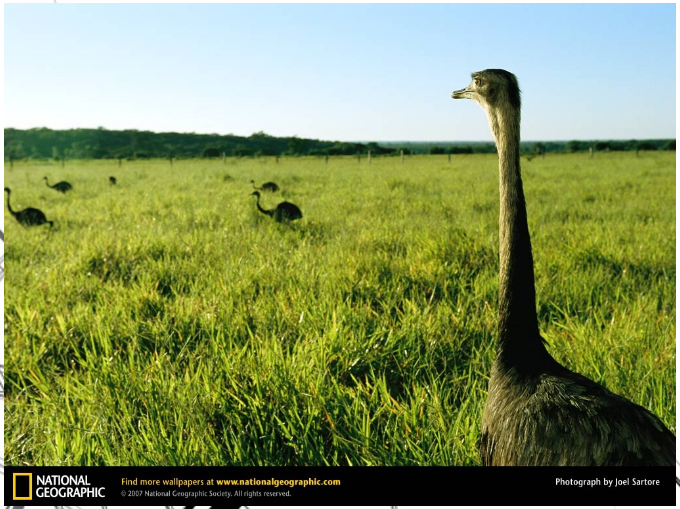
The savanna problem from a statistical physics point of view

Federico Vázquez, Cristóbal López, Maxi San Miguel

Instituto de Fisica Interdisciplinar y Sistemas Complejos, Palma de Mallorca.



Miguel Angel Muñoz

Instituto Carlos I de Fisica

Teorica y Computacional, Granada.

Justin Calabrese, Volker Grimm

Helmholtz Centre for Environmental

Research-UFZ, Leipzig.

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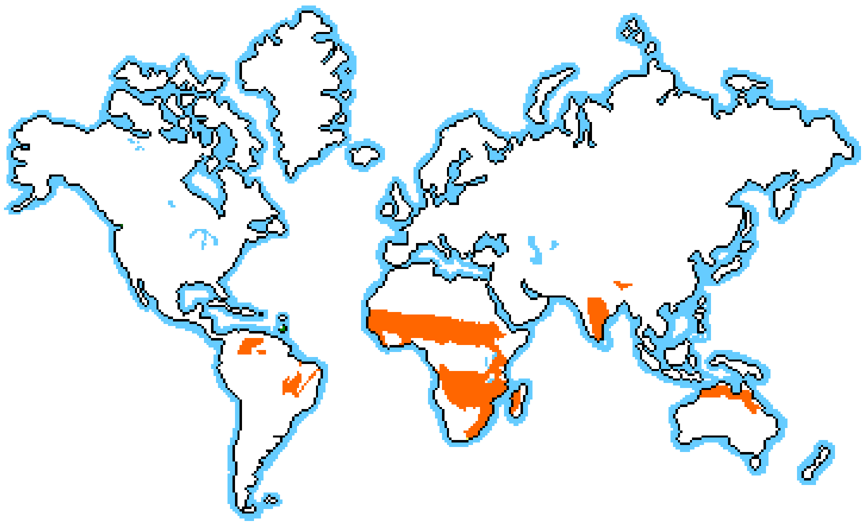
What is a Savanna?

Savanna: rolling grassland scattered with shrubs and isolated trees.

It is found between tropical rainforests and deserts.



Tarangire National Park (Tanzania)

Some properties

- They appear in regions with broad range of climatic conditions.
- Large rainfall variability: dry season (summer), wet season (winter).
- Characterized by persistent mixture of trees and grasses.

The Savanna Problem

What is unique about savannas that allow the coexistence of trees and grasses, while in other biomass one of the species dominates? What factors govern that coexistence? (Sarmiento '84, Scholes '97, Higgins '00)

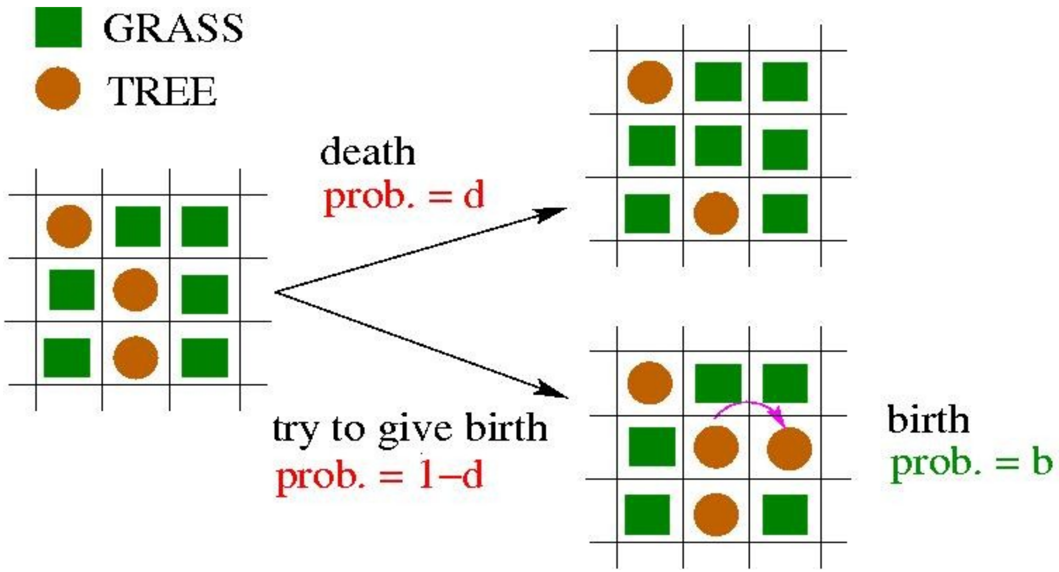
Varying environment: an explanation for tree-grass coexistence

Scholes and Walker (1993): “Savannas do not represent a stable mixture of trees and grasses, as has been suggested in the past, but an inherently unstable mixture which persists owing to disturbances such as fire, herbivore and fluctuating rainfall”.

But: are savannas really unstable under a varying environment?

Some models (Higgins '00, Sankaran '04) with climatic variability use up to 40 parameters!
Hard to see what causes coexistence.

A 'physics' model for a dynamical (fluctuating) savanna



2D square lattice.

Two main parameters:
b and d.

- Property 1: dispersion of tree seeds is limited (nearest neighbor seedling).

Property 2: Average tree cover depends on mean annual precipitations.

Long dry season $\longrightarrow$ grassland, Long rainy season $\longrightarrow$ woodland

- Birth probability *b(t)* proportional to the amount of precipitations.

More rainfalls $\rightarrow$ less competition for water $\rightarrow$ more seed establishment.

- Weather exhibits some degree of temporal correlations.

b takes the value of previous year with prob. 1-q.

$$b_{t+1} = \begin{cases} b_t \text{ with prob. } 1-q \\ \text{random value in } (0, b_{max}) \text{ with prob. } q \end{cases}$$

b_{max} is the order parameter

- Death probability d depends on tree's age.

Properties:

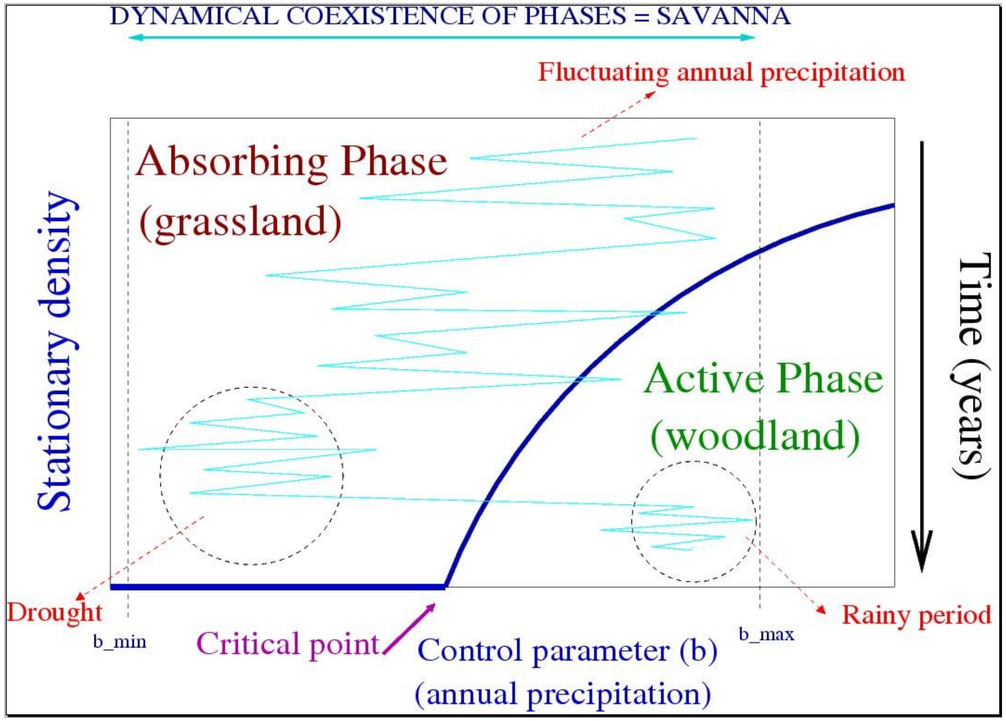
3) Juvenile trees (younger than four years of age) compete with neighboring trees to survive.

4) Established middle age trees rarely die.

5) Very old trees (older than 100 years of age) die with high probability.

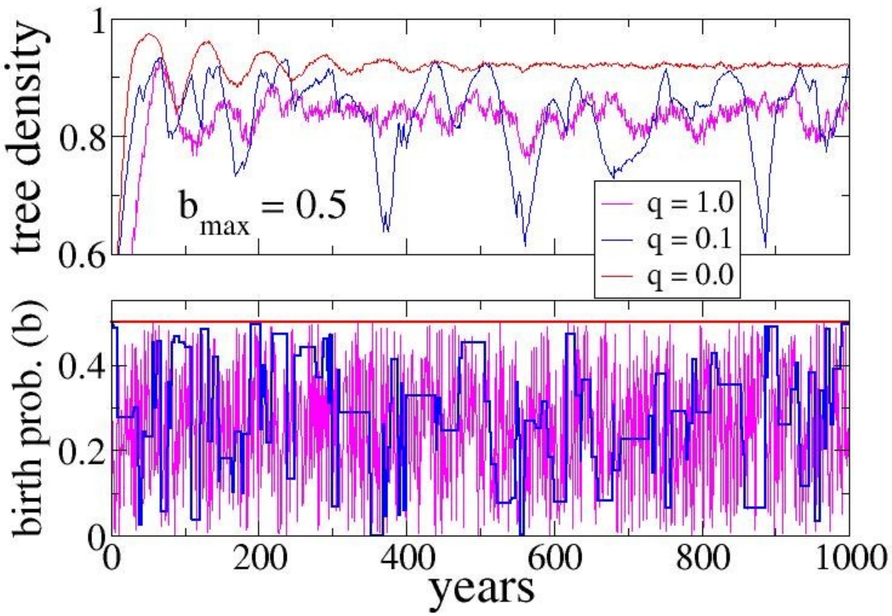
$$d = \begin{cases} 1-\exp(\delta Z_{NN}) & \text{if } 0 < \text{age} < 4 \\ 0 & \text{if } 4 < \text{age} < 100 \\ 1 & \text{if } \text{age} > 100 \end{cases}$$

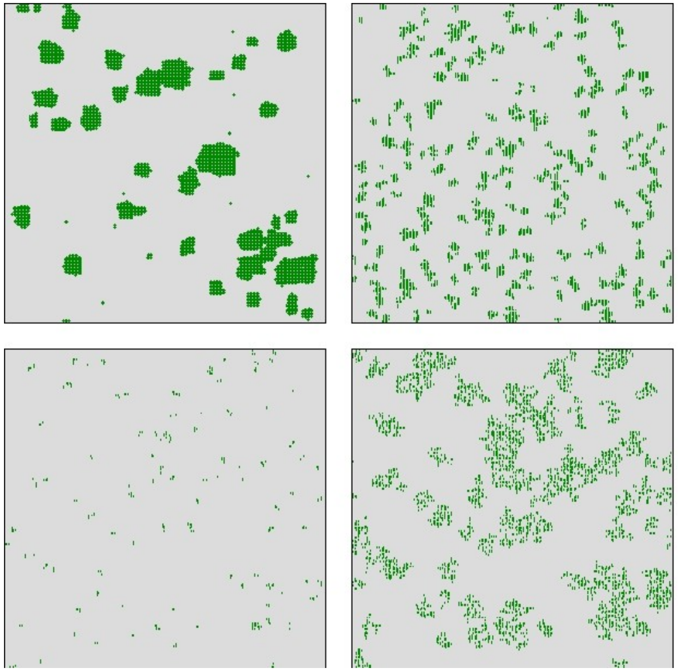
Schematic plot of the model dynamics



- b(t) fluctuates around the critical point of the pure model (b constant).
- System wanders between active and absorbing phases.

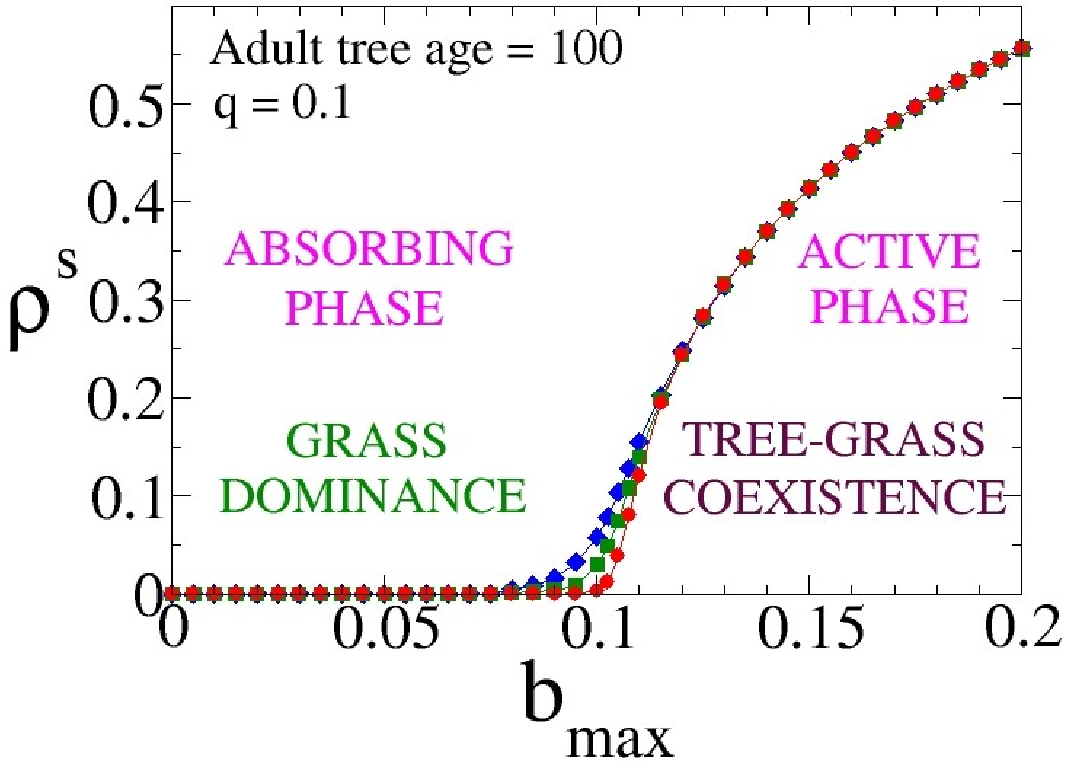
Effect of varying birth probability on time evolution



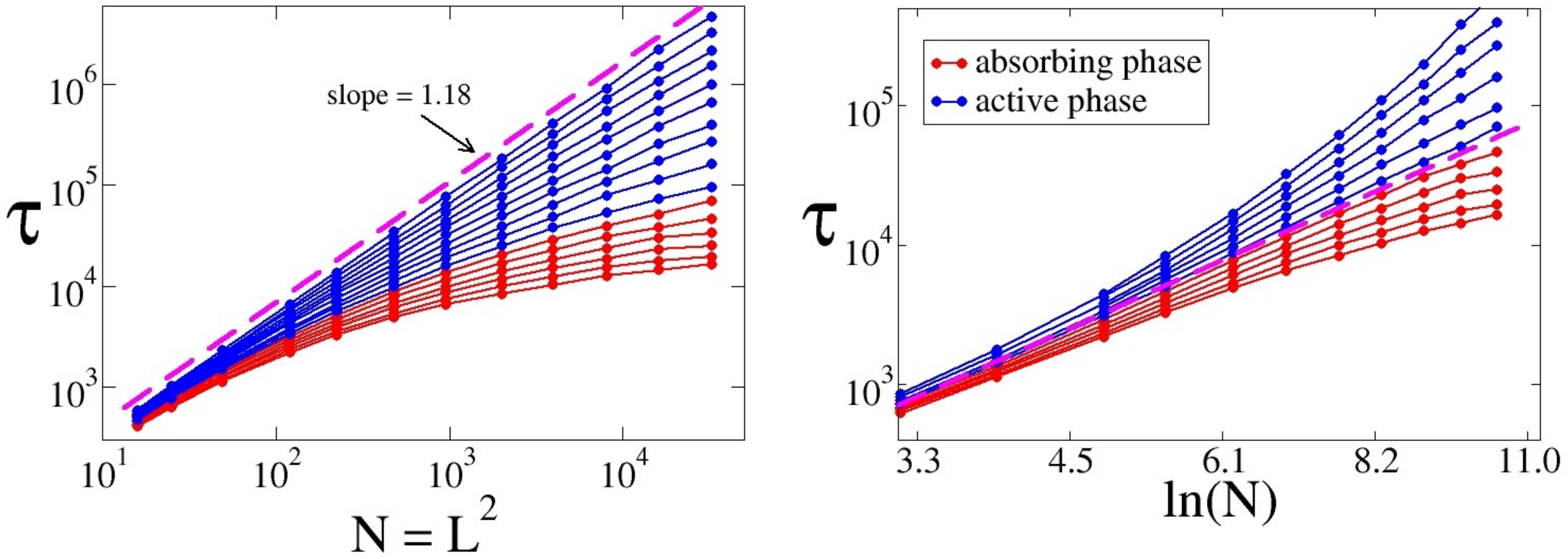


- Fixed birth rate → damped oscillations
- Varying birth rate → fluctuations (like in real savannas)
- Tree age enhances resilience

Mean density of trees in surviving runs



Savanna's mean life time: stability of the active phase



- Mean life time τ grows with system size N as $\tau \sim N^{\alpha}$, with $\alpha \sim b_{max}$.
- Active phase is stable for large N. Griffiths phases?
- Long period of small birth rates needed to reach absorbing state.

Summary/conclusions

- Savanna model with temporally varying birth rates (dry to humid weather conditions) allows dynamical coexistence of trees and grasses.
- Model exhibits large fluctuations in tree cover (grassland to woodland).
- The coexistence phase is stable in large N limit.
- Rarely long periods of droughts (bad weather conditions) lead the system to the absorbing phase (tree extinction).

- J. Calabrese, F. Vazquez, C. Lopez, M. San Miguel and V. Grimm
“The independent and interactive effects of tree-tree establishment competition and fire on the structure of savanna and dynamics. Accepted for publication in *The American Naturalist*.

- F. Vazquez, C. Lopez, J. Calabrese and M. A. Muñoz
“ Temporal Griffiths phases: a simple solution to the savanna problem”. Preprint.