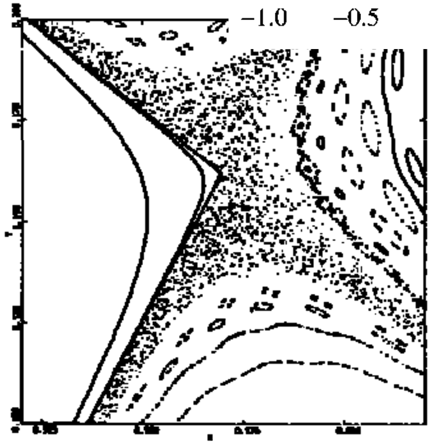
Chaotic seas, KAM tori, …

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$

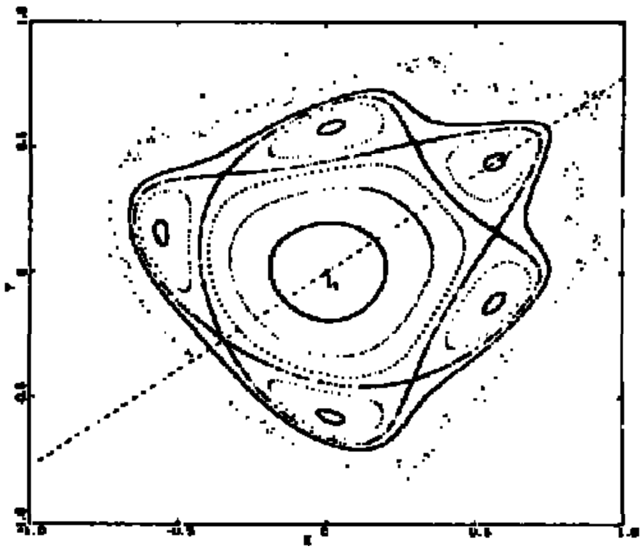


D. Rothstein, E. Henry, J. P. Gollub, Nature 401, 770 (1999)

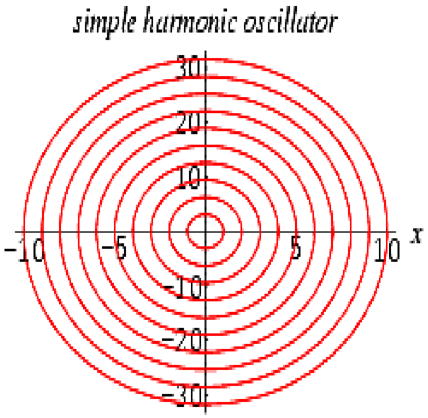


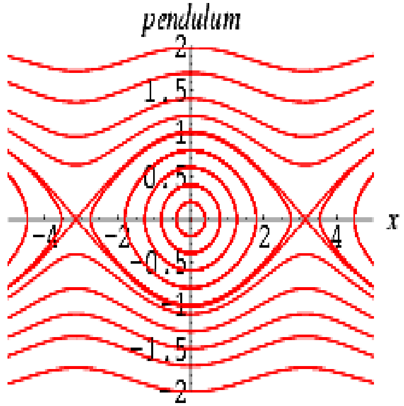
- Lagrangian dynamical system: **particle trajectories** in a given velocity field.
- **Incompresibility: symplectic structure,**
- Phase space = physical space
- Global behavior in phase space is organized by some relevant lines



$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$

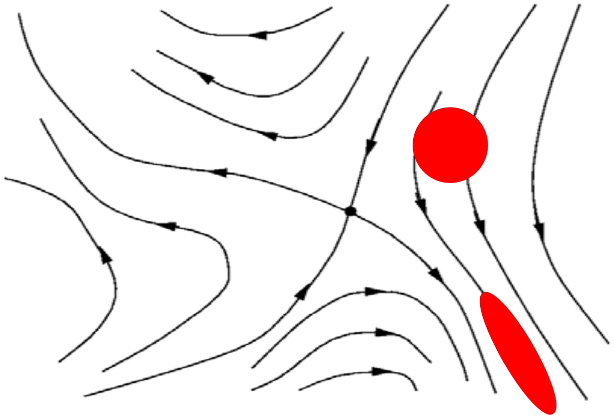
**WHAT ARE THE RELEVANT STRUCTURES ALLOWING UNDERSTANDING OF THE WHOLE FLOW?**




Trajectories of two-dimensional steady or periodic flows are organized by the fixed points, or periodic orbits of the dynamical system

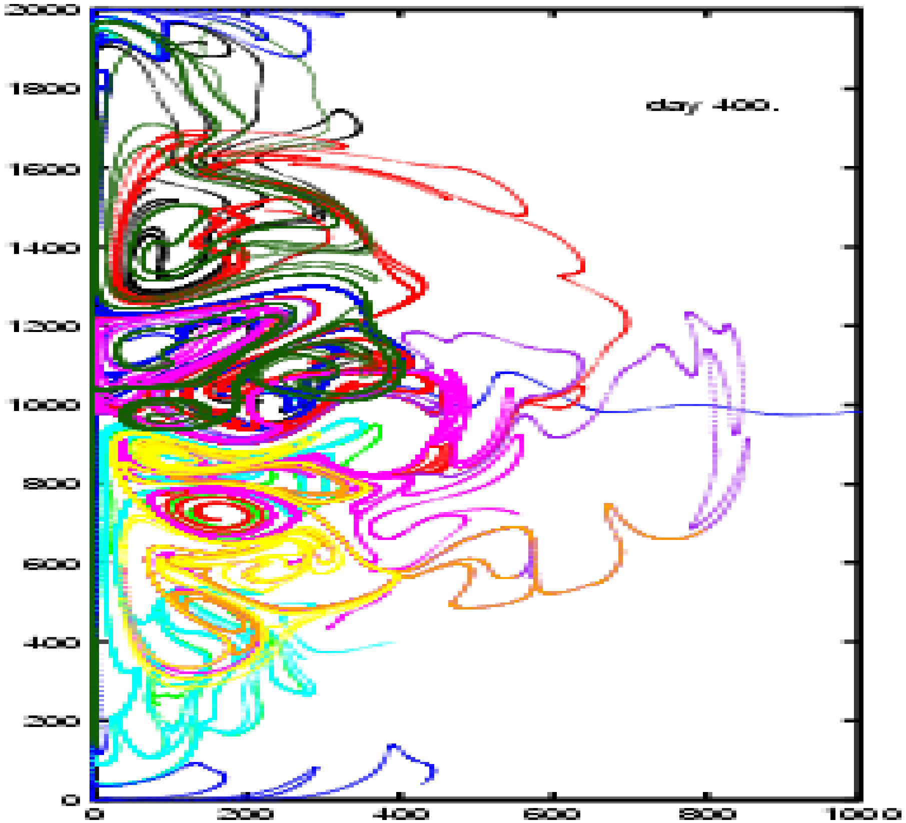
$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t))$$



If hyperbolic:
Stable and
unstable manifolds → separatrices

**Tracers tend to approach unstable manifolds**

**But**
**unsteady flows …**




From Mancho et al. 2005

Is there any particular subset of hyperbolic trajectories and manifolds organizing the dynamics (the equivalent to the fixed points in autonomous systems) ? How to select them among this mess ? **LCS**

## Identifying the relevant trajectories and manifolds in time-aperiodic dynamical systems

- Leaking, escape,or residence time methods
- Attracting or repelling material lines
- Distinguished hyperbolic trajectories and their manifolds
- **Stretching-field methods**: Finite-time Lyapunov exponents, Finite-size Lyapunov exponents, M function, …

✓ Variational approach to hyperbolic LCSs (Haller)

✓ Topological braiding (Thiffeault)

✓ Dimensions and ergodicity measures (Rypina)

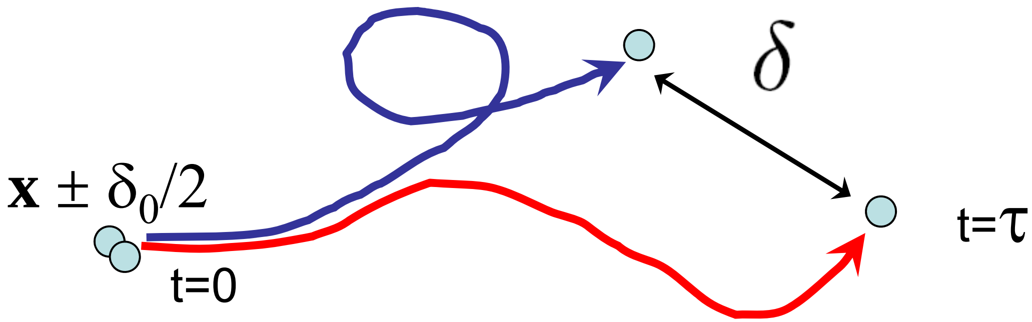
✓ …

$$\lambda(t) = \lim_{\|\delta(0)\|\to 0} \frac{1}{t} \ln \frac{\|\delta(t)\|}{\|\delta(0)\|}$$

Finite-**time** Lyapunov exponent

$$\lambda = \lim_{t\to\infty} \lambda(t)$$

Lyapunov exponent




$$\lambda(\delta_0, \delta_f) \equiv \frac{1}{\tau} \log \frac{\delta_f}{\delta_0}$$

**Finite-size Lyapunov exponent**

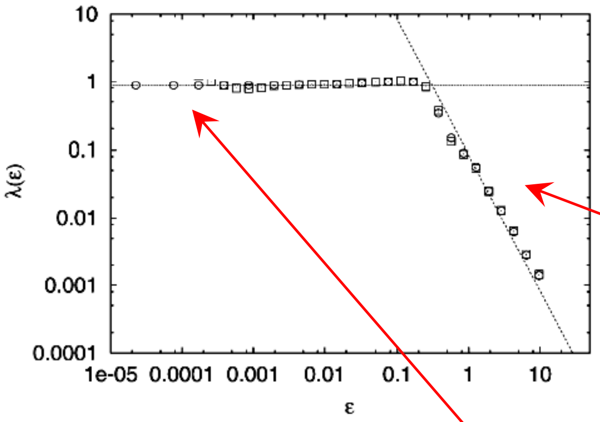
**FSLE**

**All the quantities are also functions of the initial position and time:**

$$\lambda(\mathbf{x}, t, \delta_0, \delta_f)$$

A chaotic map

Average FSLE

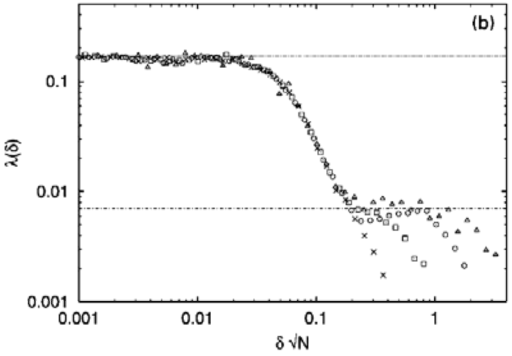



δ₀ → $\delta_0$

System with several time scales

(coupled maps)

Average FSLE




$\delta_0$

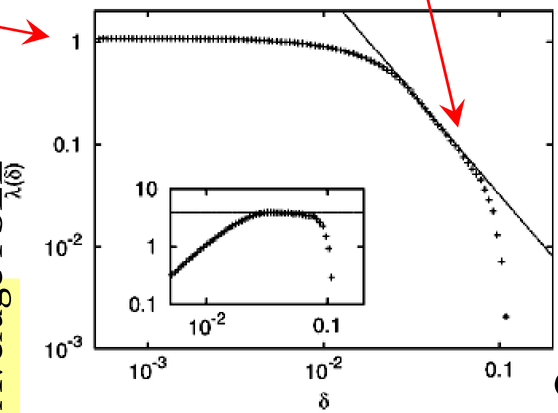
Subexponential growth (diffusion regime)

Exponential growth of separations (chaotic regime)

When $\delta_0 \to 0$,
FSLE→Lyapunov
and when t→∞,
FTLE→Lyapunov

G. Boffetta et al. / Physics Reports 356 (2002) 367–474

Average FSLE

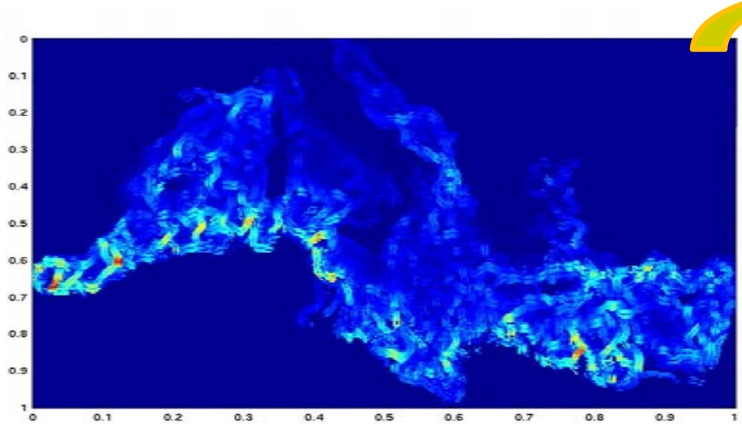


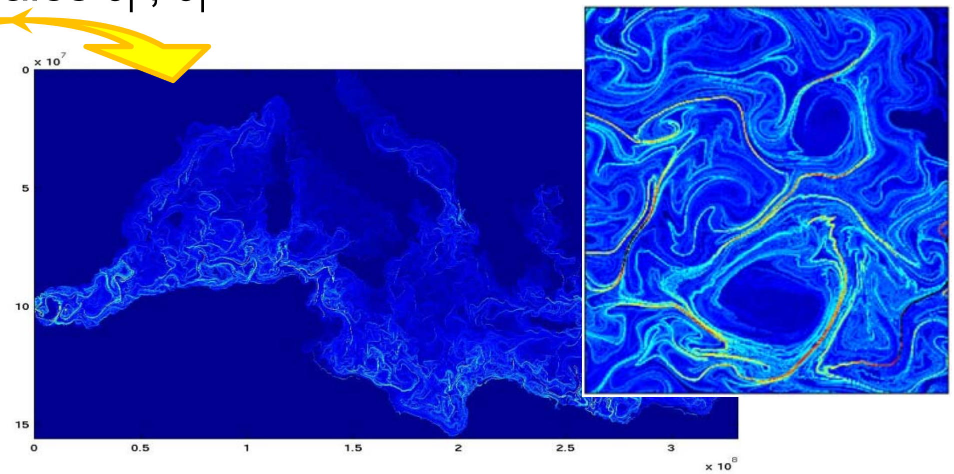

$\delta_0$

2D turbulence

**The FSLE was originally introduced to quantify dispersion from non-infinitesimal initial separations** (Aurell et al. 1997)

## Reducing scales $\delta_i$ , $\delta_f$



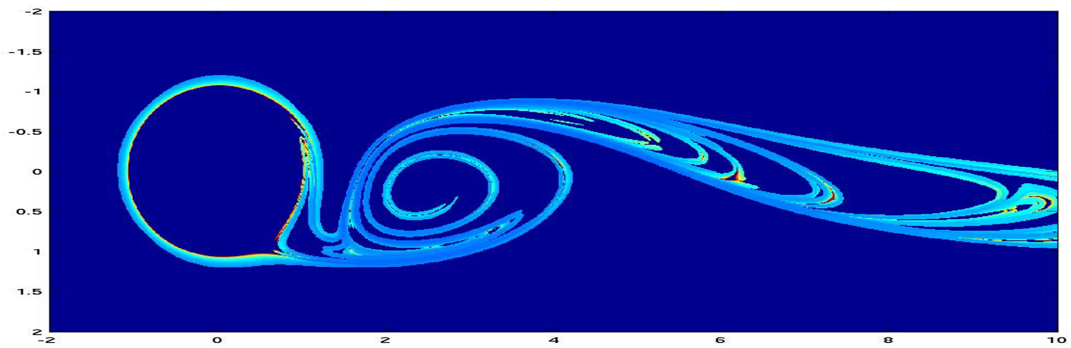


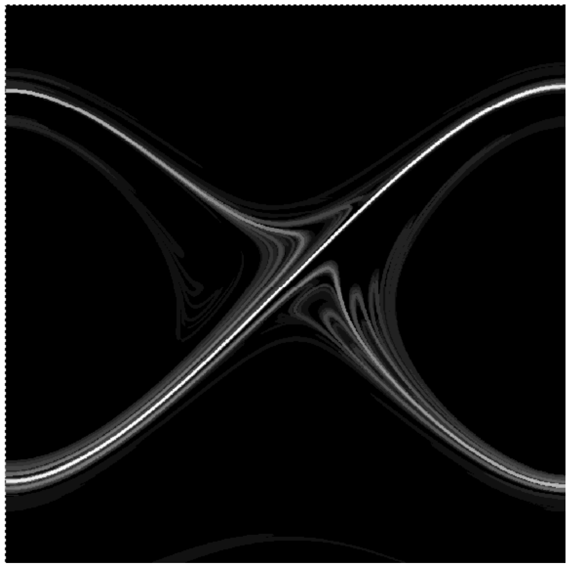
**FSLE** for small enough scales, $\leftrightarrow$ **FTLE** for large enough times

Forward in time: repelling manifolds

Backward in time: attracting manifolds

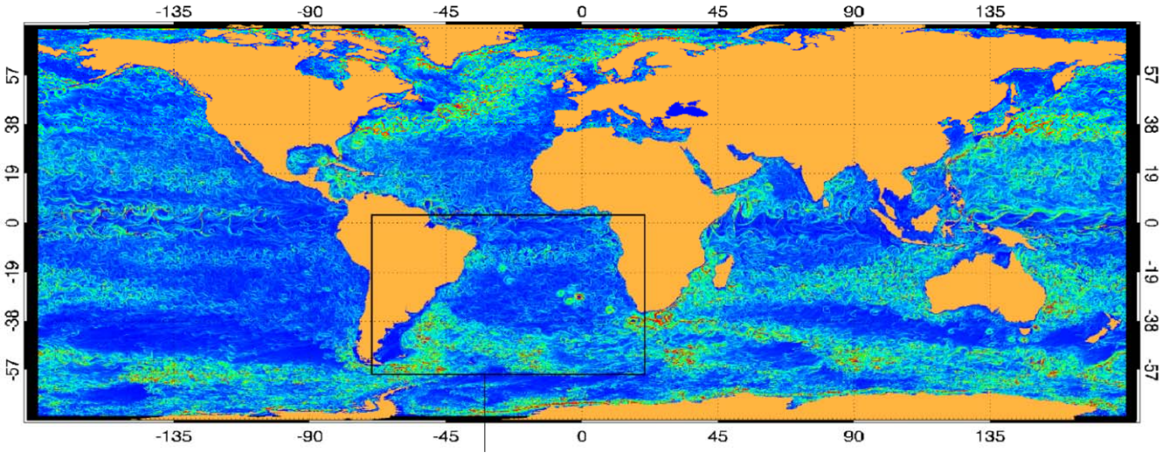
**LAGRANGIAN COHERENT STRUCTURES**

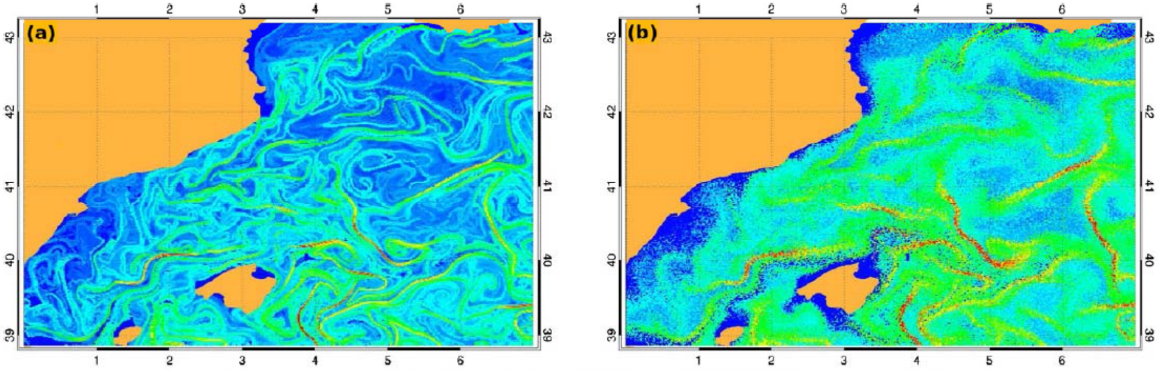




## Any advantage in using FSLE to locate LCS?

In oceanographic contexts it is usually straightforward to identify the relevant spatial scales: Rossby radius, coastal features



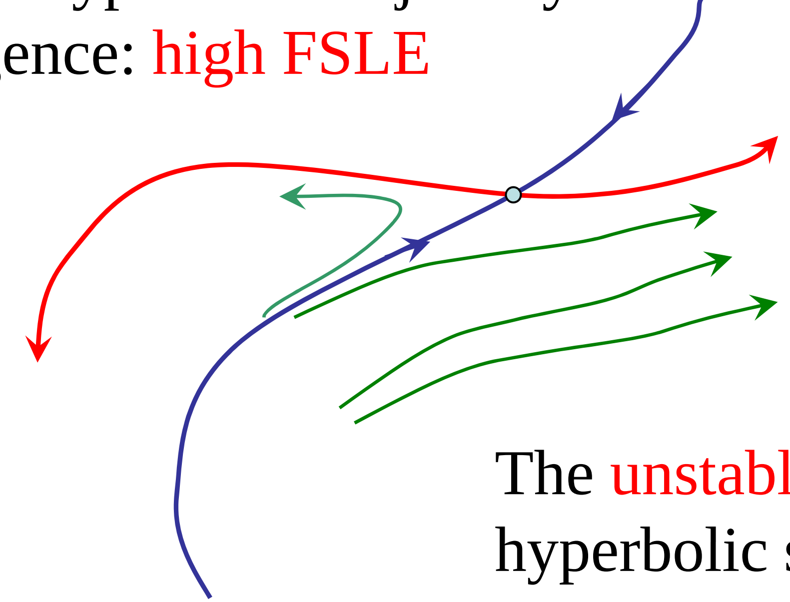



Trajectories can be nonsmooth (noise …)
I. Hernández-Carrasco et al. Ocean Mod. 36, 208 (2011)

## Disadvantage:

No theorems …

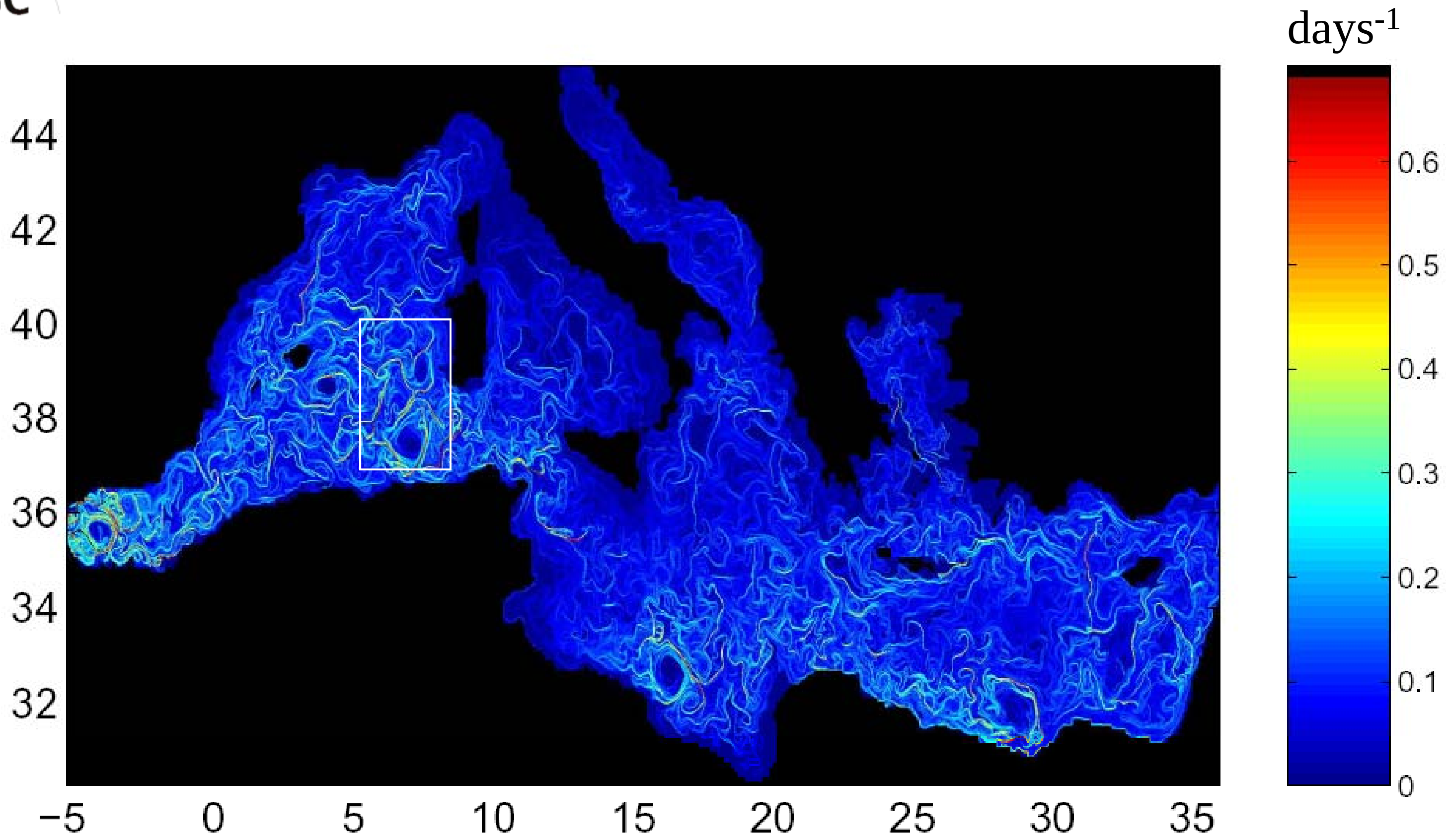
The idea is that initial conditions close to the stable manifold of a hyperbolic trajectory or set will show strong divergence: high FSLE



The unstable manifold of hyperbolic sets would be marked by high FSLE in the time backwards direction

Other types of Lyapunov exponents would display similar information, but FSLE is less afected by saturation

REMARK: these are heuristic consideration. Theorems needed (some available for FTLE)



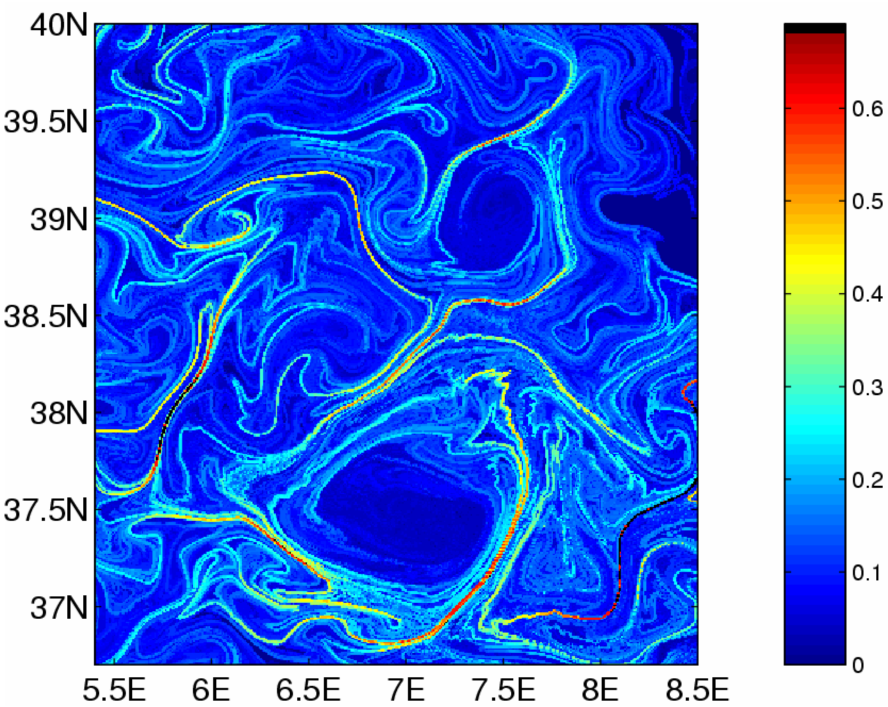

**DieCAST** model for the full MediterraneanPrimitive equations, 48 vertical levels, 1/8 º horizontal resolution, climatological forcings … → 5 years of daily velocity fields

$\delta_0 = 0.02^{\circ} \rightarrow \delta_f = 1^{\circ}$ (mesoscale transport)

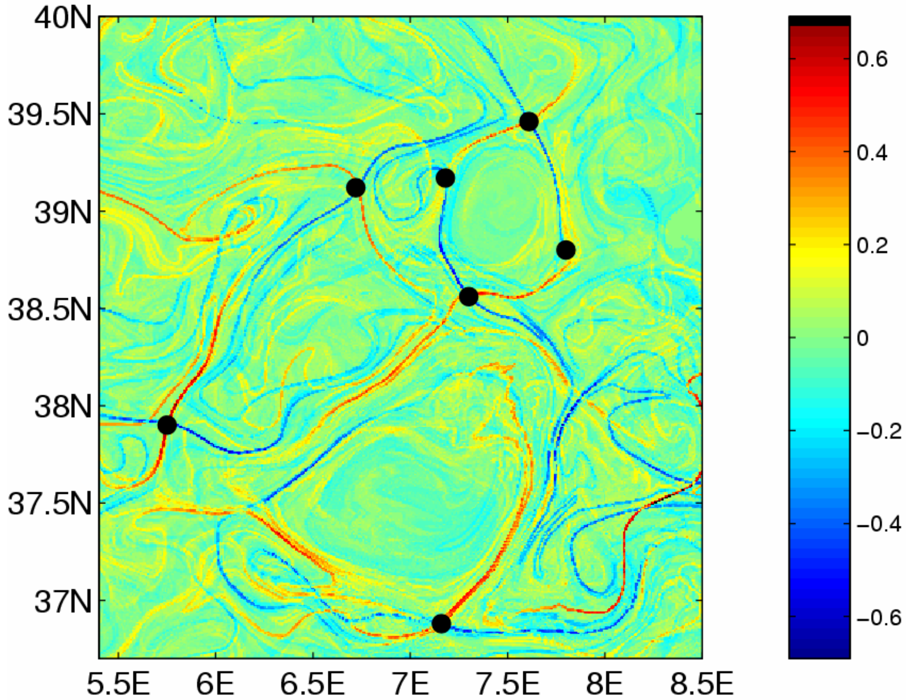
$\delta_0 \approx 2$ km $\rightarrow \delta_f \approx 110$ km twodimensional

d'Ovidio, Fernández, Hernández-García, López, Geophys. Res. Lett. 31, L17203 (2004)

FSLE from time-backwards Integrations.
Are they really unstable manifolds of hyperbolic trajectories?

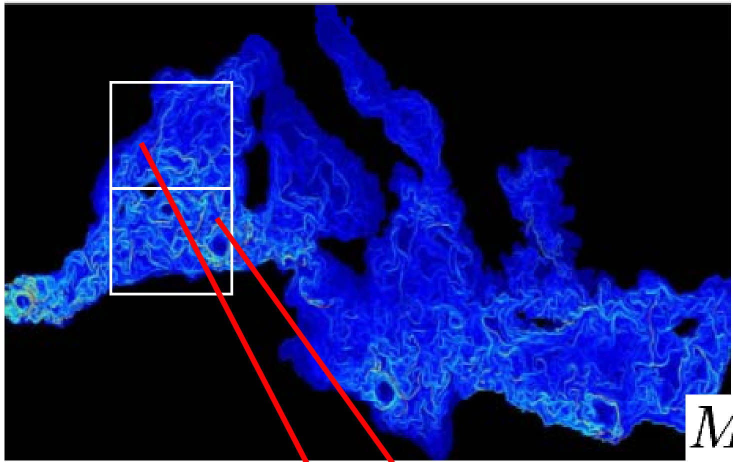


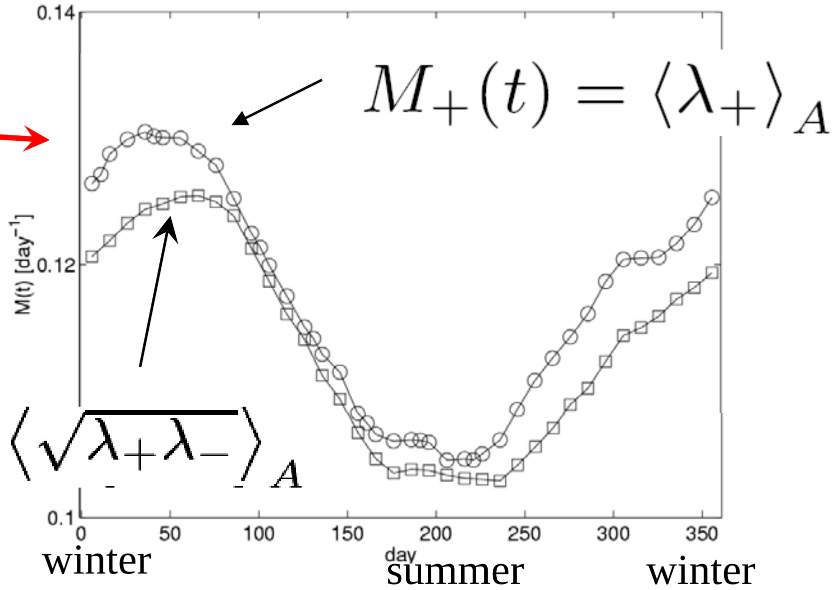

FSLE from forward and backwards integrations




The strongest lines are seen to organize tracer flow

Click figures for movies

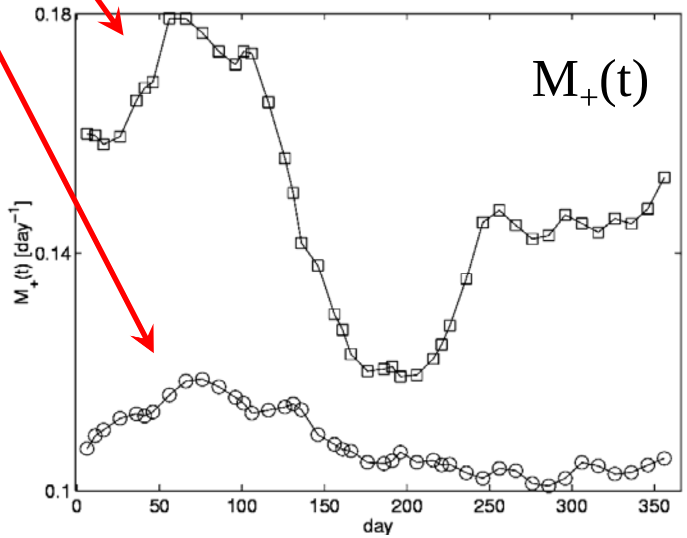


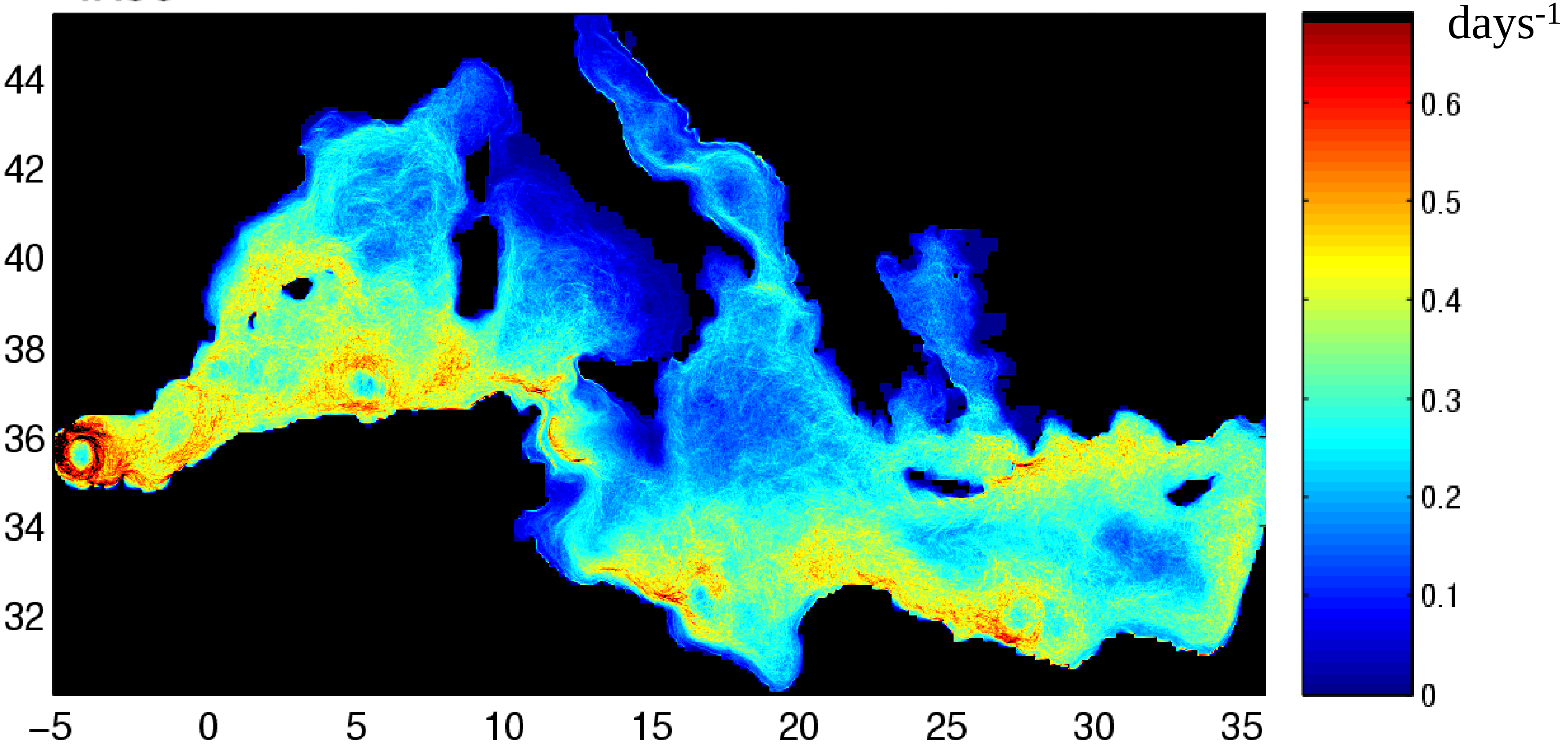



$$M_+(t) = \langle \lambda_+ \rangle_A$$

$$M_\pm(t) \equiv \langle \sqrt{\lambda_+ \lambda_-} \rangle_A$$

Mixing strenght
in different areas






$$M_+(\mathbf{x}) = \langle \lambda_+ \rangle_t$$

Local mixing strength averaged over 5 years

d'Ovidio, Fernández, Hernández-García, López, Geophysical Research Letters **31**, L17203 (1-4) (2004).

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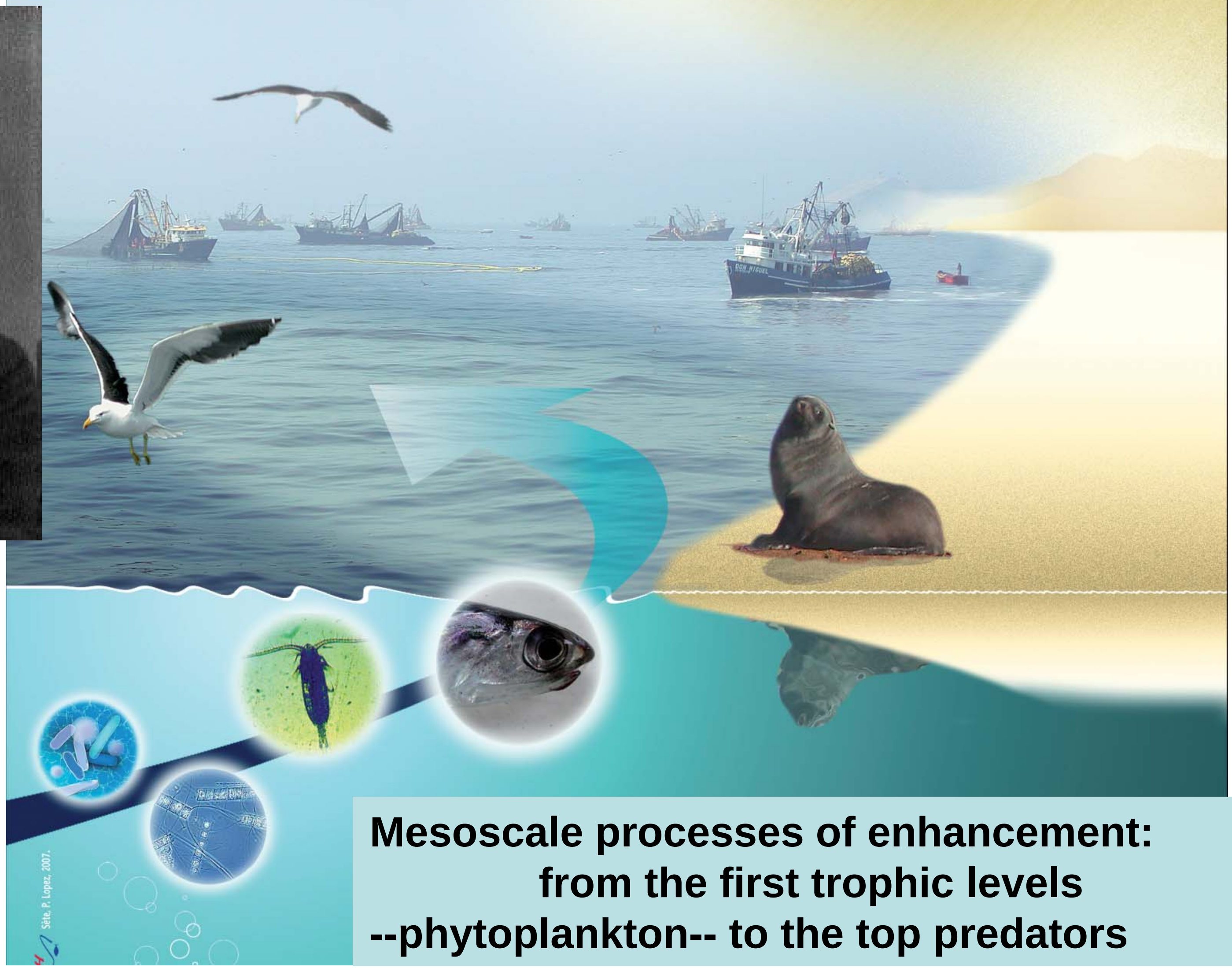
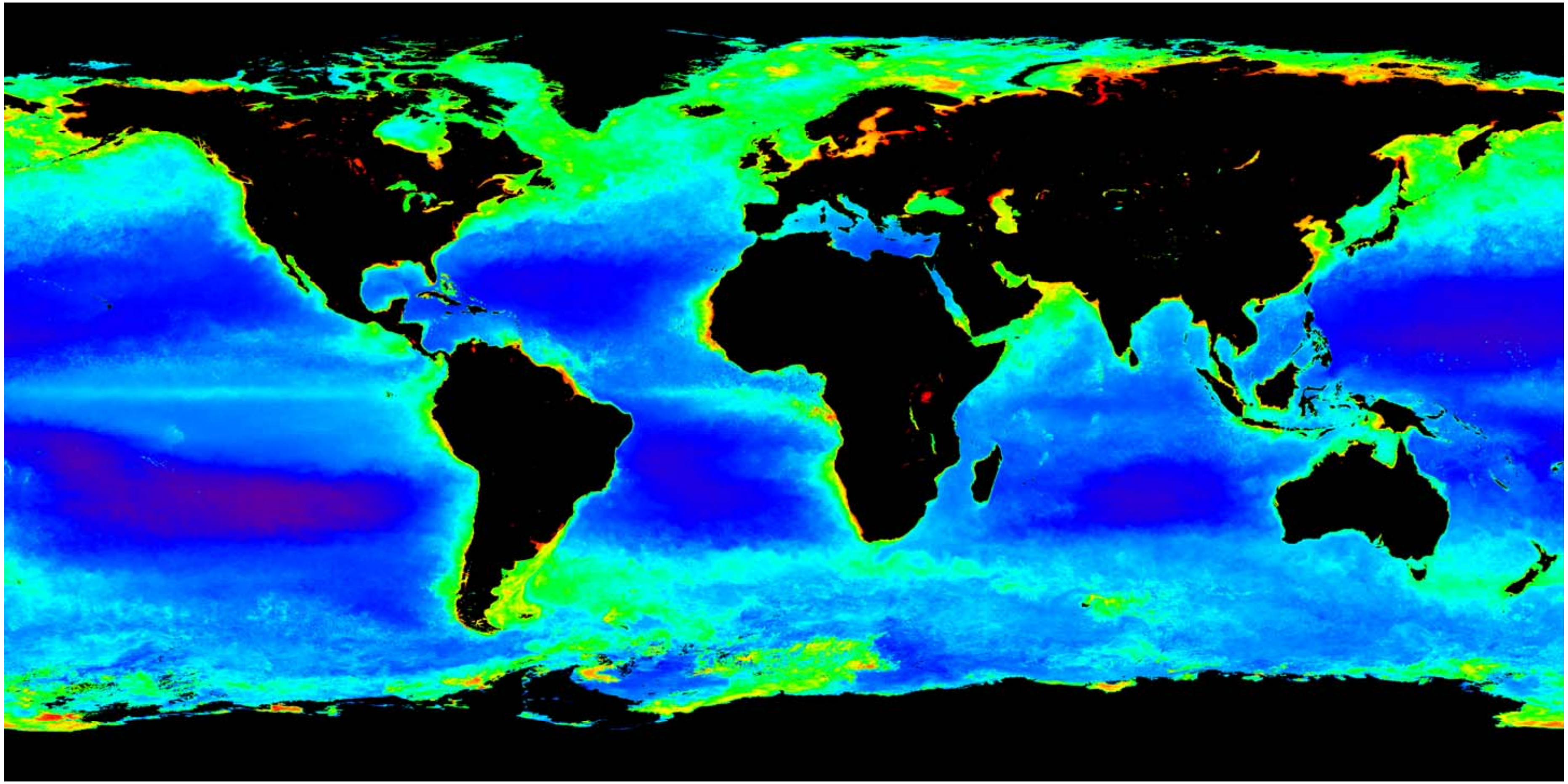
## Describing biological process with Lyapunov eyes …



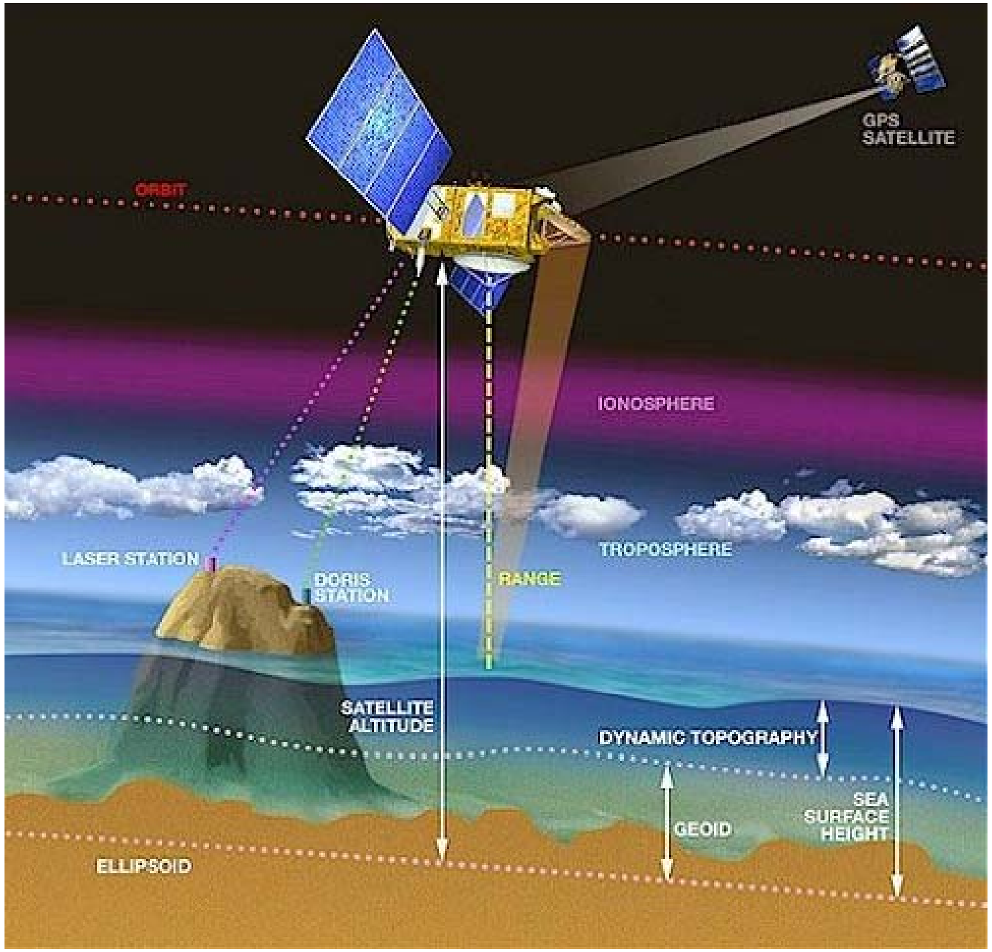
**Mesoscale processes of enhancement:**
**from the first trophic levels**
**--phytoplankton-- to the top predators**

## Chlorophyll-a (≈ phytoplankton) from space



**MODIS Image**
1 month average

## SATELLITE ALTIMETRY FROM TOPEX/POSEIDON, ERS-2, JASON, ENVISAT, …




Dynamic Topography (DT)=
Sea Surface Heigh (SSH) – Geoid (G)

SSH ≈ 3 cm

G ≈ meters …

Sea Level Anomalies (SLA) =

$SSH - \langle SSH \rangle_t = DT - \langle DT \rangle_t$

Dynamic topography determines, via the Colioris force, the velocity field (at large scales, geostrophic approximation)

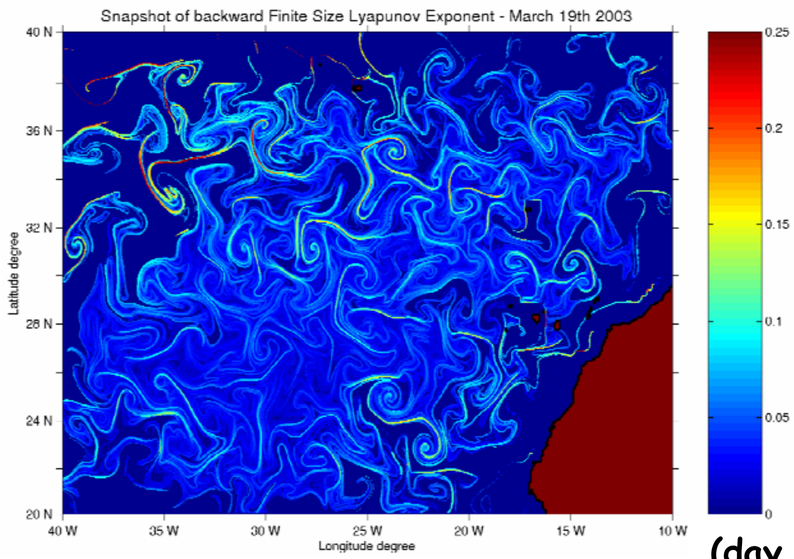
Ageostrophic components Can be estimated from scatterometer data

(Surface roughness → wind → Eckman component)

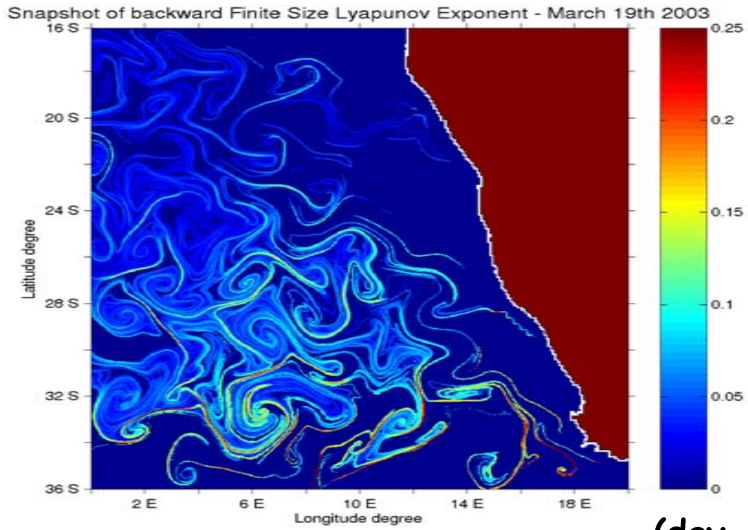
# FROM ALTIMETRY DATA

BENGUELA

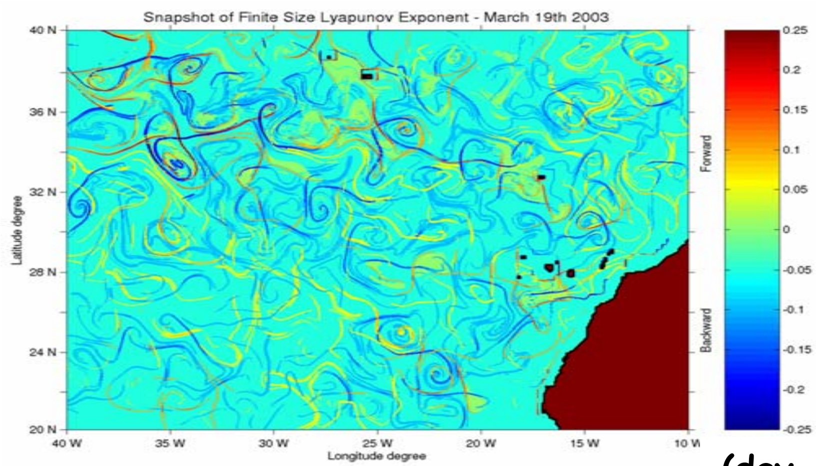
March 19 2003 snapshots



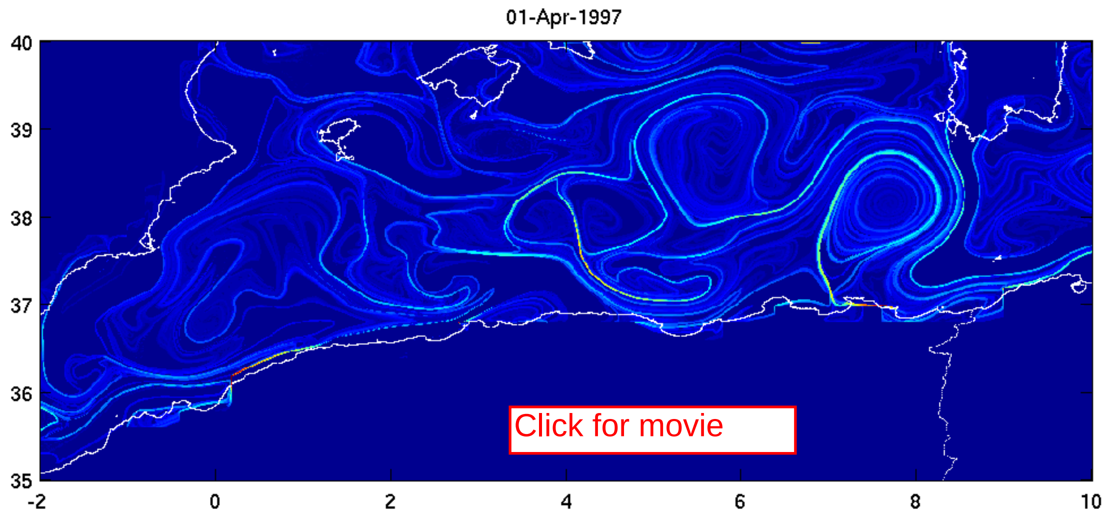

(day s$^{-1}$)




(day s$^{-1}$)




(day s$^{-1}$)




CANARY

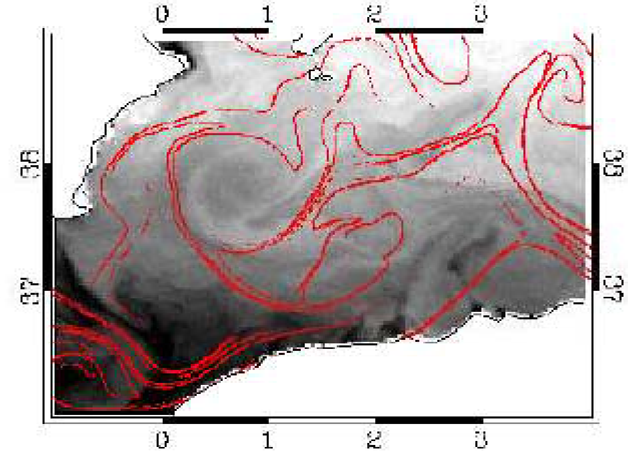
**Note the presence of SUB-MESOSCALE detail**

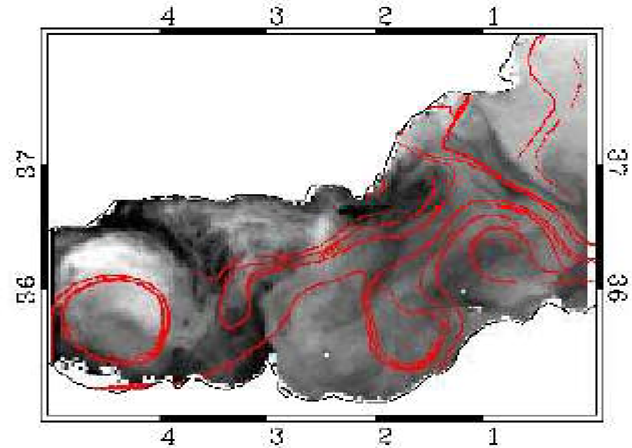
d'Ovidio et al. Deep-Sea Res. I 56, 15 (2009)
V. Rossi et al. Nonlin. Proc. Geophys. 16, 557 (2009)

## Sea Surface Temperature vs lines of FSLE > 0.1 day$^{-1}$ (LCSs)

d'Ovidio et al.
Deep-Sea Res. I (2009)

July 9 2003

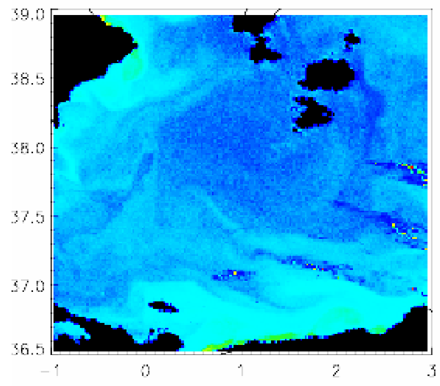


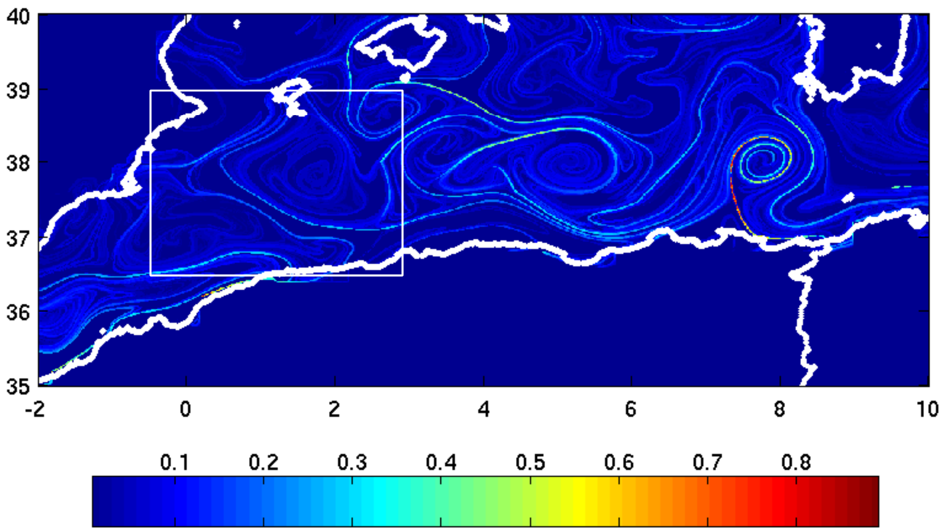




April 6 2004

Chlorophyll

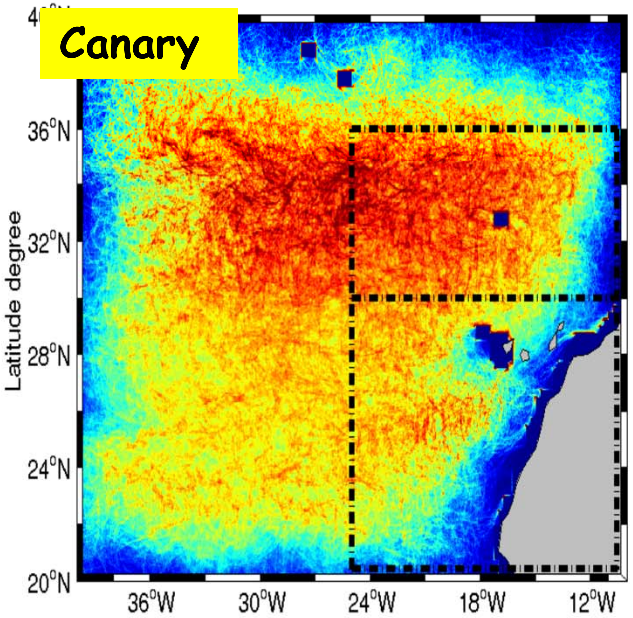
18 May 1998

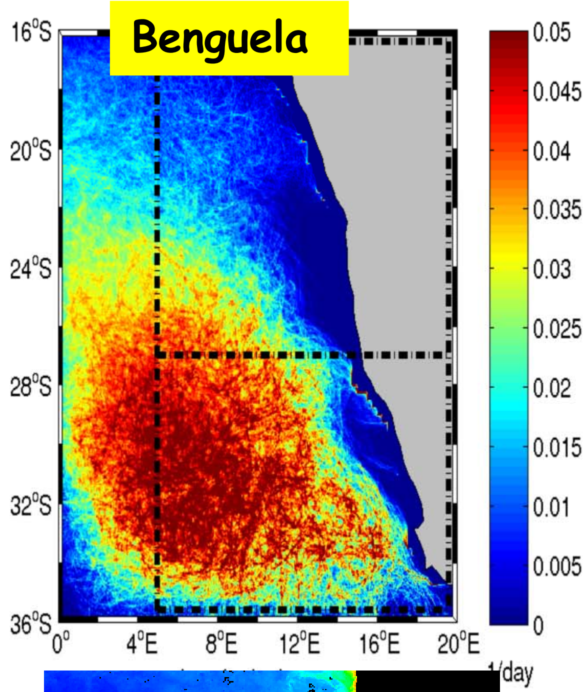


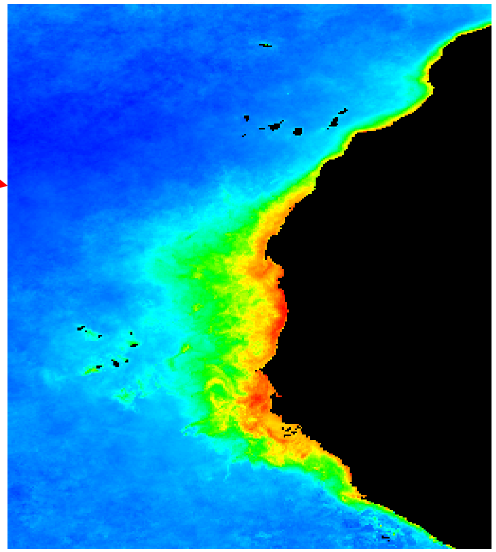

days$^{-1}$

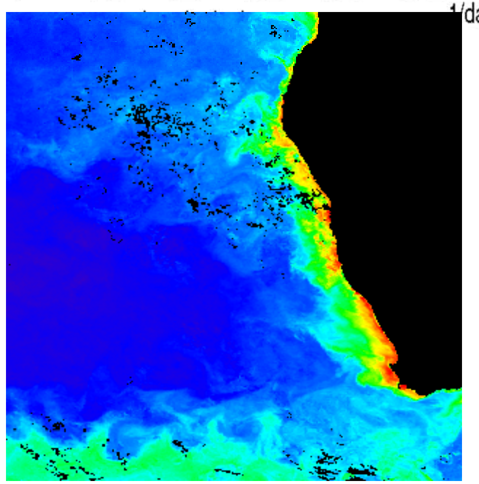
Backward FSLE ($\lambda$-): Temporal average **(a measure of horizontal MIXING)** from June 2000 till June 2005




**Phytoplankton and in the world major upwelling areas**

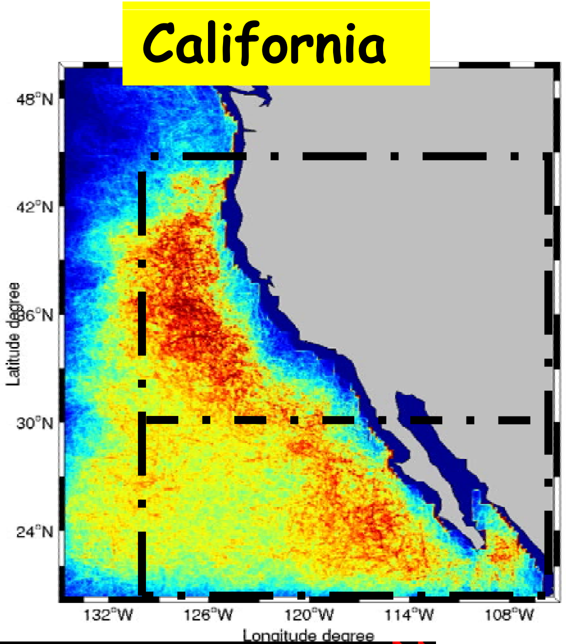


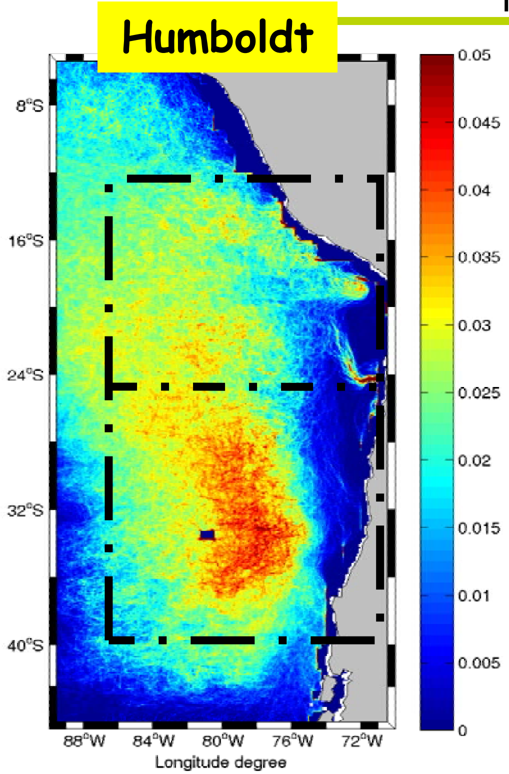


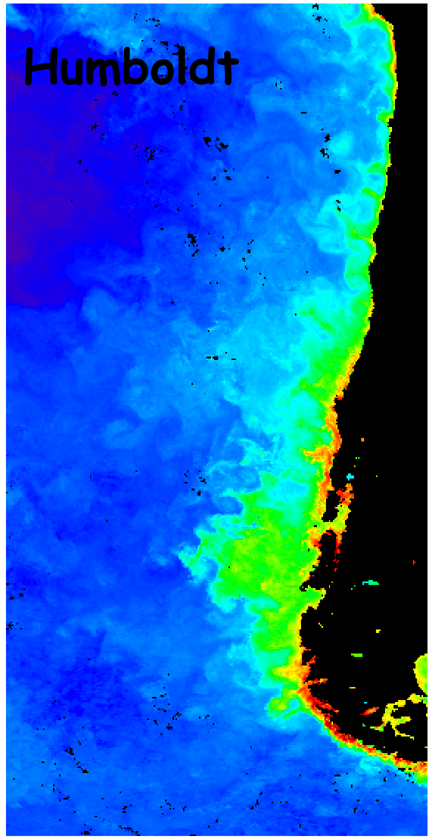
Rossi et al.,

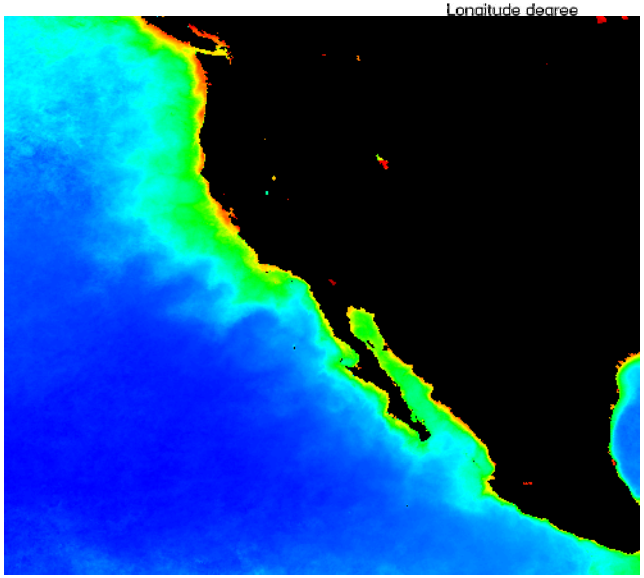
Geophys, Res. Lett. 2008

Nonlin. Proc. Geophys. 2009



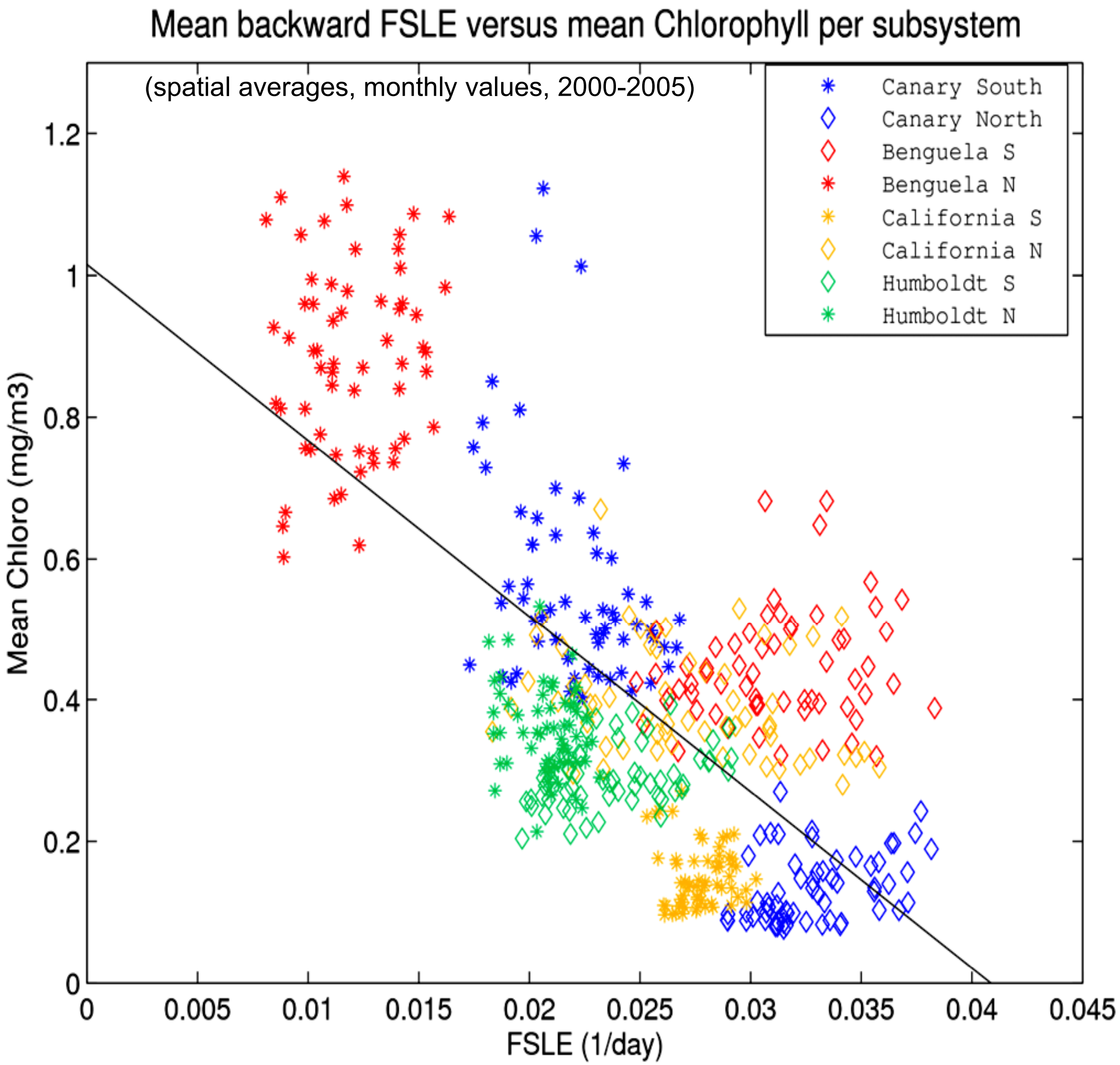



Backward FSLE (λ-): Temporal average **(a measure of** **horizontal MIXING)** from June 2000 till June 2005

Rossi et al.,
Geophys, Res. Lett. 2008
Nonlin. Proc. Geophys. 2009




- Negative correlation
- Clustering
- Less turbulent systems are characterized by: LOW FSLE / HIGH CHLOROPHYLL.
- Most turbulent systems: HIGH FSLE / LOW CHLOROPHYLL.

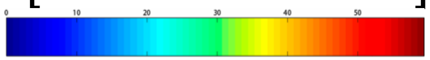
Opposite to behavior seen in less enriched systems

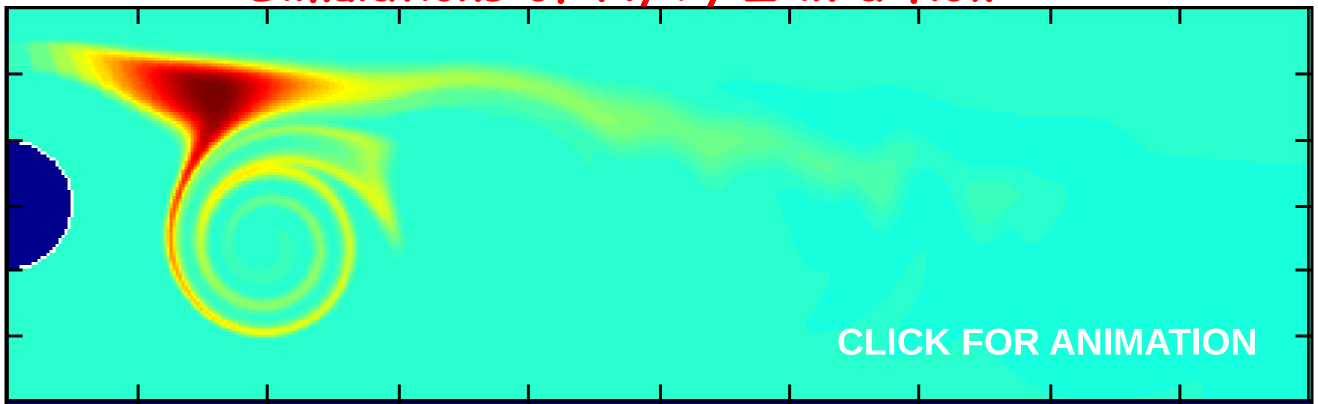
# Simulations of N, P, Z in a flow

steady state inflow
concentrations (N, P, Z)

phytoplankton

[0 14]



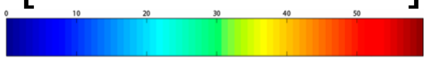


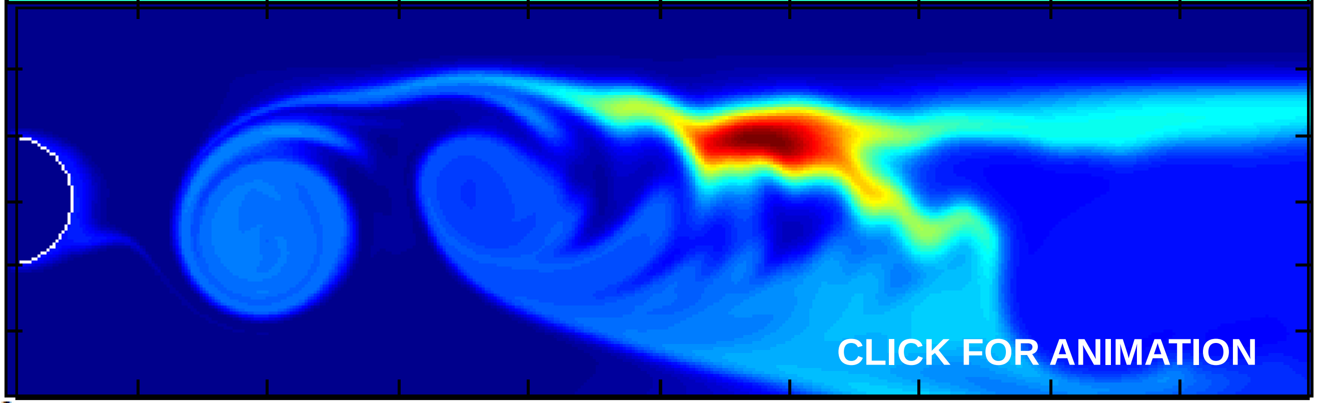

Y

low inflow
concentrations (N, P, Z)

phytoplankton

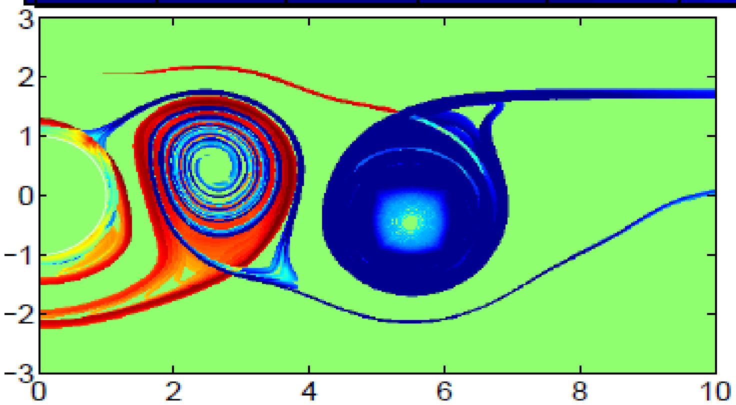
[0 45]






Y

FSLE field

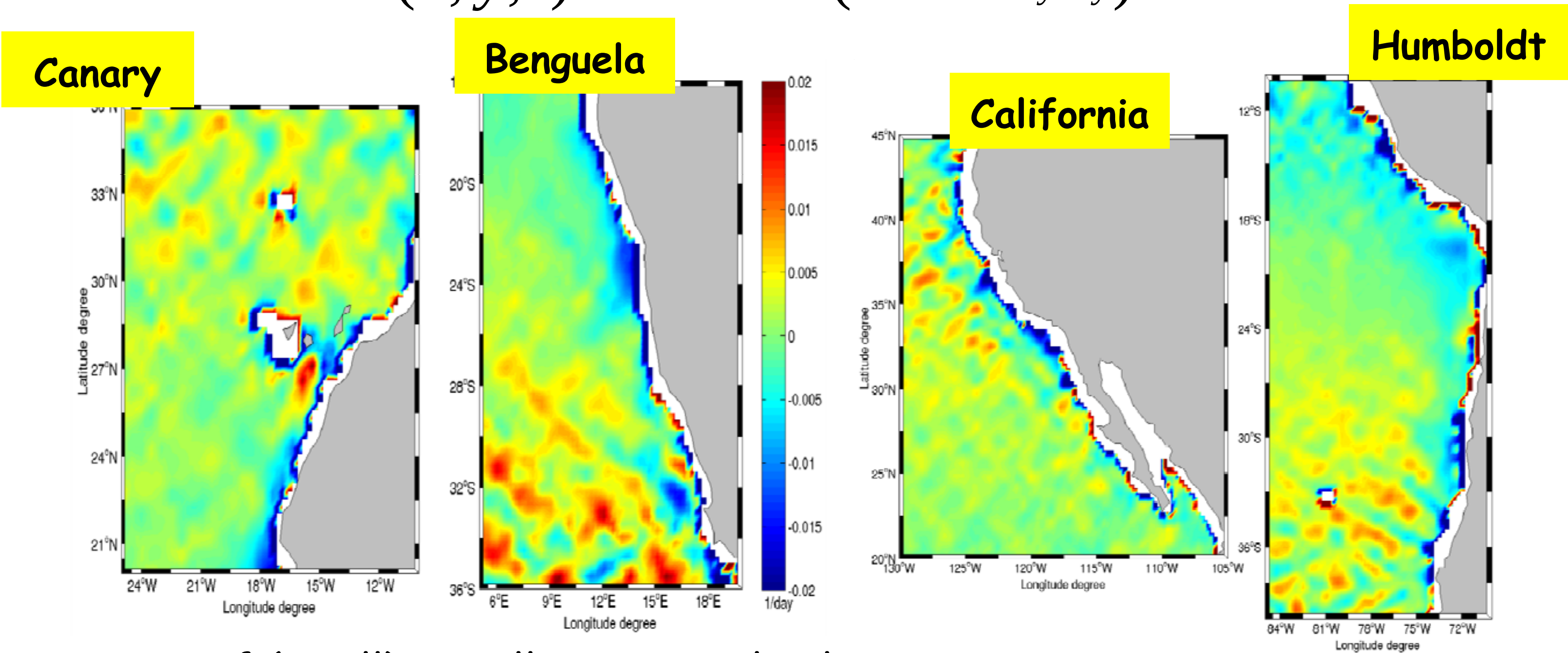



Sandulescu et al. *Nonlinear Processes in Geophysics* **14**, 443-454 (2007)

Sandulescu et al. *Ecological Complexity* **5**, 228-237 (2008)

## Temporal averages of vertical velocities from incompressibility condition

$$\Delta(x,y,t) \equiv \partial_z V_z = -(\partial_x V_x + \partial_y V_y)$$

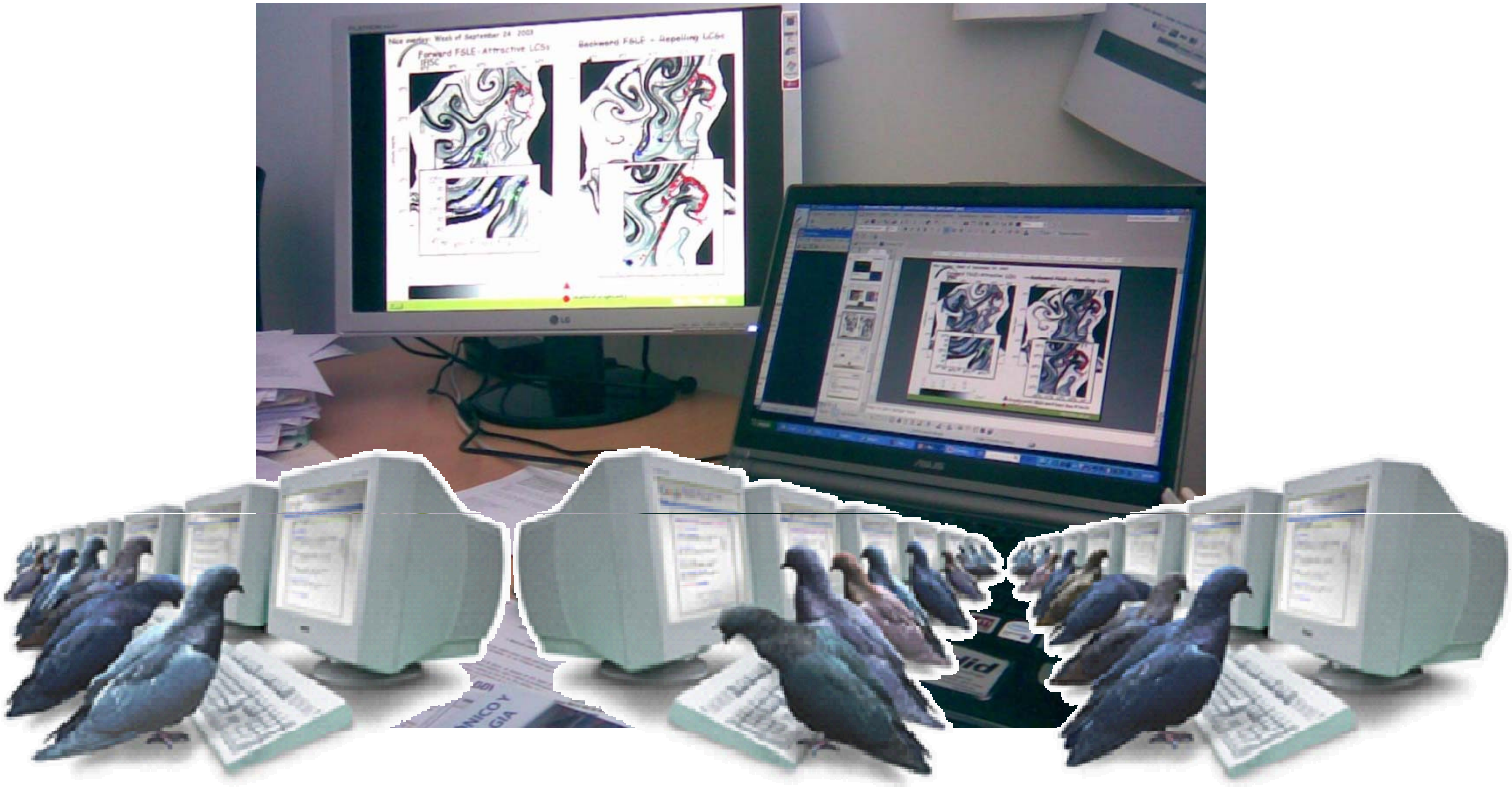



- Dominance of (small) upwelling vertical velocities in the less turbulent subsystem.
- Thus, probably the influence of horizontal stirring on plankton is only indirect: need to understand the 3d flow structure: high FSLE associated to low Eckman transport.

Rossi et al., Geophys, Res. Lett. 2008, Nonlin. Proc. Geophys. 2009

- Lagrangian Coherent Structures give the skeleton of horizontal transport
- This certainly influences abiotic quantities: temperature, nutrients, …
- This certainly influences plankton distribution
- From there, impact is expected in plankton consumers, their predators, … cascades up along the food chain …

# Do birds know about FSLE calculations?



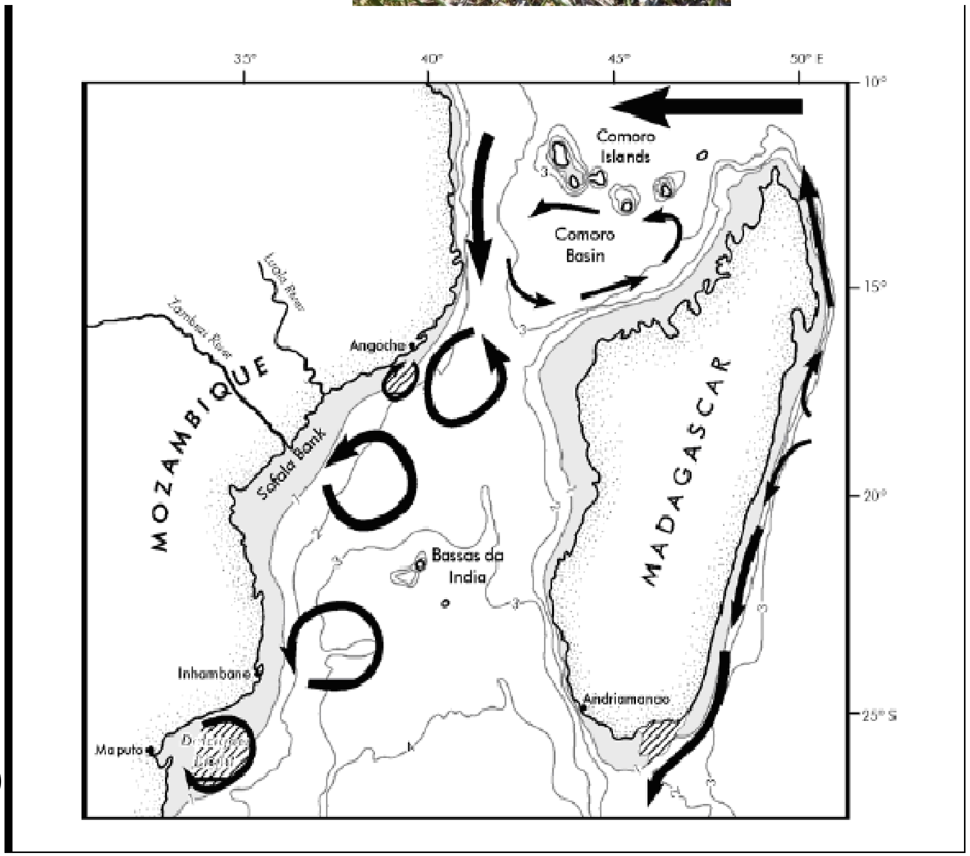
Tew Kai, Rossi, Sudre, Weimerskirch, Lopez, Hernandez-Garcia, Marsac, Garçon, PNAS 106, 8245 (2009)

# FRIGATEBIRDS in the MOZAMBIQUE CHANNEL

Particular topography (channel/islands) linked with strong mesoscale activity:

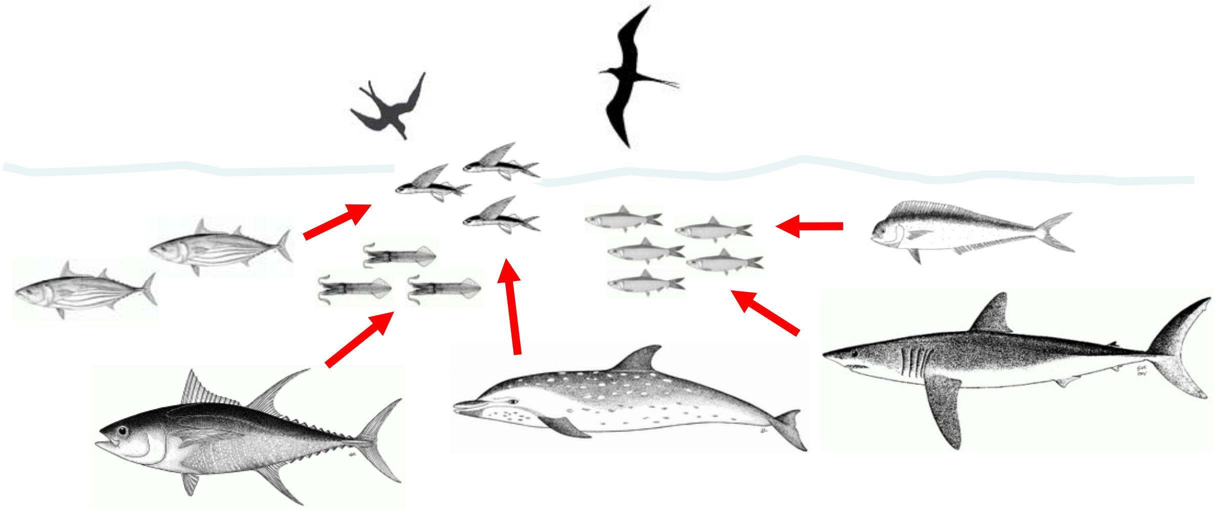
- Large anticyclonic cell at the north
- Local upwellings
- Anticyclonic and cyclonic mesoscale eddies moving southward permanently.

(De *Ruijter et al., 2004*)



**Great frigatebird (*fregata minor*):**

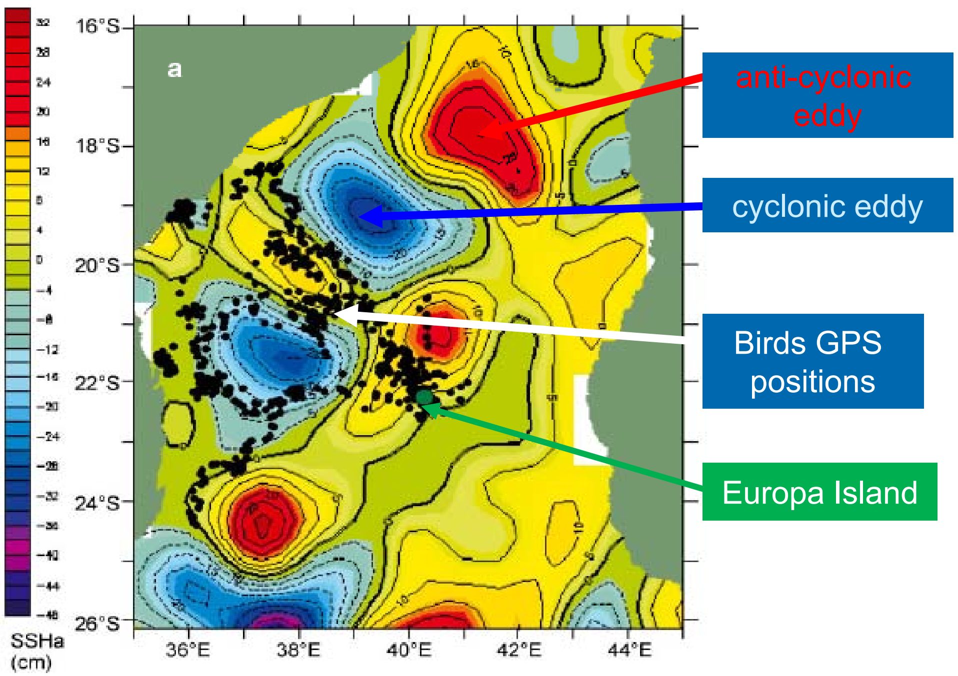
- Large seabirds (light weight < 5 kg and large wings > 2m). Use thermals to soar before gliding over long distances and time (days/nights over weeks).
- Traveling at high altitudes to locate patches of prey and come close to surface to feed (reduced flight speed indicates foraging).
- Feeding occurs only during daytime (peaks in the morning and evening).
- Unable to dive or rest on the water surface (permeable plumage) → in association with subsurface predators (tuna, …): **fisheries indicators**

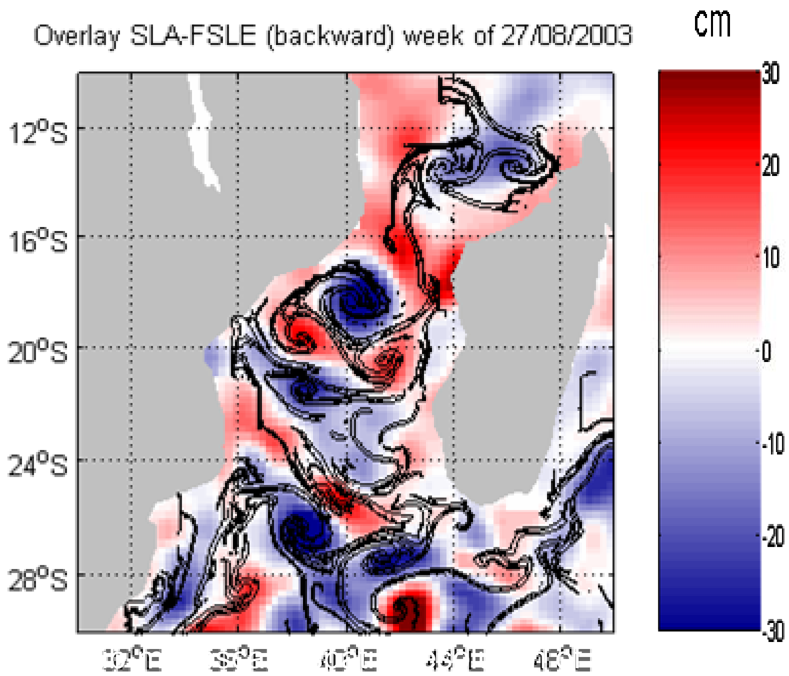


Weimerskirch et al. 2004; Jaquemet et al. 2005

## SSH (cm): Eulerian view

## Lagrangian FSLEs versus SSH




*Weimerskirch et al,2004*

The Lagrangian FSLE gives access to submesoscale structures

**We identify Lagrangian Coherent Structures with |FSLE| > 0.1 day$^{-1}$**

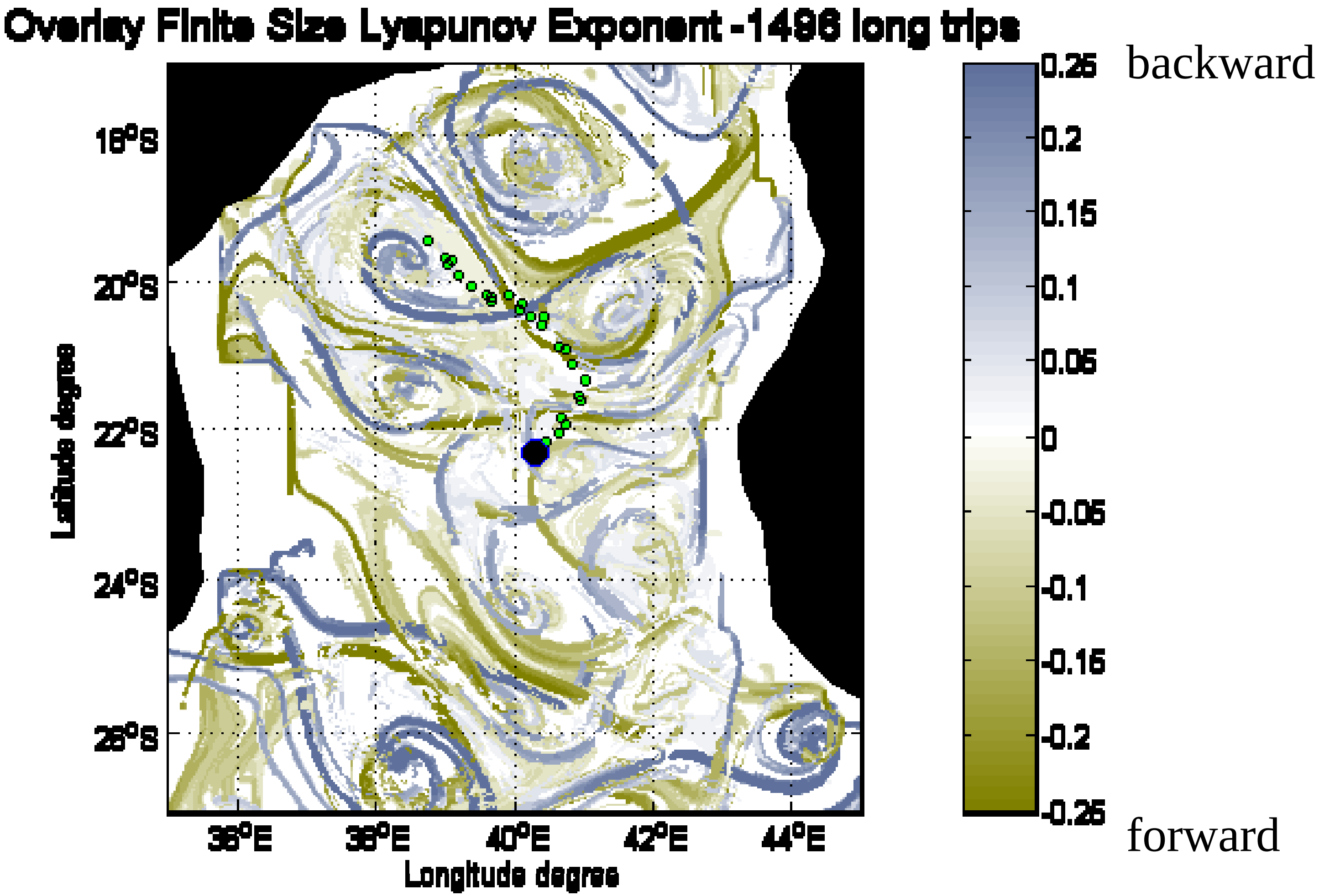
Satellite transmitter and altimeter
(total weight : 1 to 3% mass of adults, max 45g)

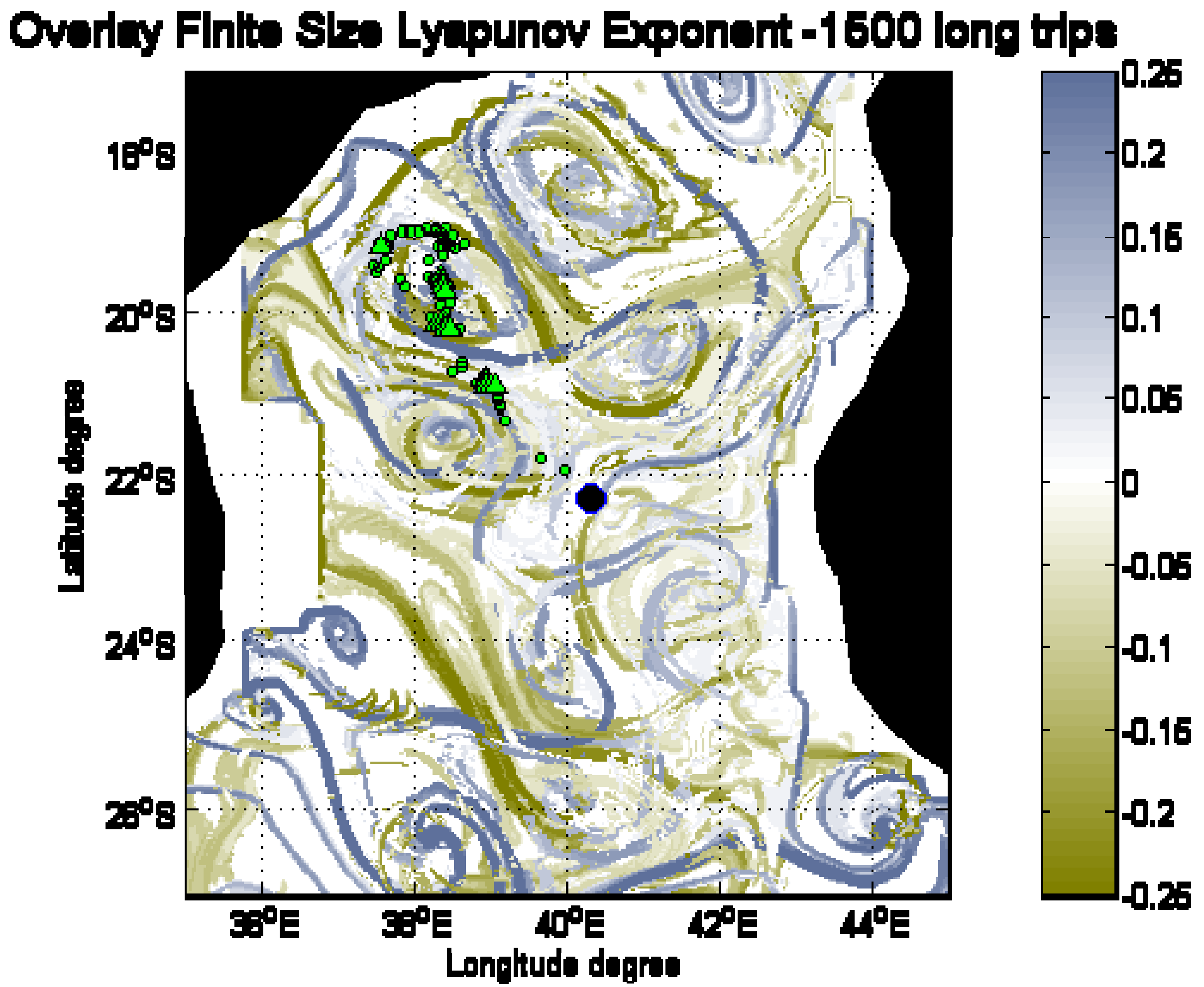
8 birds (from Europa Island community) fitted with satellite transmitter and altimeter.

Followed for their foraging trips from August 18 to September 30, 2003.

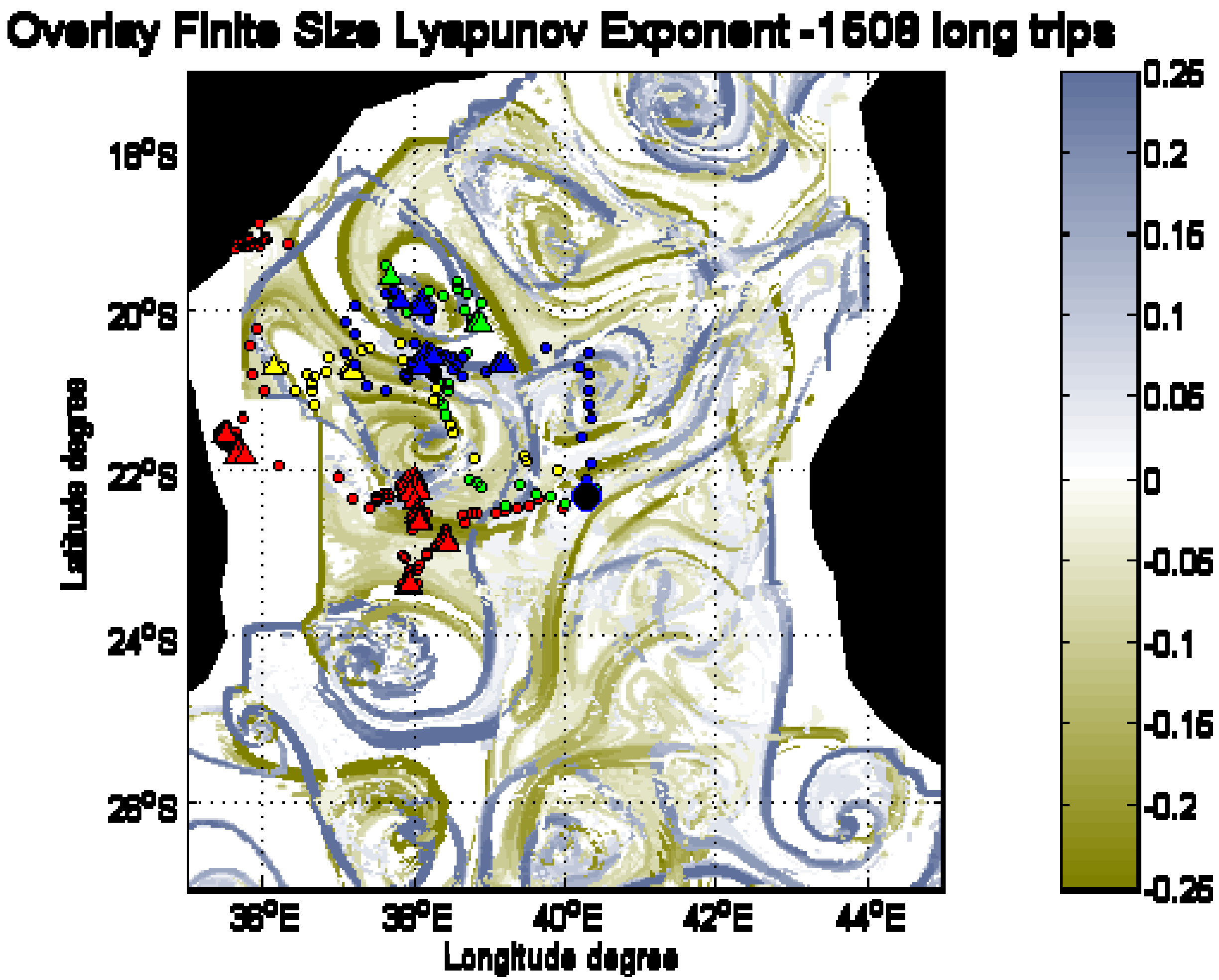
1600 Argos from 50 trips positions, distributed into 17 long trips (> 614 km) and 33 short trips.

(Weimerskirch et al., 2004

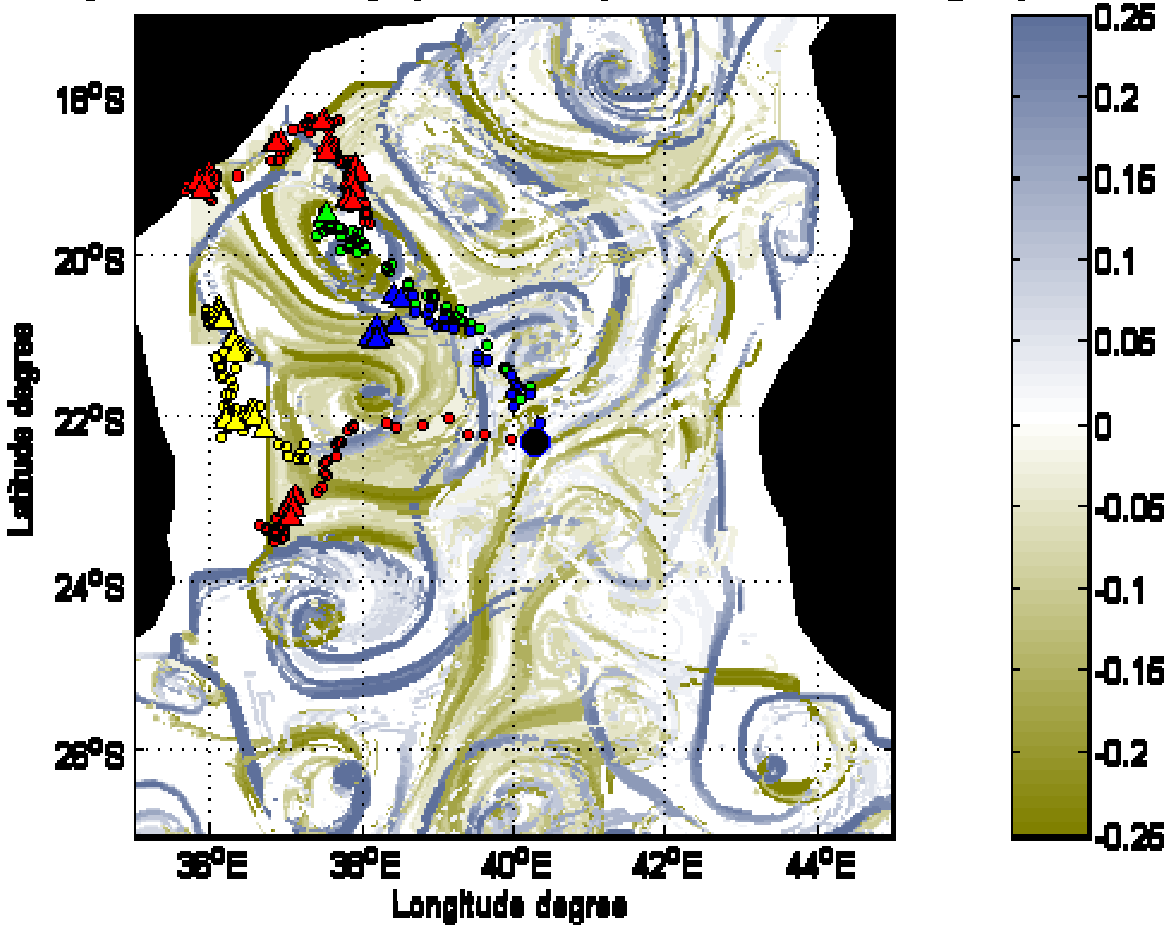

Overlay Finite Size Lyapunov Exponent -1496 long trips
16°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
backward
forward

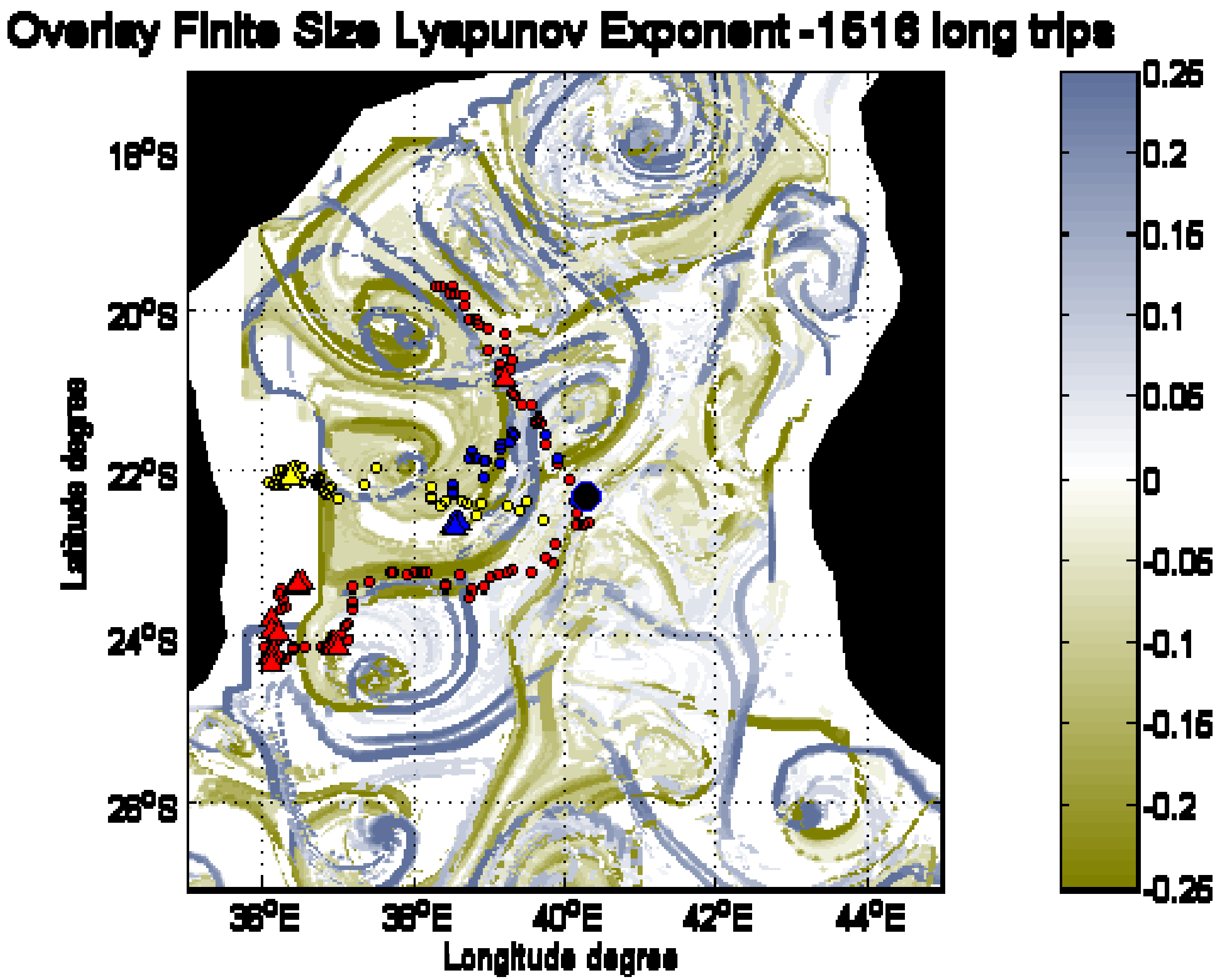


Overlay Finite Size Lyapunov Exponent -1500 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1508 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

Overlay Finite Size Lyapunov Exponent -1512 long trips




Overlay Finite Size Lyapunov Exponent -1516 long trips
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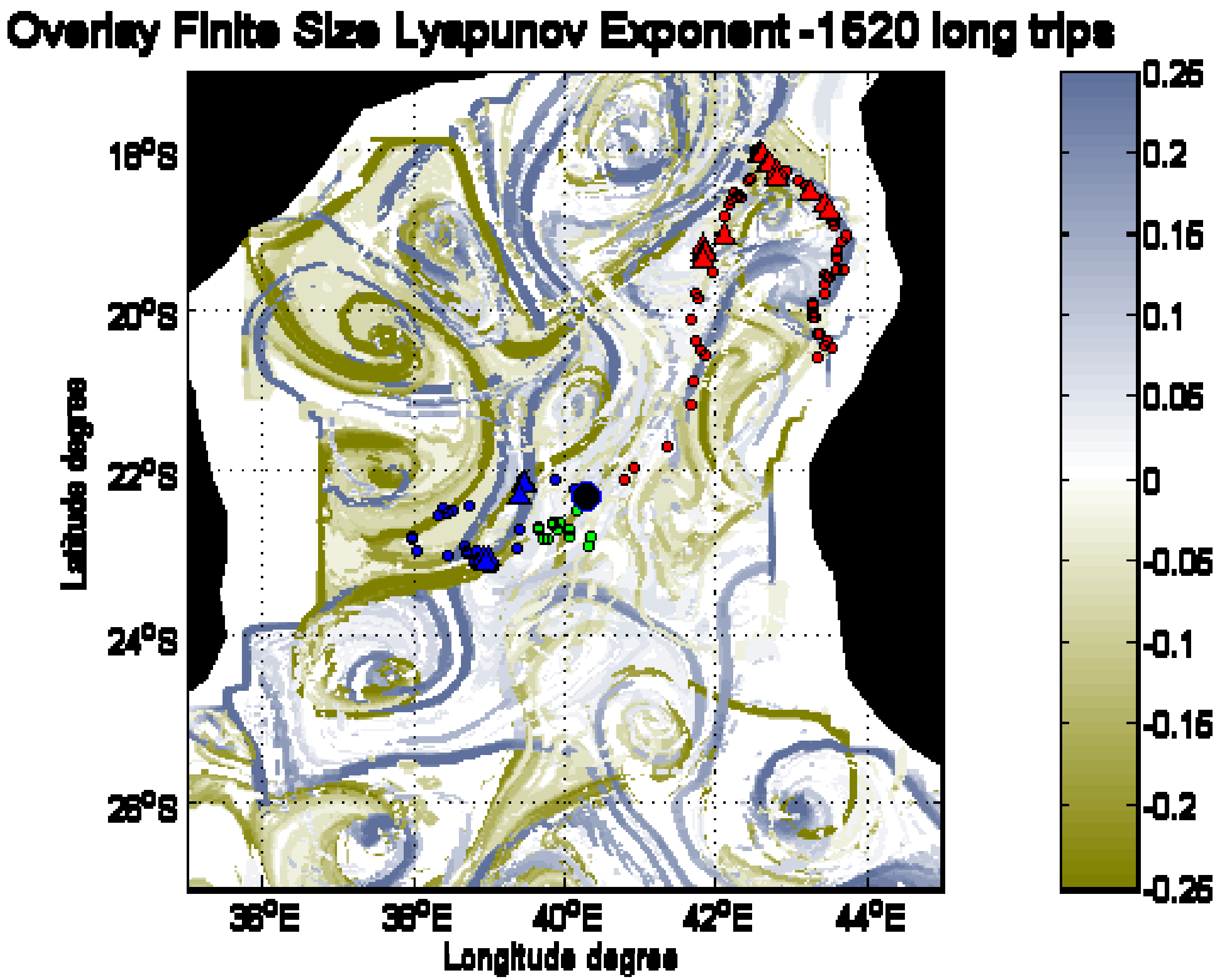
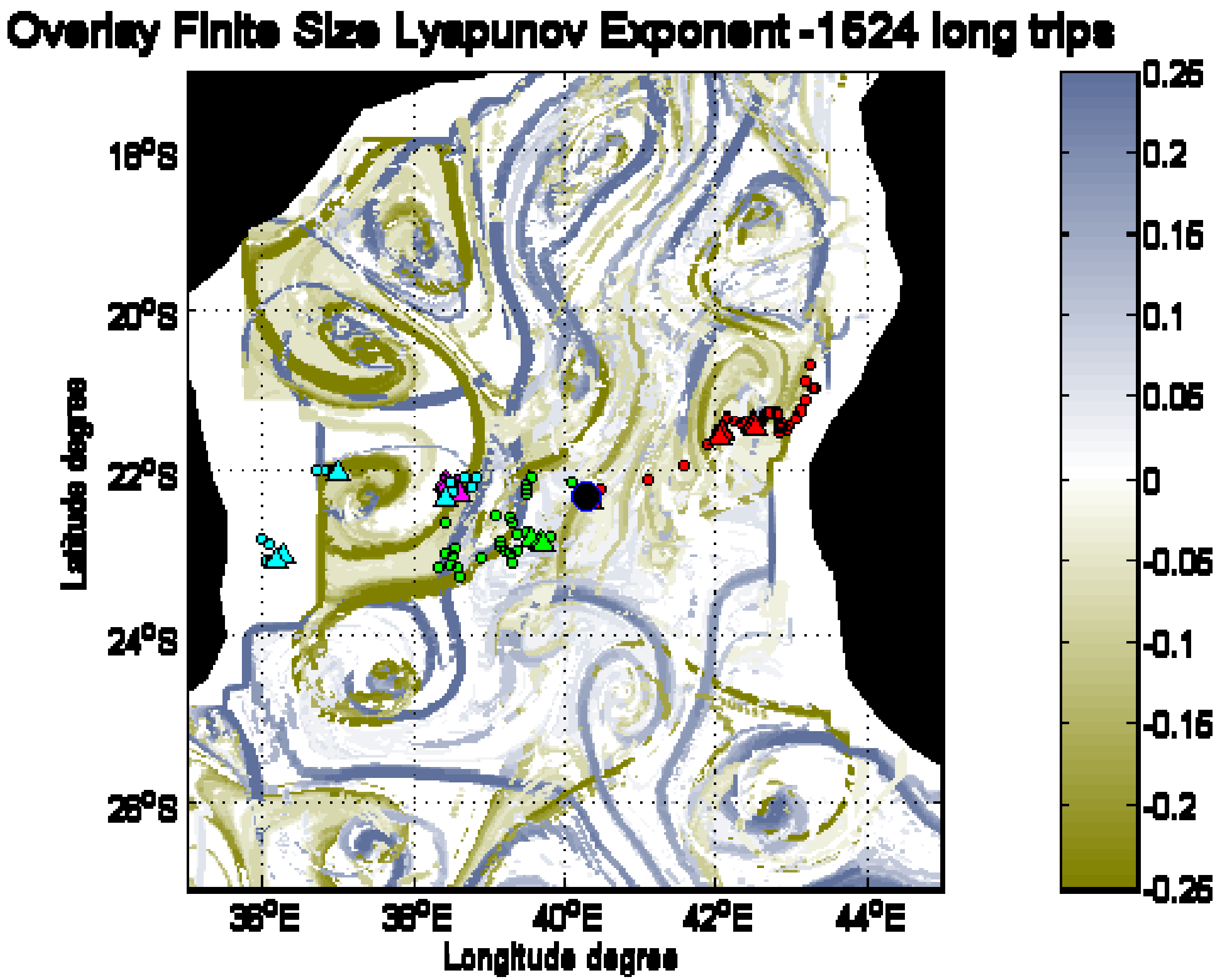
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1620 long trips
18°S
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25



Overlay Finite Size Lyapunov Exponent -1524 long trips
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20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
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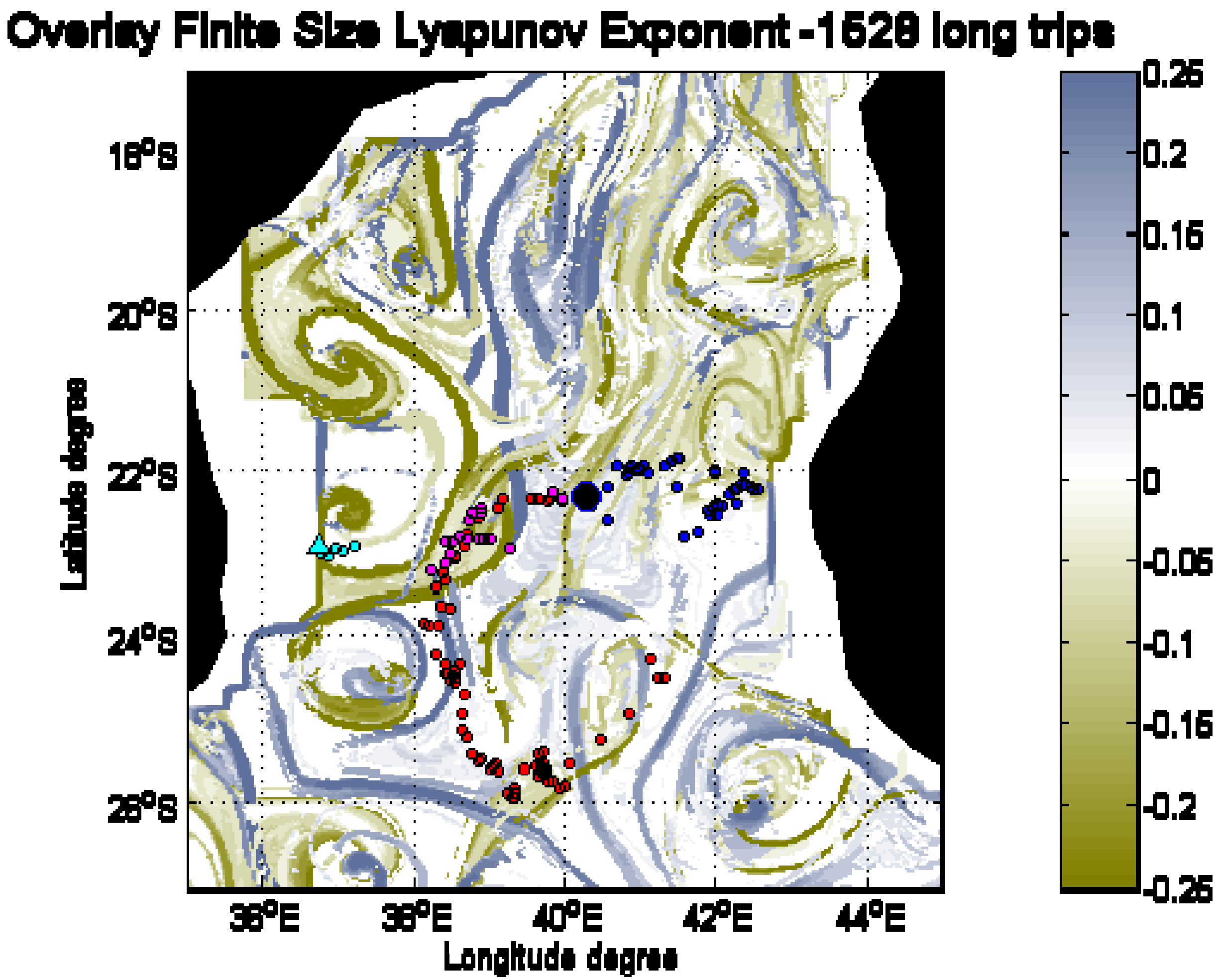
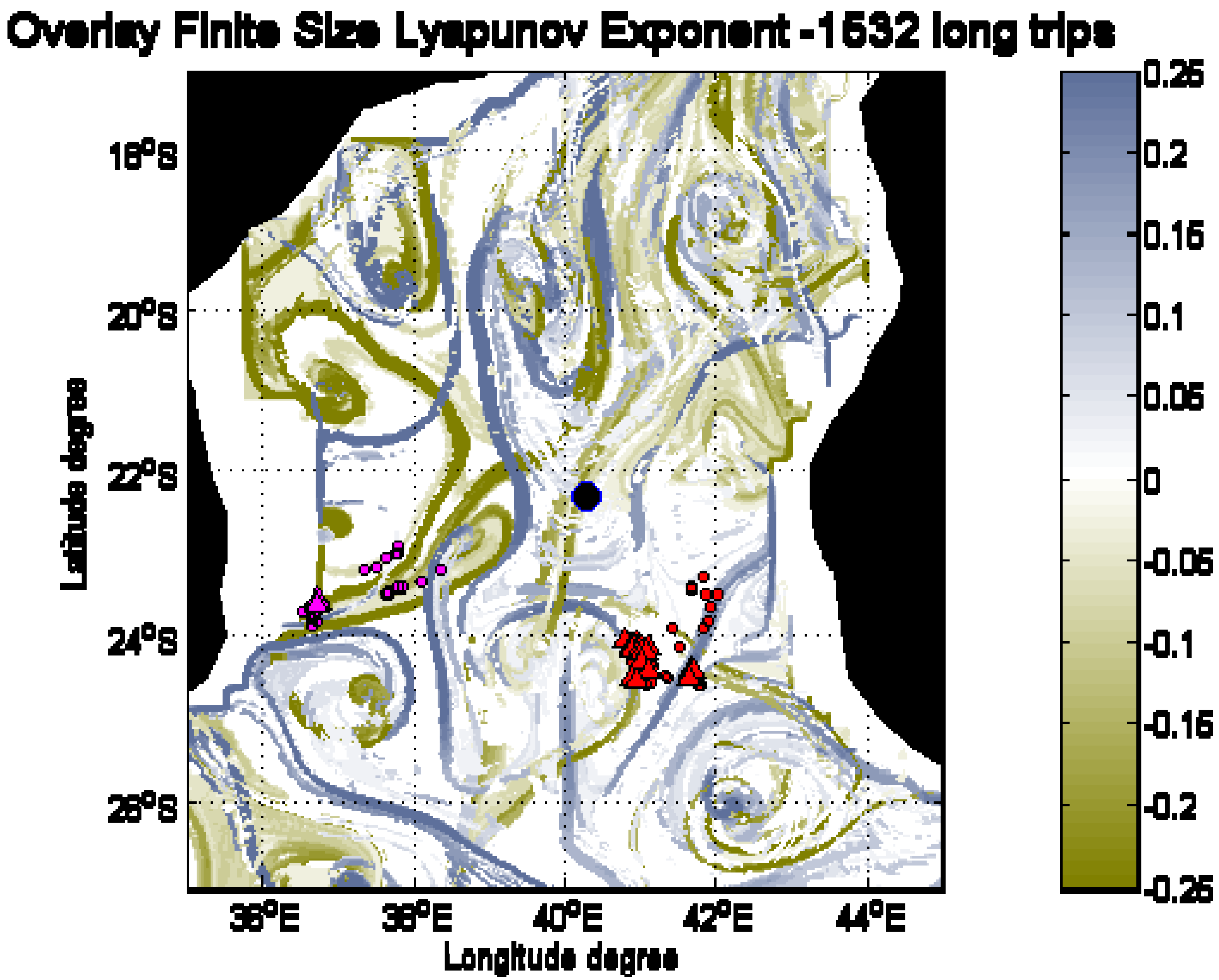
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1528 long trips
18°S
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1532 long trips
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20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
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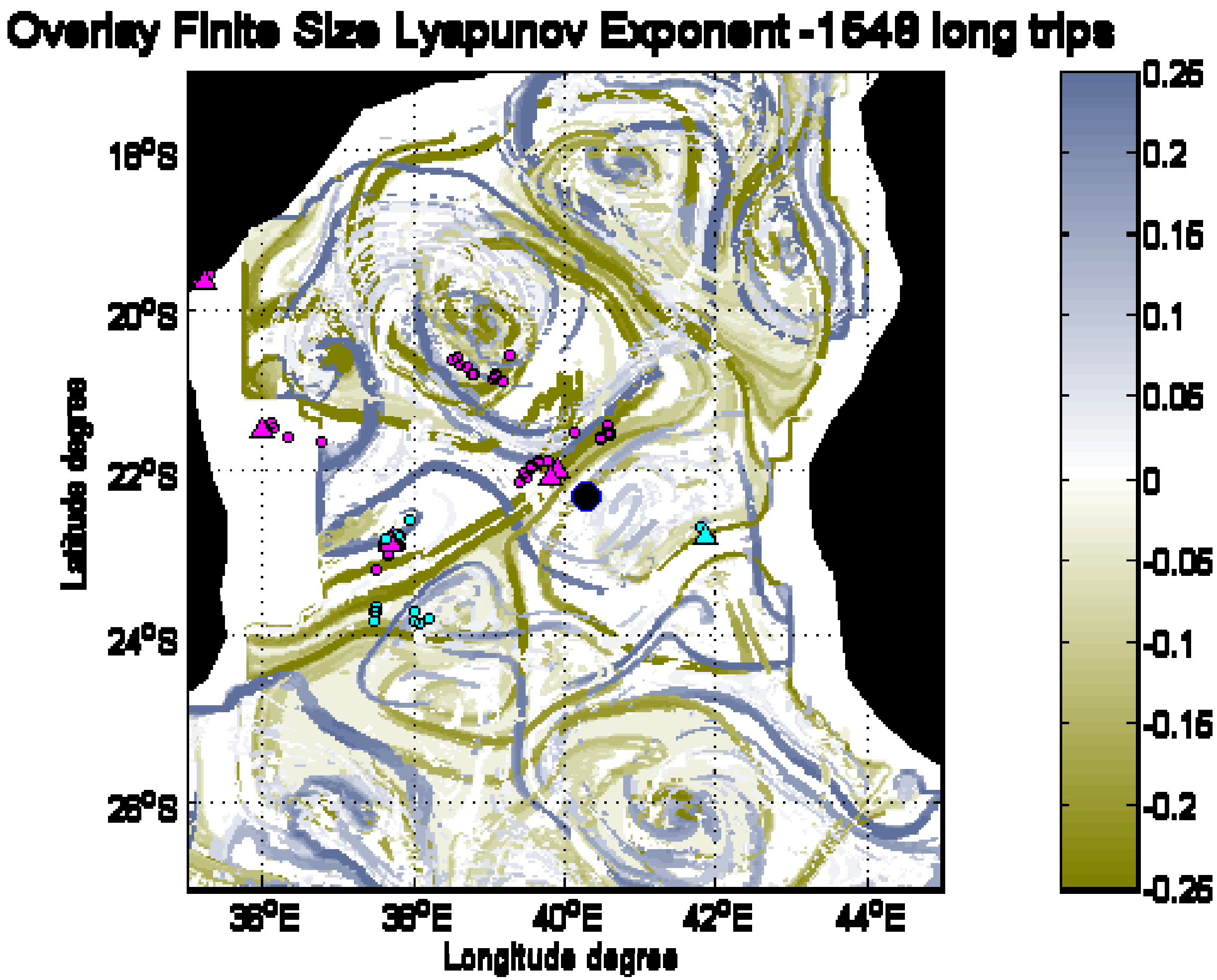
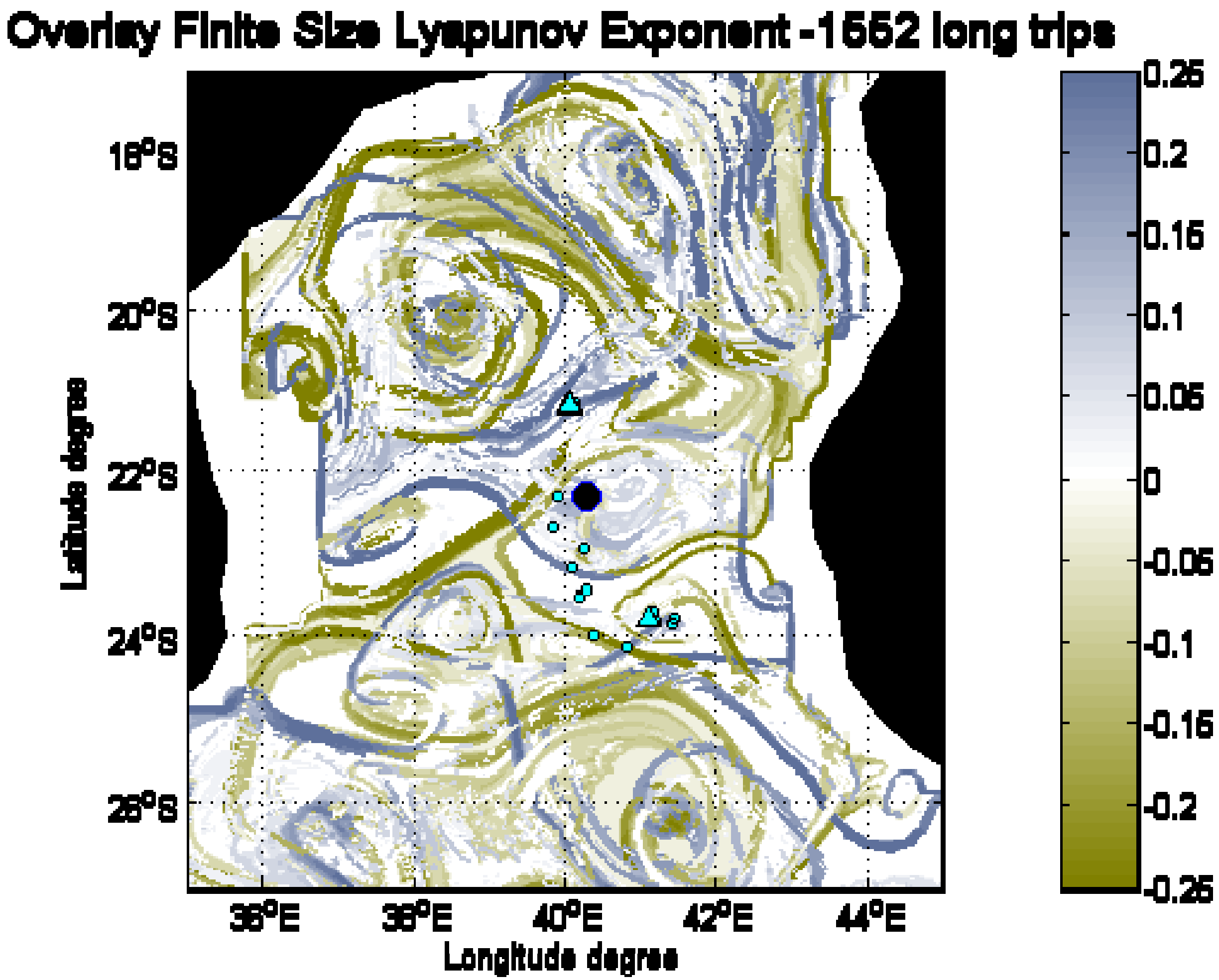
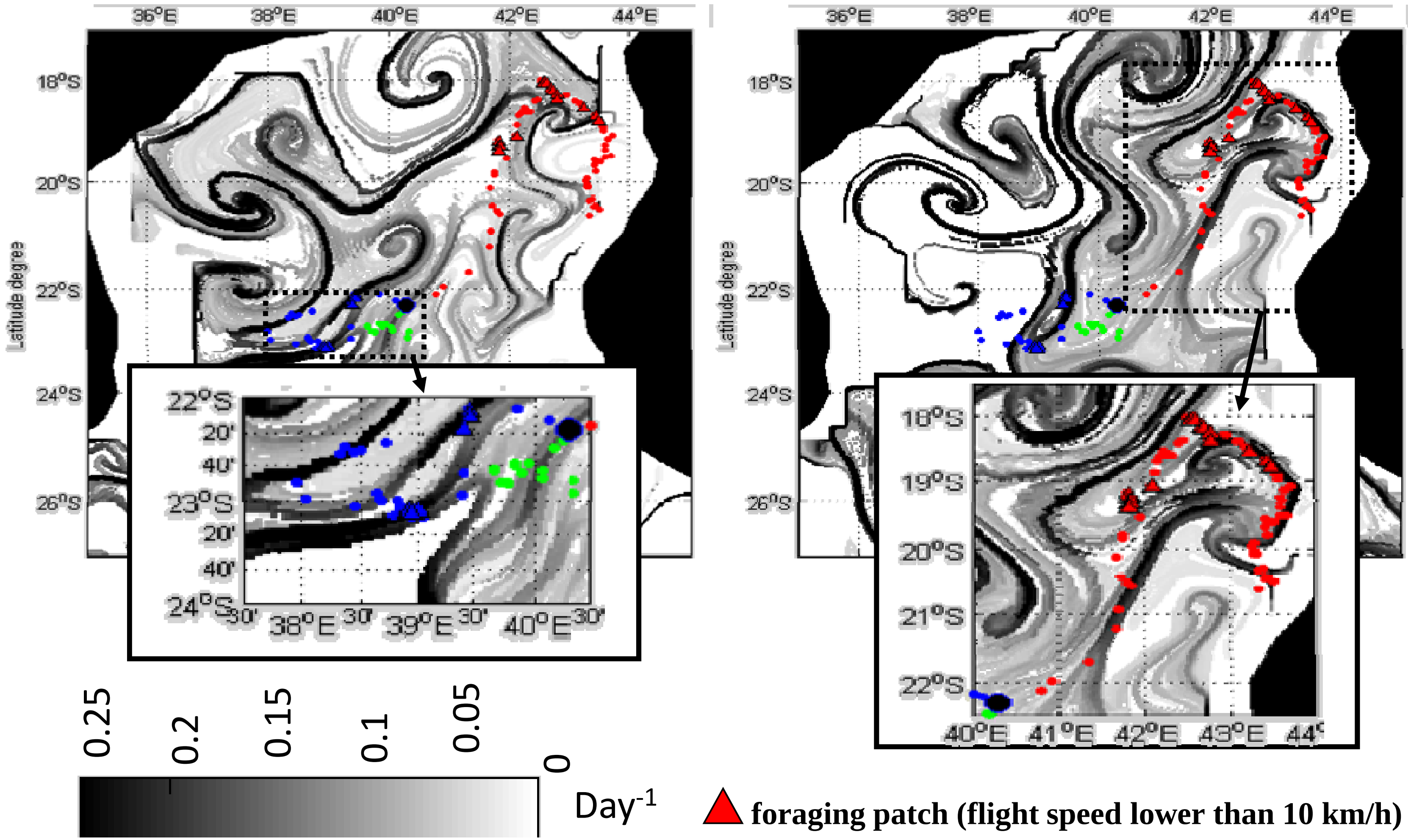
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1548 long trips
18°S
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1552 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

Week of September 24, 2003

**Backward FSLE=Attractive LCSs** **Forward FSLE = Repelling LCSs**



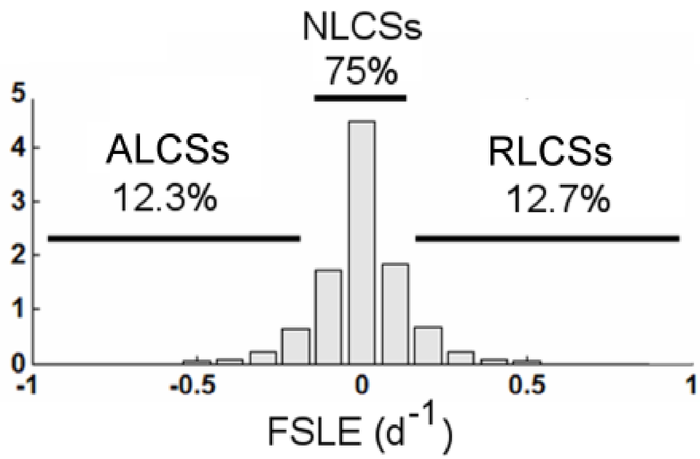

▲ foraging patch (flight speed lower than 10 km/h)

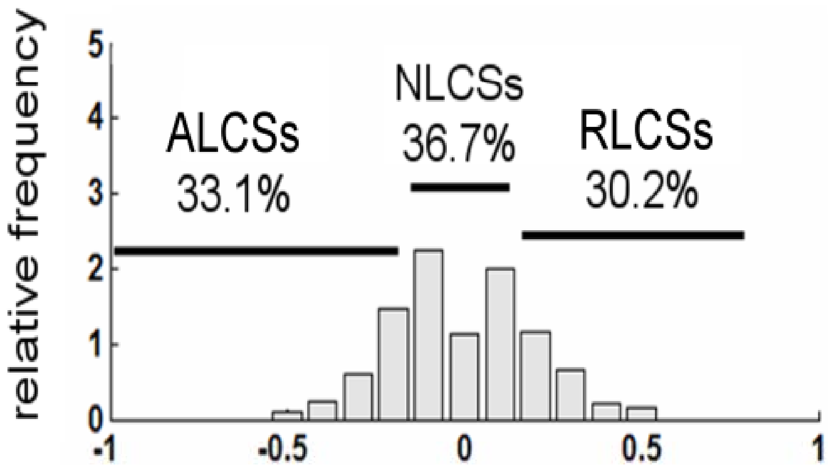
● seabird trajectory

## Histograms of FSLE values

On the whole area

On the birds positions




ALCS: attracting LCS, i.e. FSLE (backwards) < - 0.1 day$^{-1}$

RLCS: repelling LCS, i.e. FSLE (forwards) > 0.1 day$^{-1}$

NLCS: not LCS (small FSLE)

Despite LCS occupy only 25% of space, 63% of bird's positions are on them

STATISTICAL TESTS

Table 1. Absolute frequency of seabird positions on LCSs and on no Lagrangian structures for long and short trips per week and result of the G-test for goodness of fit

| Week | All trips: LCSs: \|FSLE\| > 0.1 day$^{-1}$ | All trips: \|FSLE\| < 0.1 day$^{-1}$ | Long trips: LCSs: \|FSLE\| > 0.1 day$^{-1}$ | Long trips: \|FSLE\| < 0.1 day$^{-1}$ | Short trips: LCSs: \|FSLE\| > 0.1 day$^{-1}$ | Short trips: \|FSLE\| < 0.1 day$^{-1}$ |
|---|---|---|---|---|---|---|
| 1 | 38 | 9 | 19 | 7 | 19 | 2 |
| 2 | 78 | 40 | 55 | 12 | 23 | 28 |
| 4 | 208 | 85 | 147 | 54 | 61 | 31 |
| 5 | 167 | 109 | 137 | 84 | 30 | 25 |
| 6 | 120 | 77 | 89 | 51 | 31 | 26 |
| 7 | 79 | 55 | 72 | 32 | 7 | 23 |
| 8 | 53 | 34 | 53 | 34 | — | — |
| 9 | 61 | 59 | 61 | 59 | — | — |
| 10 | 55 | 31 | 45 | 24 | 10 | 7 |
| 14 | 35 | 12 | 35 | 12 | — | — |
| 15 | 10 | 5 | 10 | 5 | — | — |
| % | 63.7 | 36.3 | 65.9 | 34.1 | 56.0 | 44.0 |
| G-test (log-likelihood ratio) | | | | | | |
| *n* | 1420 | | 1097 | | 323 | |
| *k* | 11 | | 11 | | 7 | |
| df | 10 | | 10 | | 6 | |
| G | 28.119 | | 30.613 | | 32.057 | |
| *P* | 0.00173 | | 0.001 | | 0.000 | |

One-tailed tests. Null hypothesis Ho: Seabird positions share equally LCSs (|FSLE| > 0.1 day$^{-1}$ and on no LCSs. $\alpha$ = 5%.

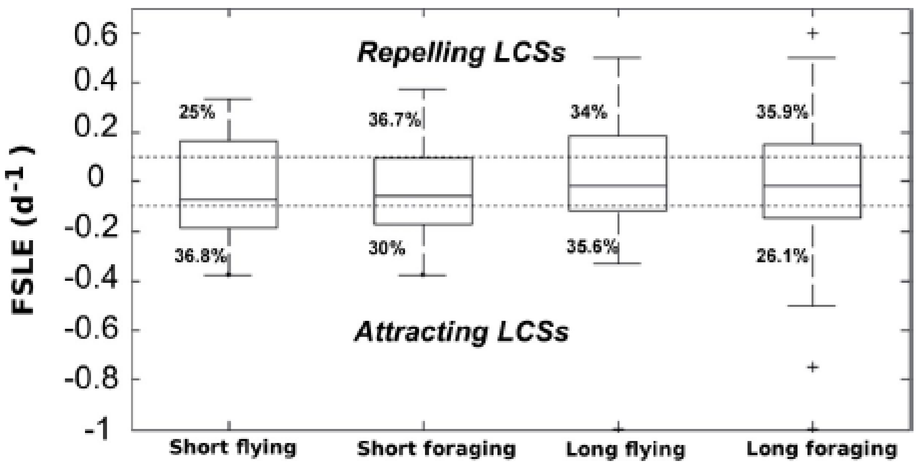



Table S2. Result of G-test statistics for comparison between frequency of bird positions on repelling or attracting LCS during flying and foraging and short and long trips

| Variable | Flying | Foraging |
|---|---|---|
| Long trips | | |
| Repelling LCS (FSLE > 0.1 day$^{-1}$) | 318 | 50 |
| Attracting LCS (FSLE<-0.1 day$^{-1}$) | 333 | 37 |
| *n* | 738 | |
| G | 2.29 | |
| *P* | 0.13021 | |
| Short trips | | |
| Repelling LCS (FSLE > 0.1 day$^{-1}$) | 76 | 9 |
| Attracting LCS (FSLE<-0.1 day$^{-1}$) | 112 | 10 |
| *n* | 207 | |
| G | 0.34 | |
| *P* | 0.55993 | |

Two-tailed tests. Null hypothesis Ho: seabirds share out equally on repelling and attracting structures when they fly or forage. $\alpha$ = 5%.

**Results of statistical tests:**

- Frigate birds fly on top of LCSs both for travelling as for foraging
- No significant difference between day and night positions
- No significant difference between come and return trip

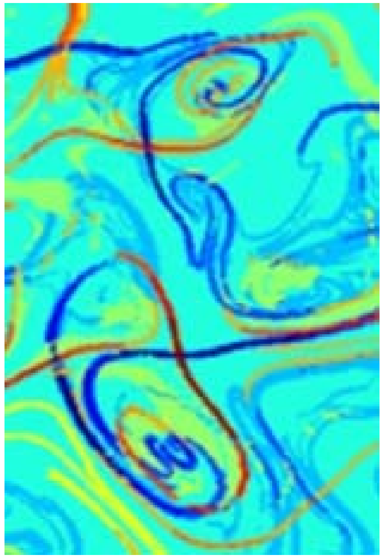
**Frigatebirds 'follow' LCSs not only to find there prey, but as biological corridors which bring them to foraging places**

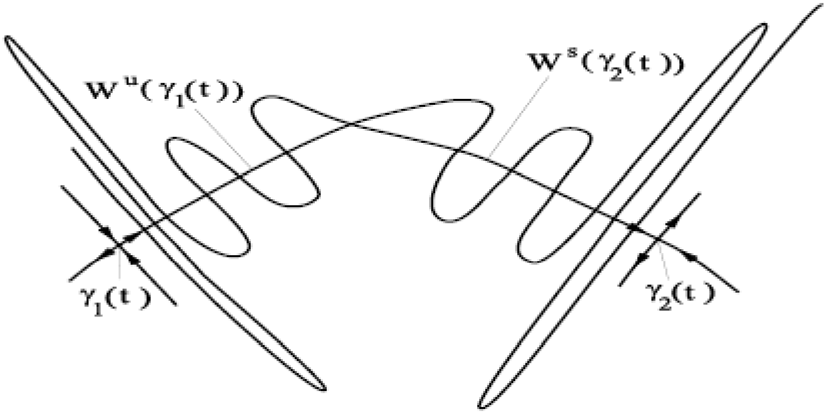
Aggregation of prey on LCSs? or aggregation of subsurface predators?
Olfatory clues (DMS produced by zooplankton) ? thermal air currents?

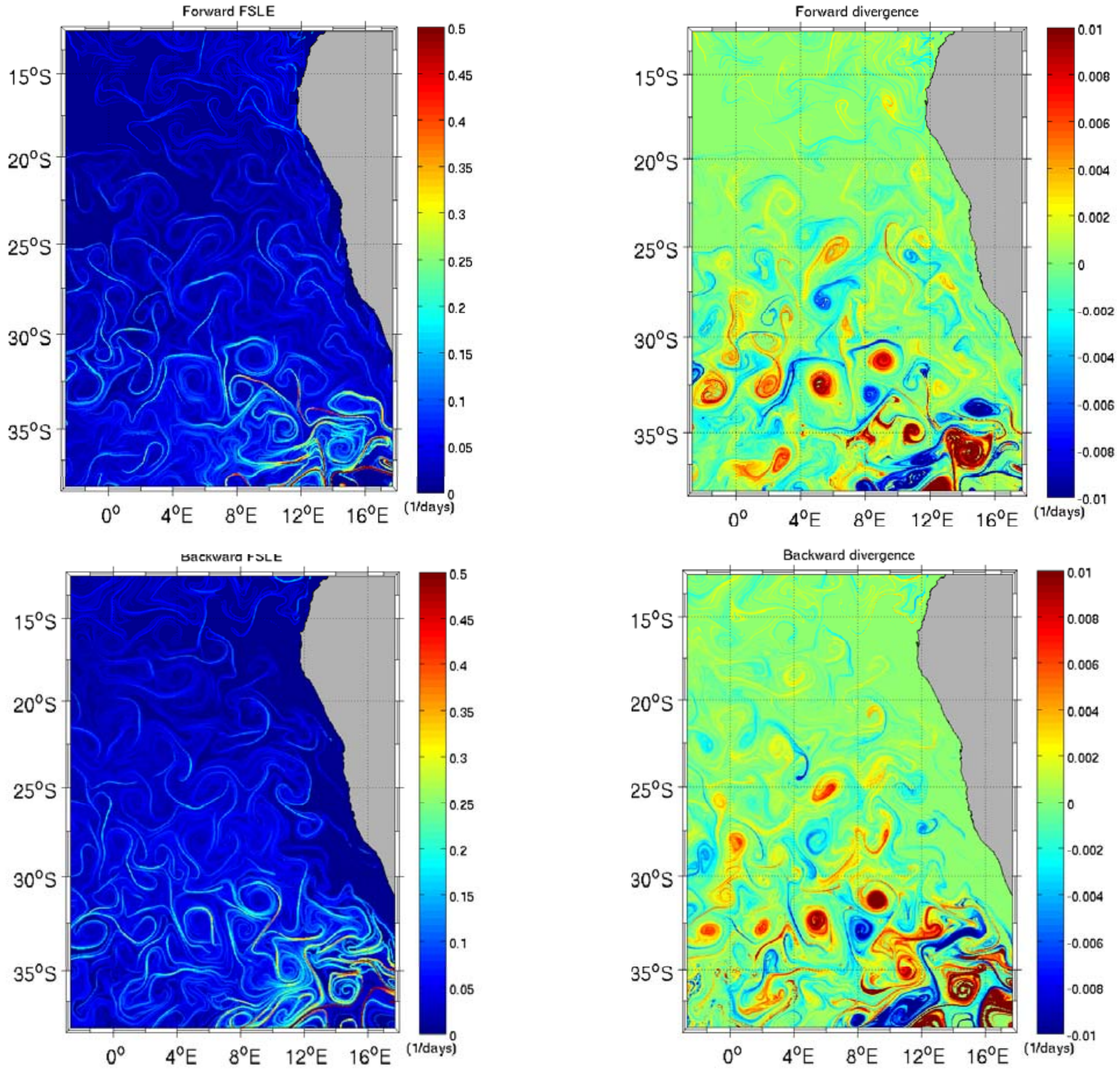
**Tew Kai et al. PNAS (2009)**

Puzzling issue: no significant difference between attracting and repelling LCSs (c.f. talk by Shane Ross)

- Tangencies between manifolds?
- Interleaving between them?
- 3d dynamics associated both to ALCS and RLCS?
- Do they simply avoid low FSLE regions?







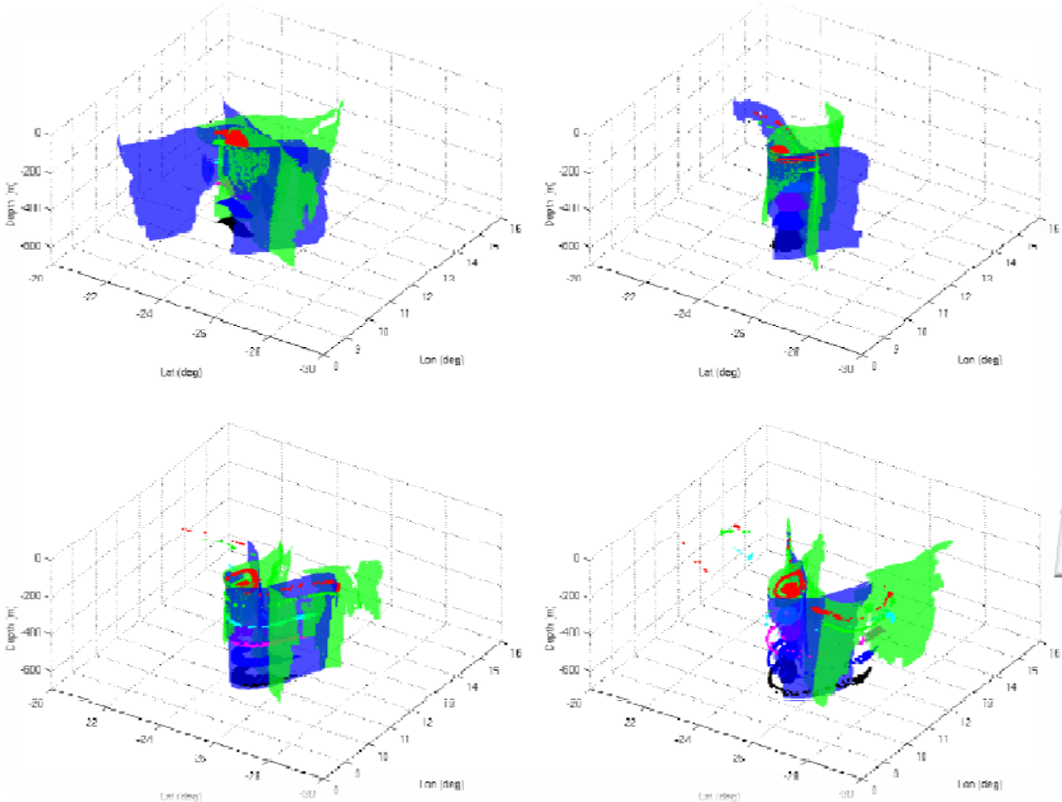

Lagrangian divergence

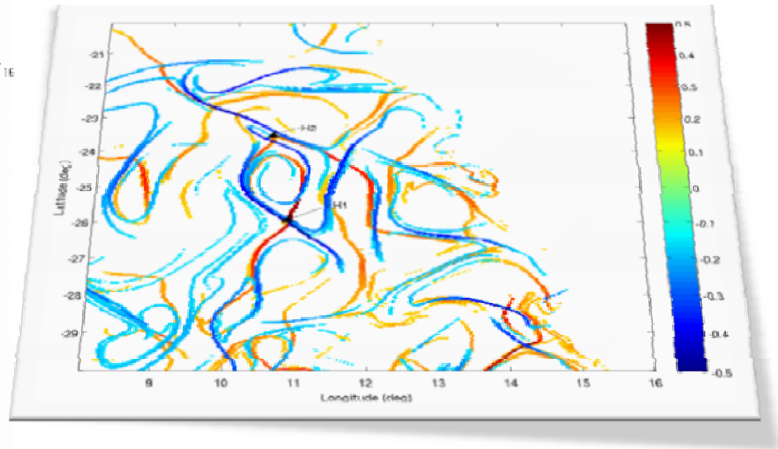
$$\frac{1}{t}\int_0^t dt(\partial_x V_x + \partial_y V_y)$$

(work in progress with Ismael Hernández-Carrasco)

(c.f. Tang, Chan, Haller, J. Appl. Met. Clim. 2011)

Threedimensional structure of an eddy in Benguela from ROMS
(work in progress with J. Bettencourt)



- Biological processes in oceans are impacted by fluid flow at all trophic levels, from primary producers to top predators
- Lagrangian Coherent Structures are a convenient way to analyze these interactions
- Tridimensional effects need to be addressed in more detail

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**http://ifisc.uib-csic.es/research/research_fluid.php**